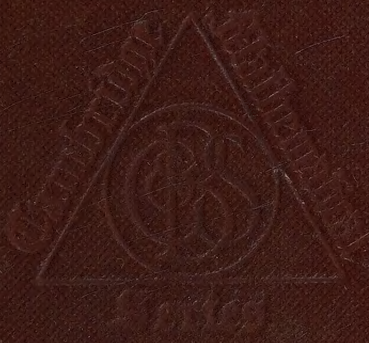

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COMMERCIAL ARITHMETIC

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PREFACE.

This book is based upon Mr. Pendlebury's *Arithmetic for Schools*, and upon the scheme suggested by the Education Department for students in Evening Continuation Classes. It also meets the requirements of pupils on the modern, or mercantile, side of secondary schools, who may be preparing for a business career.

It is assumed that students have already had a good grounding in the simple and compound rules, the reduction of weights and measures, elementary fractions, and the unitary method of solving problems; but methods which shorten work in the simple and compound rules are fully dealt with in this book.

Decimals are considered immediately after the simple rules, and are treated as an extension of the ordinary mode of representing whole numbers. Methods of approximation are explained, and are introduced in connection with practice, interest, and discount. Short methods for the mental calculation of prices are also given.

The sections dealing with such important branches of Commercial Arithmetic as Percentages and their applications, Bills of Exchange, Foreign Exchange, and Stocks and Shares, are as complete as the necessary limits of space allow.

Apprentices in building trades, attending classes held by the various authorities for Technical Instruction, will find the sections on Areas and Quantities treated more fully than is usual in books on Arithmetic.

The questions for written work are carefully graduated, and are such as are met with in actual business. Throughout the book, examples have been taken from papers set at Oxford and Cambridge Local Examinations, and by the Education Department, the London Chamber of Commerce, the Society of Arts, the College of Preceptors, the Institute of Chartered Accountants, and other examining bodies.

September, 1898.

In the Second Edition a chapter on Mixtures was inserted. In the Third Edition parts of the sections on Multiplication, Decimals, and Approximations have been altered.

C. P

February, 1904.

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20 hundredweight	..... 1 ton.

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12 inches	make 1 foot (ft.)
3 feet	..... 1 yard (yd.)
5½ yards	..... 1 rod, or pole (po.)
40 poles	..... 1 furlong (fur.)
8 furlongs, or 1760 yards	..... 1 mile (m.)
Also 100 links, or 22 yards	..... 1 chain. (ch.)

Square Measure.

144 sq. inches	make 1 sq. foot (sq. ft.)
9 sq. feet	..... 1 sq. yard (sq. yd.)
30¼ sq. yards	..... 1 sq. rod, sq. pole, or perch (per.)
40 perches	..... 1 rood (ro.)
4 roods	..... 1 acre (ac.)
640 acres	..... 1 sq. mile (sq. m.)
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1728 cubic inches	make 1 cubic foot (cub. ft.)
27 cubic feet	..... 1 cubic yard (cub. yd.)

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4 gills	make 1 pint (pt.)
2 pints	..... 1 quart (qt.)
4 quarts	..... 1 gallon (gall.)
36 gallons	..... 1 barrel.

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4 gills	make 1 pint (pt.)
2 pints	..... 1 quart (qt.)
4 quarts	..... 1 gallon (gall.)
2 gallons	..... 1 peck (pk.)
4 pecks	..... 1 bushel (bush.)
8 bushels	..... 1 quarter (qr.)

Measure of Number.

12 units	make 1 dozen
20 units	..... 1 score
12 dozen	..... 1 gross
12 gross	..... 1 gt. gross
Also 24 sheets of paper	..... 1 quire
20 quires	..... 1 ream
10 reams	..... 1 bale

Measure of Time.

60 seconds	make 1 minute (min.)
60 minutes	..... 1 hour (hr.)
24 hours	..... 1 day
7 days	..... 1 week
28, 29, 30, or 31 days	..... 1 calendar month
12 calendar months	..... 1 year
365 days	..... 1 common year
366 days	..... 1 leap year
100 years	..... 1 century.

TABLE OF METRIC WEIGHTS AND MEASURES.

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10 millimetres	make 1	centimetre (cm.)
10 centimetres	... 1	decimetre (dm.)
10 decimetres	... 1	METRE (m.)
10 metres	... 1	decametre (Dm.)
10 decametres	... 1	hectometre (Hm.)
10 hectometres	... 1	kilometre (Km.)
10 kilometres	... 1	myriametre (Mm.)

Square Measure.

100 sq. millimetres	make 1	sq. centimetre
100 sq. centimetres	... 1	sq. decimetre
100 sq. decimetres	1	sq. metre or 1 centiare
10 centiares	... 1	deciare
10 deciares or 100 sq. metres	... 1	ARE or 1 sq. decametre
10 ares	... 1	decare
10 decares or 100 sq. decametres	... 1	hectare or 1 sq. hectometre
100 sq. hectometres	1	sq. kilometre

Cubic Measure.

1000 cu. millimetres	make 1	cu. centimetre
1000 cu. centimetres	... 1	cu. decimetre
1000 cu. decimetres or 10 decisteres	... 1	STERE or 1 cu. metre.
10 steres	... 1	decastere

Measure of Capacity.

10 millilitres	make 1	centilitre (cl.)
10 centilitres	... 1	decilitre (dl.)
10 decilitres or 1000 cu. centimetres	... 1	LITRE (l.)
10 litres	... 1	decalitre (Dl.)
10 decalitres	... 1	hectolitre (Hl.)

Measure of Weight.

10 milligrammes	make 1	centigramme (cg.)
10 centigrammes	... 1	decigramme (dg.)
10 decigrammes	... 1	GRAMME (g.)
10 grammes	... 1	decagramme (Dg.)
10 decagrammes	... 1	hectogramme (Hg.)
10 hectogrammes	... 1	kilogramme (Kg.)
10 kilogrammes	1	myriagramme (Mg.)
10 myriagrammes	... 1	quintal (Q.)
10 quintals	1	tonne (T.)

GOLD STANDARD COUNTRIES.

	Standard Coin.	Grains of Fine Gold.	Exchange Value.	Sub-divisions.
United Kingdom . .	Sovereign	113·0016	—	—
United States . . .	Dollar (gold) .	23·2199	49·29*d.*	100 cents.
German Empire . .	Mark . .	5·5313	11·745*d.*	100 pfennige.
Austria-Hungary (*a*).	Florin . .	9·4099	19·985*d.*	100 kreutzer.
Holland and Dutch Colonies	Florin or Guilder .	9·3335	19·84*d.*	100 cents.
Norway, Sweden, and Denmark	Krone .	6·2227	13·2*d.*	100 öre.
Turkey	Pound .	102·0799	216·8*d.*	100 piastres.
Egypt	Pound .	114·7781	243·773*d.*	100 piastres.
France	Franc . .	4·4803	9·51*d.*	100 centimes.
Italy	Lira . .	4·4803	9·51*d.*	100 centesimi.
Belgium	Franc . .	4·4803	9·51*d.*	100 centimes.
Spain	Peseta .	4·4803	9·51*d.*	100 centimos.
Switzerland	Franc . .	4·4803	9·51*d.*	100 centimes.
Greece	Drachma	4·4803	9·51*d.*	100 lepta.
Bulgaria	Leva . .	4·4803	9·51*d.*	100 stotinkis.
Roumania	Ley . .	4·4803	9·51*d.*	100 banis.
Servia	Dinar . .	4·4803	9·51*d.*	100 paras.
Portugal (*a*) . . .	Milreis .	25·0885	53·278*d.*	1000 reis.
Brazil (*a*)	Milreis .	12·682	26·935*d.*	1000 reis.
Russia (*a*)	Rouble . (old) .	17·92	38·060*d.*	100 copeks.
Argentina (*a*) . . .	Peso . .	22·402	47·575*d.*	100 centesimi
Uruguay (*a*) . . .	Peso . .	24·015	51·004*d.*	100 centesimi.

(*a*) Currency at present consists of inconvertible paper.

SILVER STANDARD COUNTRIES.

	Standard Coin.	Grains of Fine Silver.	Exchange Value.	Sub-divisions.
India . . .	Rupee . . .	165·0	(Except in the case of India, the exchange value in gold depends upon the market price of silver.)	16 annas.
Russia . .	Rouble . .	277·722		100 copeks.
Mexico . .	Dollar . . .	377·168		100 cents.
China (Shanghai)	Tael (weight)	516·105		1000 cash.
Japan . . .	Yen . . .	374·400		100 sen.
Persia . .	Kran . .	(varies)		20 shahis.
Siam . . .	Tical . . .	264·301		32 phainungs.
	[Trade Dollar	378·0		100 cents.]

COMMERCIAL ARITHMETIC.

PART I.

SIMPLE ADDITION.

1. In the addition of columns of figures we begin at the right-hand column by adding up the units. The words employed, either orally or mentally, should be as few as possible.

Example. Add together 739, 62604, 85, 91876, *and* 8753.

We arrange the numbers as in the margin.

The mental steps are as follows:—

(i) 9, 14, 18, 27; write down **7** and carry 2.
(ii) 7, 14, 22, 25; write down **5** and carry 2.
(iii) 9, 17, 23, 30; write down **0** and carry 3.
(iv) 11, 12, 14; write down **4** and carry 1.
(v) 10, 16; write down **16.**

739
62604
85
91876
8753
164057

It is usual to begin at the bottom of each column and add upwards.

2. The correctness of the result may be tested by adding again, beginning at the top of each column.

The result of addition is called the **sum** or **amount** of the given numbers.

3. The mark + placed between numbers denotes that they are to be added together, and is called the **sign of addition**, or **plus.**

4. The calculations of totals such as are given on the next page are very often required in commercial work.

EXERCISE I.

Add together (i) the vertical columns; (ii) the horizontal rows. Verify the gross total, which will be equal to the sum of the totals of the vertical columns or that of the horizontal rows.

	A	B	C	D	E	F	G	H	I	J	Total.
(1)	38756	19077	394	8520	17955	16391	18365	5417	18764	375	
(2)	43958	6771	1643	2469	17282	897	1886	43986	19467	2489	
(3)	10195	205	1997	19843	7639	95483	13489	6105	3847	31725	
(4)	80631	19482	67935	12367	17632	11835	16596	36569	963	43897	
(5)	58392	15587	210	14678	18215	7624	19998	26573	1234	32916	
(6)	9505	7495	770	9576	13688	69840	9876	19845	56789	7238	
(7)	19425	7079	2657	159	9228	32752	9987	13789	12578	987	
(8)	24293	6304	4689	98756	6590	34728	17963	13897	6431	43172	
(9)	12120	5725	15698	1314	3205	6679	16821	14798	23478	9682	
(10)	2897	673	25793	2074	1096	34728	15945	16793	9678	396	
(11)	36095	488	3784	7400	819	39642	33369	17693	1678	18927	
(12)	3891	3285	892	329	2356	32752	36734	13697	94321	4859	
(13)	9687	12424	1405	8963	7485	1983	1324	13796	178	63287	
(14)	16075	1488	457	9572	18919	74321	1432	16954	16789	4969	
(15)	6537	4525	1689	16228	22279	47983	1243	17494	9763	18765	
(16)	13276	23473	2657	21555	2076	17218	98763	18763	17286	9876	
(17)	7790	615	2502	10075	1899	23109	1369	4689	18346	329	
(18)	4073	17449	3535	18090	20639	39642	25963	5943	456	38654	
Total											Gross Total.

SIMPLE SUBTRACTION.

5. Instead of the common process in subtraction, the student will often find it convenient to determine how much must be *added* to the subtrahend to make up the minuend.

Example. Subtract 27568 *from* 32102.

We are to find what number added to 27568 will make 32102. The mental steps are as follows:—

(i) 8 and **4** make 12; write down **4**;

(ii) 1 to 6 make 7, and **3** make 10; write down **3**;

(iii) 1 to 5 make 6, and **5** make 11; write down **5**;

(iv) 1 to 7 make 8, and **4** make 12; write down **4**;

(v) 1 to 2 make 3.

32102	Minuend
27568	Subtrahend
4534	Difference

Here the operation stops, and we find the difference to be 4534.

6. The correctness of the result may be tested by adding the difference to the subtrahend; the result should be the minuend.

7. The mark − (called **minus**) placed between two numbers denotes that the second number is to be taken from the first.

8. The process of Art. 5 is especially useful when it is required to subtract from a number the sum of several numbers.

Example. In one operation take the sum of 19782, 2346 *and* 6257 *from* 61384.

We add each column of the subtrahends and subtract each sum from the corresponding top part of the line. Thus:—

(i) 11; 18 and **6** make 24; write down **6** and carry 2.

(ii) 10; 16; 21 and **7** make 28; write down **7** and carry 2.

(iii) 9; 13; 15 and **8** make 23; write down **8** and carry 2.

(iv) 11; 14; 20 and **1** make 21; write down **1** and carry 2.

(v) 3; 5 and **1** make 6; write down **1**.

Hence the result is 11876.

61384
6257
23469
19782
11876

EXERCISE II.

Perform *mentally* the operation indicated below :—

(1)	4176832 – 3747898.	(2)	3741698 – 2097899.
(3)	5681423 – 5673829.	(4)	9876123 – 4388678.
(5)	3107369 – 2198070.	(6)	4169025 – 2199896.
(7)	7684192 – 6998437.	(8)	4712386 – 3298049.
(9)	6993481 – 3499763.	(10)	7416931 – 6189708.
(11)	9841672 – 3982895.	(12)	3281074 – 3197685.

In one operation take the sum of

(13) 7632, 3675, 3297, 6324, 6369, 7624, 3746 and 8367 from 56123.

(14) 1358, 7621, 6845, 4194, 1890, 1889, 9631 and 4729 from 66538.

(15) 9875, 5432, 1234, 9783, 4713, 7248, 3697 and 8846 from 73615.

(16) 4261, 3150, 9716, 6427, 1234, 5678, 9357 and 9184 from 81023.

(17) 1897, 1364, 8887, 7776, 6841, 9372, 5284 and 4819 from 95129.

(18) 9817, 6314, 7848, 6794, 2476, 8756, 7539 and 4837 from 62130.

(19) 3210, 7893, 9123, 4573, 3698, 4397, 5793 and 7931 from 85206.

(20) 6324, 3746, 4398, 7435, 7479, 8735, 4857 and 9478 from 91341.

SIMPLE MULTIPLICATION.

9. If two or more numbers be multiplied together the result is called the **product**, and the numbers multiplied are called **factors** of the product.

Thus 6, being the result of multiplying 2 by 3, is called the *product* of 2 and 3 ; and 2 and 3 are called the *factors* of 6.

Multiplication is denoted by an *inclined* cross $\times$, called **the sign of multiplication.**

10. The following example will explain the usual method of the multiplication of numbers greater than 12.

Example. Multiply 5763 *by* 154.

This means that we take the first number 154 times over. But $154=100+50+4$. If therefore we multiply 5763 by 4, then by 50, and then by 100, and add the results together, we shall get the product required. We arrange the work as in the margin.

```
  5763
   154
-------
 23052 =   4 times
288150 =  50 times
576300 = 100 times
-------
887502 = 154 times.
```

It is customary to omit the final zeros of the 2nd and 3rd lines, placing each product *with its last digit under that multiplying digit which produces it.* Hence we see the meaning of the common rule.

11. Products are usually obtained as in Art. 10, but it would be better for some purposes if we were to multiply *first* by 100, then by 50, and *last* of all by 4. We should thus get the different parts of the result in the order of their greatness and importance.

The operation would then appear as in the margin; the successive products being placed with their right-hand digits in a column.

```
  5763  Multiplicand
   154  Multiplier
-------
 57630
288150
 23052
-------
887502  Product
```

NOTE.—In the multiplication of decimals it will often be found convenient to adopt the method of Art. 11.

12. If the multiplier *ends in* 0, or a series of 0's, these 0's may be ignored in forming the successive products; but we must then place the same number of 0's at the end of the final result.

Example. Multiply 7894 by 70; *and by* 700.

```
  7894          7894
    70           700
-------       -------
552580        5525800
```

In the first case we simply multiply by 7 and add a 0 to the product; and in the second case we add two 0's.

13. If one or more of the digits of the multiplier be 0, these 0's may be ignored in forming the successive products; but the pupil must be particularly careful to place each product so that its last digit comes under the multiplying figure.

Example. *Multiply* 2154321 *by* 300203.

```
   2154321
    300203
  ---------
6462963      = 300,000 times 2154321
   4308642   =     200.....
      6462963=       3.....
-----------
646733627163
```

In order to become quick at mental arithmetic the pupil should make for himself, and learn, the extended multiplication table as far as 20 times 20.

EXERCISE III.

Perform the operations indicated below :—

(1)	3617305×9800.	(2)	4720670×39000.
(3)	5630750×52700.	(4)	2154321×9068.
(5)	3174925×7008.	(6)	4703006×5207.
(7)	7078050×8070.	(8)	6300760×5060.
(9)	7008540×9040.	(10)	5080600×70090.
(11)	7380060×50904.	(12)	8000070×60800.

SHORT METHODS IN MULTIPLICATION.

14. For some multipliers special methods may be adopted.

(i) *To multiply by* 9.

We know that $9 = 10 - 1$. Hence, if we multiply the number by 10 we take it *once* too often; if therefore we subtract the number itself from this product, we clearly get the same as if we had multiplied the number at once by 9.

But the product of a number by 10 is obtained by merely placing a 0 at the end of the number. Hence multiplication

by 9 is reduced to simple subtraction of one number from another.

For example, 10 times 6729403 = 67294030,
and once ..... = 6729403,
∴* by subtraction, 9 times ..... = 60564627.

(ii) *To multiply by* 99, 999, 9999, 99999, *etc.*
These may all be treated in the same way as 9; for

$$99 = 100 - 1,$$
$$999 = 1000 - 1,$$
$$9999 = 10000 - 1,$$
$$99999 = 100000 - 1, \text{ etc.}$$

Hence we place at the end of the number as many 0's as there are 9's in the multiplier, and from the result we subtract the number itself.

(iii) *To multiply by a number differing but little from* 10, 100, 1000, 10000, *etc.*

Example. To multiply 763021 *by* 99996.

We know that $99996 = 100000 - 4$. Hence in this case we place five 0's at the end of the multiplicand, and from the result we subtract 4 *times* the multiplicand.

Thus 100000 times 763021 = 76302100000,
4 ..... = 3052084,
∴ 99996 ..... = 76299047916.

(iv) *To multiply by* 5.
We know that 10 is twice 5. Hence if we multiply by 10 the result is twice as much as it should be. We therefore divide this result by 2.

Thus 10 times 763021 = 7630210,
∴ 5 ..... 763021 = 3815105.

One advantage of this process is that we obtain the figures of the result in their proper order, beginning at the left hand.

* The sign ∴ is used to denote *therefore.*

(v) *To multiply by* 25.

We know that 100 is 4 times 25. Therefore, to multiply by 100, we place two 0's at the end of the multiplicand, and divide the result by 4.

Thus $$763021 \times 25 = \frac{76302100}{4}$$
$$= 19075525.$$

(vi) *To multiply by* 125.

In this case, for similar reasons, we place three 0's at the end of the multiplicand, and divide the result by 8.

Thus $$763021 \times 125 = \frac{763021000}{8}$$
$$= 95377625.$$

(vii) *To multiply by* 625.

In this case we place four 0's at the end of the multiplicand, and divide the result by 16.

Thus $$763021 \times 625 = \frac{7630210000}{16}$$
$$= 476888125.$$

(viii) *To multiply by any number from* 11 *to* 19 *in one line.*

The following process may be adopted :—

Multiply by the units' digit, and at each step add the digit on the right hand in the multiplicand, until the last digit on the left hand is reached; the last digit is then brought down and the digit last carried added to it.

Example. Multiply 68176 *by* 13.

The mental steps are as follows :—

(i) $3 \times 6 = 18$; write down **8** and carry 1.
(ii) $3 \times 7 = 21$; and 1 carried make 22; and 6 make 28; write down **8** and carry 2.
(iii) $3 \times 1 = 3$; and 2 carried make 5; and 7 make 12; write down **2** and carry 1.
(iv) $3 \times 8 = 24$; and 1 carried make 25; and 1 make 26; write down **6** and carry 2.
(v) $3 \times 6 = 18$; and 2 carried make 20; and 8 make 28; write down **8** and carry 2.
(vi) 2+6 (the last digit)=8; write down **8**;

$\therefore \quad 68176 \times 13 = 886288.$

EXERCISE IV.

Using the shortest method, multiply

(1)	768395 by 9.	(2)	274876 by 99.
(3)	823548 by 999.	(4)	657429 by 9999.
(5)	528754 by 97.	(6)	487651 by 994.
(7)	348916 by 9995.	(8)	475628 by 9980.
(9)	325739 by 9987.	(10)	732854 by 5.
(11)	584397 by 25.	(12)	854693 by 125.
(13)	369502 by 625.	(14)	769425 by 3125.

(15) 527864 by 250.

Multiply in *one* line

(16)	3726954 by 11.	(17)	4637845 by 12.
(18)	5938621 by 13.	(19)	6892147 by 14.
(20)	5789214 by 15.	(21)	6843297 by 16.
(22)	5743296 by 17.	(23)	6934578 by 18.

(24) 8912456 by 19.

15. The pupil should keep a careful watch for such a relation between the digits of the multiplier as is used in the following examples, where one digit (or set of digits) in the multiplier is contained in another set of digits.

Example i. *Multiply* 78932 *by* 12111.

We notice that $12111 = 12100 + 11$,
and that $12100 = 1100$ times 11.

We may arrange the work in *two* lines, thus:—

```
   78932
   12111
  ------
  868252 (i)
955077200 (ii)
---------
955945452
```

The line (i) is 11 times 78932; the line (ii) is 1100 times (i), and therefore 12100 times 78932.

Example ii. *Multiply* 78932612 *by* 5678109.

We notice that the multiplier = 9 + 8100 + 5670000. We may arrange the work in *three* lines, thus :—

```
                78932612
                5678109
              ----------
              710393508 (i)
           639354157200 (ii)
        447547910040000 (iii)
        ----------------
        448187974590708
```

The line (i) is 9 times the multiplicand; (ii) is 900 times (i) and therefore 8100 times the multiplicand; (iii) is 700 times (ii) and therefore 5670000 times the multiplicand. Consequently the sum of these three lines is the product required.

Example iii. *Multiply* 78932612 *by* 567981.

We notice that the multiplier = 900 + 81 + 567000. The work may be arranged in *three* lines, thus :—

```
                78932612
                567981
              ----------
              710393508  =    900 times the multiplicand
             6393541572  =     81 times .....
        44754791004      = 567000 times .....
        ----------------
        44832223896372 = 567981 times .....
```

Note.—At each step the right-hand figure of the product must be placed under the right-hand figure of the part which is the multiplier at that step.

EXERCISE V.

Multiply in *two* lines

(1)	37234 by 648.	(2)	62089 by 287.
(3)	12895 by 279.	(4)	73985 by 255.
(5)	84527 by 183.	(6)	39874 by 568.
(7)	58479 by 364.	(8)	36874 by 1089.
(9)	46789 by 13212.	(10)	562875 by 945.
(11)	253769 by 763.	(12)	202842 by 324.
(13)	632047 by 525.	(14)	369875 by 832.

(15) 527698 by 9108.

EXERCISE VI.

Multiply in *three* lines

(1)	361428 by 48126.	(2)	463189 by 64164.
(3)	236975 by 54273.	(4)	529875 by 72369.
(5)	693716 by 83296.	(6)	387695 by 63672.
(7)	141651 by 42856.	(8)	816849 by 27309.
(9)	729834 by 32864.	(10)	329754 by 36448.
(11)	698327 by 81936.	(12)	731865 by 25525.

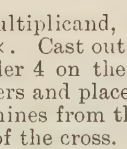
16. The correctness of a *product* may be tested by the process called **casting out the nines**, which is as follows:—

Divide the sum of the digits of one factor by 9, *and find the remainder; divide the sum of the digits of the other factor by* 9, *and find the remainder; multiply the two remainders together, and divide the result by* 9. *The remainder after this last division should be the same as the remainder when we divide the sum of the digits of the product by* 9.

If these remainders are different, the product is certainly wrong; if they are the same, we cannot say that the product is certainly right, but it is very likely to be so.

Example. Multiply 875 *by* 427, *and prove the result by "casting out the nines."*

```
  875
  427
-----
 3500
 1750
 6125
-----
373625
```



Cast out the nines from the multiplicand, and place the remainder 2 on the left hand of the ×. Cast out the nines from the multiplier, and place the remainder 4 on the right hand of the ×. Multiply these two remainders and place the result 8 at the top of the ×. Next cast out the nines from the product, and place the remainder 8 at the bottom of the cross. Since the two remainders agree we can conclude that the product is correct.

SQUARES.

17. The product obtained by the continued multiplication of a number by itself is termed a **power** of the number; and the number of times the number is multiplied is called the **index** of the power. Thus 10×10 is the *second power*, or the **square** of 10. It is indicated by 10^2, the *index* being 2.

18. The following principles may here be enunciated :—

i. *The square of the sum of two numbers is equal to the sum of their squares,* **plus** *twice their product.*

$$\begin{aligned}(35)^2 = (30+5)^2 &= (30)^2 + (5)^2 + 2 \times 30 \times 5 \\ &= 900 + 25 + 300 \\ &= 1225.\end{aligned}$$

ii. *The square of the difference of two numbers is equal to the sum of their squares,* **minus** *twice their product.*

$$\begin{aligned}(47)^2 = (50-3)^2 &= (50)^2 + (3)^2 - 2 \times 50 \times 3 \\ &= 2500 + 9 - 300 \\ &= 2209.\end{aligned}$$

iii. *The product of the sum and the difference of two numbers is equal to the difference of their squares.*

$$\begin{aligned}42 \times 38 = (40+2)\ (40-2) &= (40)^2 - (2)^2 \\ &= 1600 - 4 \\ &= 1596.\end{aligned}$$

These principles may be remembered in their algebraical form ;—

i. $(a+b)^2 = a^2 + b^2 + 2ab$;
ii. $(a-b)^2 = a^2 + b^2 - 2ab$;
iii. $(a+b)\ (a-b) = a^2 - b^2$.

EXERCISE VII.

Square *mentally* the following numbers :—

(1) 21.	(2) 41.	(3) 33.	(4) 53.	(5) 47.
(6) 56.	(7) 74.	(8) 89.	(9) 68.	(10) 85.

FACTORS AND PRIME NUMBERS.

19. Any number which divides another without a remainder is called a **measure** of that number.

Thus 8 is a measure of 48.

20. Any number which contains another an exact number of times is called a **multiple** of that number.

Thus 48 is a multiple of 8.

21. All numbers are multiples of unity, and each contains itself once. Any number can therefore be divided by itself and by unity, but if it cannot be divided without remainder by *any other* number it is said to be **prime**, and is called **a prime number.**

We may therefore define a prime number as *a number which has no factor or divisor but itself and unity.*

Numbers which are not prime are called **composite.**

It will be seen that 2, 3, 5, 7, 11, 13, 17, etc., are prime numbers, and that 4, 6, 8, 9, 10, 12, 14, 15, etc., are composite.

22. Such a number as 12 is the product of 6 and 2, also of 4 and 3, and also of 2, 2 and 3; but the last of these sets, namely 2, 2 and 3, has a property which the others have not, for 2, 2 and 3 are all *prime.*

No number can be separated into *prime* factors in more than one way. When therefore we speak of separating a number into factors, we shall generally mean *prime* factors.

Thus $12 = 2 \times 2 \times 3 = 2^2 \times 3$;

and, similarly, $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$.

23. The prime factors of a number can be found by trying the prime numbers in succession, beginning with 2. When we have found one factor, we divide the number by it, and then treat the quotient in a similar way; and we continue this process until we arrive at a divisor which gives a quotient less than itself.

For example, in the case of the number 150, we try divisors 2, 3, 5, 7, 11 in succession; but when we try 13, we get a quotient 11 and a remainder 7. The quotient 11 being less than the divisor 13, we need try no further. All prime factors of 150 lie therefore between 2 and 11 inclusive.

What numbers should be tried will readily be seen after a little practice, but it will be well to remember the following **rules of divisibility**, the proofs of which will be useful exercises for the pupil.

(i) **2** is a factor of all numbers whose last digit can be divided by 2.

(ii) **4** is a factor if the number composed of the last 2 digits can be divided by 4.

(iii) **8** is a factor if the number composed of the last 3 digits can be divided by 8.

(iv) **5** is a factor if the last digit be either 0 or 5.

(v) **10** is a factor if the last digit is 0.

(vi) **3** is a factor if the sum of the digits can be divided by 3.

(vii) **9** is a factor if the sum of the digits can be divided by 9.

(viii) **6** is a factor if both 2 and 3 are factors.

(ix) **12** is a factor if both 3 and 4 are factors.

(x) **11** is a factor if, when the 1st, 3rd, 5th, etc., digits are added together, and also the 2nd, 4th, 6th, etc., the difference between these sums is 0, or can be divided by 11.

(xi) Divisibility by **7** can be determined only by actual division.

The prime numbers up to 100 are 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

24. We will now apply these rules.

Example. Find the prime factors of 133056.

(i) 2 is a factor because the last digit 6 can be divided by 2; hence $133056 = 2 \times 66528$.

(ii) 3 is a factor of 66528, because it divides the sum of the digits, viz. 27; thus $133056 = 2 \times 3 \times 22176$.

(iii) 11 is a factor of 22176, because the sum of the 1st, 3rd, and 5th digits is 9, and the sum of the 2nd and 4th digits is also 9; the difference of these sums being 0, it follows that 22176 is divisible by 11. Hence

$$133056 = 2 \times 3 \times 11 \times 2016.$$

(iv) 9 is a factor of 2016, because the sum of the digits, viz. 9, is divisible by 9. Thus we have

$$133056 = 2 \times 3 \times 11 \times 9 \times 224.$$

(v) 4 is a factor of 224, because the number formed by the last two digits, viz. 24, can be divided by 4. Thus we have

$$133056 = 2\times3\times11\times9\times4\times56$$
$$= 2\times3\times11\times9\times4\times8\times7 = 2^6\times3^3\times7\times11.$$

The work shown above illustrates the application of the rules, but the process of separating a number into factors may be stated briefly thus:—

8) 133056
8) 16632
9) 2079
3) 231
7) 77
11

$\therefore\ 133056 = 8\times8\times9\times3\times7\times\mathbf{11}$
$= 2^6\times3^3\times7\times11.$

EXERCISE VIII.

Separate each of the following numbers into its prime factors:—

(1)	60.	(2)	78.	(3)	102.	(4)	210.
(5)	252.	(6)	315.	(7)	525.	(8)	612.
(9)	715.	(10)	854.	(11)	1331.	(12)	1456.
(13)	3675.	(14)	4620.	(15)	5250.	(16)	11025.

SIMPLE DIVISION.

25. The process of finding how often one number is contained in another is called **division.**

26. The number to be divided is called the **dividend.**

27. The number by which it is divided is called the **divisor.**

28. The result of division is called the **quotient.**

29. The mark $\div$ is the **sign of division.**

30. If the divisor be such a number as is easily factorized, we divide successively by the factors.

Example i. *Divide* 3726589 *by* 35, *and find the remainder.*

We see that $35 = 5\times7$. We first divide 3726589 by 5; the quotient is 745317, and the remainder is 4 units. We now divide 745317 by 7; the quotient is 106473, and the remainder is 6 *sets of* 5.

Hence the complete remainder after division by 35

$= 6$ *sets of* 5 and 4 *units*
$= 6\times5$ *units* $+ 4$ *units*
$= 34$ *units.*

Hence, to find the complete remainder we *multiply the second remainder by the first divisor, and add the first remainder.*

The work is stated as below:—

$$35\left\{\begin{array}{l}5\ )\ 3726589\\ 7\ )\ 745317\ldots4\\ \hline 106473\ldots6\end{array}\right\}34$$

Example ii. *Divide* 3762905 *by* 105, *by short division, and find the remainder.*

$105=3\times5\times7$. We therefore divide successively by the factors 3, 5, 7.

$$105\left\{\begin{array}{l}15\left\{\begin{array}{l}3\ )\ 3762905\\ 5\ )\ 1254301\ldots2\\ 7\ )\ 250860\ldots1\\ \hline 35837\ldots\ldots1\end{array}\right\}\begin{array}{l}5\\ \\ \\ \\ \end{array}\\ \\ \\ \\ \end{array}\right\}20$$

After division by 3 and 5 the complete remainder is 5. To find the complete remainder after the next division, we consider 15 as our first divisor and 5 as our first remainder.

We may proceed in this way, whatever be the number of successive divisors.

31. The following short methods can be used for certain divisors.

(i) *To divide by* 10, 100, 1000, 10000, *etc.*

Strike out as many figures from the right of the dividend as there are 0's at the right of the divisor; these figures form the remainder.

(ii) *To divide by* 5, 25, 125.

We may multiply the number by 2, 4, 8, and divide the product by 10, 100, 1000, respectively.

EXERCISE IX.

Using the shortest method, divide

(1)	64328946 by 15.	(2)	74638201 by 36.
(3)	63942527 by 63.	(4)	87654321 by 378.
(5)	16752600 by 256.	(6)	18374121 by 105.
(7)	25716953 by 162.	(8)	67936497 by 648.
(9)	48695687 by 792.	(10)	61738972 by 5.
(11)	27493862 by 50.	(12)	37893178 by 500.
(13)	73246837 by 25.	(14)	76851738 by 250.
(15)	68931634 by 2500.	(16)	61738978 by 125.
(17)	15433697 by 625.	(18)	18374121 by 3125.

CONTRACTED METHOD OF DIVISION.

(The Italian Method.)

32. When the pupil has had sufficient practice in long division, he may shorten the process by omitting the various products, and writing down only the successive differences.

Example. Divide 13877136 *by* 3276.

Adopting this principle, the work would appear thus :—

```
3276 ) 13877136 ( 4236
        7731
        11793
        19656
```

(**a**) The first difference, 773, may be obtained thus :—
Multiplying 3276 by 4 we get

(i) $4 \times 6 = 24$. We cannot take 24 from 7, so we increase 7 to 27; we then subtract 24 from 27, and write down the remainder 3.

(ii) $4 \times 7 = 28$; to this we add 2, in order to make up for having increased the 7 to 27 in (i); thus 28 becomes 30. As before, we cannot subtract 30 from the digit 7 above it, so we increase 7 to 37; and subtracting 30 from 37, we get a remainder 7, which we write down.

(iii) $4 \times 2 = 8$; to this we add 3 to make up for having increased the 7 to 27 in (ii); thus 8 becomes 11. As before, we increase the digit 8 above it to 18, and subtracting 11 from 18 we write down the remainder 7.

(iv) $4 \times 3 = 12$; this becomes 13 when increased by 1; subtracting 13 from the 13 in the dividend, we get no remainder.

Consequently the whole difference is 773. To this we affix the next digit 1 of the dividend, and proceed as before.

(**b**) To obtain the first difference by the process of Art. 5, the mental steps would be as follows :—

(i) $4 \times 6 = 24$; and 3 make 27; write down **3** and carry 2.
(ii) $4 \times 7 = 28$; and 2 carried make 30; and **7** make 37; write down **7** and carry 3.
(iii) $4 \times 2 = 8$; and 3 carried make 11; and **7** make 18; write down **7** and carry 1.
(iv) $4 \times 3 = 12$; and 1 carried make 13.

The difference, as before, is 773.

EXERCISE X.

Perform the operations indicated below:—

(1)	$568089968 \div 113$.	(2)	$478921614 \div 258$.
(3)	$562809390 \div 286$.	(4)	$234566950 \div 362$.
(5)	$218492568 \div 504$.	(6)	$634289535 \div 891$.
(7)	$462184590 \div 745$.	(8)	$234561519 \div 789$.
(9)	$321055835 \div 865$.	(10)	$476203005 \div 923$.
(11)	$347478176 \div 1032$.	(12)	$856286832 \div 1156$.
(13)	$218477856 \div 1184$.	(14)	$561084230 \div 1231$.
(15)	$849628956 \div 1334$.	(16)	$632450301 \div 1562$.
(17)	$342980706 \div 2841$.	(18)	$732146548 \div 3284$.
(19)	$342890804 \div 3321$.	(20)	$103470632 \div 4283$.

33. The correctness of a *quotient* may be ascertained by multiplying the quotient by the divisor, and adding the remainder (if any) to the product. The result so obtained will be the same as the *dividend*, if the quotient be correct.

34. The relation between divisor, dividend, and quotient in *exact* division may be written in either of the two forms

$$\text{Dividend} \div \text{Divisor} = \text{Quotient},$$

and $$\text{Quotient} \times \text{Divisor} = \text{Dividend}.$$

In *inexact* divison

$$\text{Quotient} \times \text{Divisor} + \text{Remainder} = \text{Dividend}.$$

DECIMALS.

35. The characteristic of the Arabic, or common, system of notation is that each digit has a **local** value depending upon its position relative to the others, as well as an **intrinsic** value depending only upon itself. In the number 5555, for example, all the digits have the same *intrinsic* value 5, but the first represents 5 *thousands*, the second 5 *hundreds*, the third 5 *tens*, and the fourth 5 *units*.

The local value of a digit increases therefore ten times for each place as we move from right to left, and diminishes to one tenth for each place as we go from left to right. The last digit always represents *units*, and the notation is sufficient so long as we are concerned with whole units only. When however we consider *parts* of a unit, the system does

not go far enough, but we shall find that we may represent any fraction whatever by placing digits to the *right* of the units' place, and extending to them the same *Principle of Local Value.*

For example, a digit 5 placed to the right of the units' place may be considered to represent $\frac{5}{10}$ of a unit, and a digit 5 placed to the right of this to represent $\frac{5}{100}$ of the unit, and so on.

The position of a digit determines also what is sometimes spoken of as its **order.**

Thus in 62758, the digit 6 is said to be of the order 4, because it denotes 6 times 10^4; similarly, 2 is of the order 3, 7 of the order 2, 5 of the order 1, and 8 of the order 0.

36. In the case of a whole number, since the last digit always denotes units, we can at once from their position tell what is denoted by each of the other digits; but when the principle is extended to the right of the units' place we must distinguish the digits in some special manner. We do this by placing *a point* immediately after the units' place.

If, for example, in 62758 the units' digit is 2, we write the number thus—62·758; and we then see at once that it represents

$$60+2+\tfrac{7}{10}+\tfrac{5}{100}+\tfrac{8}{1000}.$$

In 62·758 the digit 7 is said to be of the order -1, because it denotes 7 times $\frac{1}{10}$ or 10^{-1}; 5 is of the order -2, 8 of the order -3, and so on.

37. If there are no whole units in the number, we may place a zero before the point, or we may begin with the point itself.

For example, we write such a number in the form 0·768, or in the form ·768.

38. An expression of the form ·768 is called a **Decimal,** and the point is called a **Decimal Point.**

The digits after the point represent the numerators of vulgar fractions whose denominators are 10, 100, 1000, etc. respectively, and the expression itself represents the sum of these fractions.

We read a decimal by naming the digits in order. Thus, ·15 is "*decimal, one, five,*" not "*decimal, fifteen.*"

39. An expression such as 24·768, part of which represents a whole number, and part a decimal, is called a *mixed number.*

40. A decimal in which a digit, or a set of digits, is repeated continually is called a **Recurring**, or a **Periodic**, or a **Circulating** decimal; and the repeated digit, or set of digits, is called the **Period** of the decimal.

In the decimal equivalent to $\frac{8}{3}$ the period is 6; in $\frac{1}{7}$ it is 142857; in $\frac{21}{22}$ it is 54.

41. Recurring decimals are abbreviated thus:—

$\frac{8}{3}$ is written $2\cdot\dot{6}$,

$\frac{1}{7}$ ..... $\cdot\dot{1}42857\dot{}$,

and $\frac{21}{22}$ ..... $\cdot9\dot{5}\dot{4}$;

the period being marked by dots over its first and last digits.

42. Such a decimal as $\cdot\dot{1}42857\dot{}$, in which all the digits recur, is called a **pure** recurring decimal; and one such as $\cdot9\dot{5}\dot{4}$, in which one or more digits do not recur, is called a **mixed** recurring decimal.

TO MULTIPLY A DECIMAL BY 10, 100, 1000, etc.

43. Since $\cdot768 = \frac{7}{10} + \frac{6}{100} + \frac{8}{1000}$;

$\therefore 10 \times \cdot768 = 7 + \frac{6}{10} + \frac{8}{100} = 7\cdot68$,

and $100 \times \cdot768 = 70 + 6 + \frac{8}{10} = 76\cdot8$.

Again, $\cdot0638 = \frac{0}{10} + \frac{6}{100} + \frac{3}{1000} + \frac{8}{10000}$;

$\therefore 10 \times \cdot0638 = \frac{6}{10} + \frac{3}{100} + \frac{8}{1000} = \cdot638$,

and $100 \times \cdot0638 = 6 + \frac{3}{10} + \frac{8}{100} = 6\cdot38$.

Hence, *to multiply a decimal by* 10, 100, 1000,....., *we move the decimal point* 1, 2, 3,..... *places respectively to the right.*

EXERCISE XI.

Multiply each of the following decimals successively by 10, 100, and 1000.

(1)	4·6.	(2)	7·7.	(3)	·5.	(4)	·8.
(5)	4·19.	(6)	15·79.	(7)	·87.	(8)	·15.
(9)	1·05.	(10)	·04.	(11)	·09.	(12)	5·345.
(13)	·628.	(14)	·015.	(15)	·005.	(16)	·5283.
(17)	2·006.	(18)	·1579.	(19)	14·003.	(20)	·0528.
(21)	·0005.	(22)	·005781.	(23)	·000057.	(24)	·000001.

TO DIVIDE A DECIMAL BY 10, 100, 1000, etc.

44. Since $54{\cdot}321 = 50 + 4 + \frac{3}{10} + \frac{2}{100} + \frac{1}{1000}$,

$\therefore\ 54{\cdot}321 \div 10 = 5 + \frac{4}{10} + \frac{3}{100} + \frac{2}{1000} + \frac{1}{10000} = 5{\cdot}4321$,

and $54{\cdot}321 \div 100 = \frac{5}{10} + \frac{4}{100} + \frac{3}{1000} + \frac{2}{10000} + \frac{1}{100000}$

$= {\cdot}54321$.

Again, $\cdot543 = \frac{5}{10} + \frac{4}{100} + \frac{3}{1000}$,

$\therefore\ {\cdot}543 \div 10 = \frac{5}{100} + \frac{4}{1000} + \frac{3}{10000}$

$= \frac{0}{10} + \frac{5}{100} + \frac{4}{1000} + \frac{3}{10000}$

$= {\cdot}0543$,

and $\cdot543 \div 100 = \frac{0}{10} + \frac{0}{100} + \frac{5}{1000} + \frac{4}{10000} + \frac{3}{100000}$

$= {\cdot}00543$.

Hence, *to divide a decimal by* 10, 100, 1000,....., *we move the decimal point* 1, 2, 3,..... *places respectively to the left; and, if in any case there be no digits to the left of the point, we imagine* 0's *to be there.*

EXERCISE XII.

Divide each of the following by 10, 100, 1000 successively.

(1)	104·6.	(2)	167·7.	(3)	157·9.
(4)	361·1.	(5)	15·79.	(6)	14·0003.
(7)	37·0007.	(8)	28·03205.	(9)	17·05706.
(10)	43·76201.	(11)	30·0012.	(12)	83·00156.
(13)	4·19.	(14)	1·05.	(15)	5·345.
(16)	2·006.	(17)	1·1003.	(18)	5·372.
(19)	·5.	(20)	·87.	(21)	·628.
(22)	·04.	(23)	·015.	(24)	·005.

45. The following property of decimals must be noticed:—

$\cdot760 = \frac{7}{10} + \frac{6}{100} + \frac{0}{1000} = \frac{7}{10} + \frac{6}{100} = {\cdot}76$,

and $\cdot7600 = \frac{7}{10} + \frac{6}{100} + \frac{0}{1000} + \frac{0}{10000} = \frac{7}{10} + \frac{6}{100} = {\cdot}76$.

Hence *we may place any number of zeros at the end of a decimal, or we may remove them from the end, without altering the value of the decimal.*

We may similarly place a decimal point followed by any number of zeros at the end of a whole number without affecting its value, thus $3 = 3{\cdot}0000...$, and $126 = 126{\cdot}00000....$

APPROXIMATIONS.

46. For many purposes a sum of money is expressed with sufficient accuracy if it is correct to the nearest penny; in no case can we give or receive a fraction of a *farthing*.

If we ask the time, we are usually content to be told the number of minutes past, or to, the hour; we do not generally care to know the number of seconds.

Again, in stating the distance of a remote place, we are generally accurate enough if we state the number of miles as nearly as possible, disregarding yards, feet and inches. In the case of a fixed star, a million miles might be considered so small a portion of the whole distance as to be negligible.

A number, or measure, is said to be *approximately* correct when it is sufficiently correct for the purpose to be served.

If a weight is known to be 9·45843 grains, and if for a particular purpose we need not trouble ourselves with anything less than *one-thousandth* part of a grain, we may for this purpose consider the weight simply as 9·458 grains.

When, however, we omit all figures after a certain decimal place, we must not ignore entirely the *first* of the omitted figures. The approximation, as far as three decimal places, in the case of 9·45843 is, as we have said, 9·458; but in the case of 9·45853 the approximation would be 9·459, not 9·458. In the latter case 9·459 is ·00047 too much, but 9·458 would be ·00053 too little. The *error* in taking 9·459 would therefore be *less* than in taking 9·458, and 9·459 is therefore the nearer, or more accurate, approximation. In fact an approximation, as far as a certain number of decimal places, does not mean that the figures which follow the decimal point are identical with the figures in the corresponding places of the true value, but that those figures make a number nearer the true value than the same number of any other figures would make. For example, a correct approximation to 62·19999642, as far as 5 decimal places, would be 62·20000, although no figure in ·20000 is the same as the corresponding figure in ·19999.

We may state this in the form of a rule thus:—If the first of the omitted figures be 9, 8, 7, 6 or 5 we increase the last figure retained by 1; and if the first omitted figure be 4, 3, 2, 1, or 0, we make no such change, but we leave the last retained figure unaltered. Thus, if we retain 4 decimal places only,

3·45624**9**9 becomes 3·4562,

but 3·45625**0**1 becomes 3·4563.

Whole numbers may be considered in a similar way. In tables of statistics, for example, we may find the hundreds, tens and units ignored. In such a case the nearest approximation to

37624392 would be 3762**4**000,

and that to 84304663 would be 8430**5**000,

and these approximations would be the numbers recorded in the tables.

EXAMPLES. XII. (a).

State the approximate values of the following numbers, retaining only two decimal places:

(1) 37·4132.	(2) 13·2051.	(3) 24·3643.
(4) 8·4174.	(5) 4·5738.	(6) 21·3626.
(7) 57·7112.	(8) 19·8939.	(9) 7·6381.
(10) 23·4235.	(11) 47·3843.	(12) 10·2957.
(13) 3·5968.	(14) 15·9982.	(15) 399·9997.
(16) 246·7254.		

Write down the following numbers correct to the nearest *thousand:*

(17) 36325.	(18) 134369.	(19) 91638.
(20) 78869.	(21) 45234.	(22) 273724.
(23) 88315.	(24) 9073.	(25) 147471.
(26) 56943.	(27) 47124.	(28) 239968.
(29) 323586.	(30) 142172.	(31) 14206.
(32) 189924.	(33) 197572.	(34) 30791.
(35) 125441.	(36) 96702.	(37) 399903.

ADDITION AND SUBTRACTION OF DECIMALS.

47. We will now consider how to find the approximate *sum* of two or more numbers, or the approximate *difference* of two numbers, *i.e.* the sum or the difference correct as far as a certain decimal place.

Suppose, for example, that we require the sum and the difference of 3·79841159321 and 5·62018078024, *correct* to *four* decimal places. In each case we retain *six* decimal places (two more than the *four* which are to be correct), and we write the numbers in column as in the margin.

	5·620181
	3·798412
sum =	9·4186
difference =	1·8218

In neither case do we write any figure in the 5th and 6th decimal places of the result. The figure in the 5th place of the sum would be 9, and we take account of it by writing 6 instead of 5 in the 4th place; and for a similar reason we write 8 instead of 7 in the 4th place of the difference.

NOTE We retain in each number two decimal places more than we want in the result, because we cannot be sure of having the *fourth* figure correct unless we know whether the *fifth* figure is one of the numbers 0, 1, 2, 3 and 4, or one of the numbers 5, 6, 7, 8 and 9.

48. When a result is correct to *three* decimal places it differs from the true value by less than ·001, *i.e.*, it is correct to the nearest *thousandth*; if it is correct to *four* decimal places, it differs from the true value by less than ·0001, *i.e.*, it is correct to the nearest *ten-thousandth*.

49. A recurring decimal may be expressed in more than one form, according to the digit with which we begin the period. Thus in ·5$\dot{1}\dot{4}\dot{3}$, which is the same as ·5143143143143 ....., we may consider the period to be either 143, or 431, or 314, and we may write the decimal as either ·5$\dot{1}\dot{4}\dot{3}$, ·51$\dot{4}\dot{3}\dot{1}$, or 514$\dot{3}\dot{1}\dot{4}$.

Example. Add together 5·76$\dot{2}$, ·54$\dot{9}$, 1·20$\dot{4}$.

We first alter the decimals so that the periods may begin at the same, *i.e.*, at the third decimal place in all. They thus become 5·7626, ·5495$\dot{4}$, 1·204.

If we now carry on each decimal to the 8th place, it will be seen that the 9th column after the decimal point will be the same as the 3rd; the 10th the same as the 4th, etc.; and similarly, the 15th the same as the 3rd and 9th; the 16th the same as the 4th and 10th, and so on: thus the *columns* recur in periods of six. It follows, therefore, that the digits at the feet of these columns will recur; that is, the decimal part of the sum will also be periodic, and its period will consist of six digits.

5·76262626. .
·54954954...
1·20444444...

It will be seen, too, that the number of columns that recur, and therefore the number of digits in the periodic part of the result, must be the L.C.M. of the numbers of digits in the given periods.

Next we notice that to the sum of the 8th column something is carried from the sum of the 9th. In order therefore that the 8th place in the result may be correct, we carry on each decimal to *two* places more; but these two places may be ignored when the addition has been performed.

The result is $7\cdot5\dot{1}\dot{6}\dot{6}2025$.

5·76262626	26
·54954954	95
1·20444444	44
7·51662025	

NOTE.—Two facts must be remembered :—

(i) The sum of any number of recurring decimals is also a recurring decimal; (ii) The number of digits in its period is the L.C.M. of the numbers of digits in the given periods.

The sum will however reduce to a terminating decimal when its period consists of 9's only. For example, $\cdot0\dot{9}=\cdot1$, $\cdot069=\cdot07$, and $3\cdot\dot{9}=4$.

50. To subtract one recurring decimal from another we proceed on the same principle as in addition.

EXERCISE XIII.

Find to within one-hundredth the sum or

(1) 23·7645, 136·489, 14·03675, 8·37865.
(2) 361·15785, 73·86576, 13·248754, 10·68752.
(3) ·56275, ·287523, ·468259, ·875625.
(4) ·4186541, 10·387527, 15·8764, ·00457.
(5) 3·5174275, 5·39875, 4·05, ·005625.
(6) 10·0265, 8·297535, ·005875, ·68715.
(7) 38·1782543, ·0125, 19·185265, 103·54375.
(8) 13·06545, 5·0078525, 7·387528, 85·45625.
(9) 7·25298, 10·076987, 12·0265, 13·875.
(10) ·05685, 13·25875, 14·48625, 15·625.

(11) $2\cdot188\dot{1}$, $\cdot0\dot{8}$, $3\cdot1\dot{6}$.
(12) $7\cdot50\dot{0}\dot{1}$, $\cdot800\dot{2}\dot{0}$, $\cdot7\dot{8}\dot{9}$, $85\cdot1\dot{2}$.
(13) $2\cdot1\dot{0}\dot{1}$, $\cdot243\dot{1}\dot{8}\dot{3}$, $\cdot123\dot{6}$, $45\cdot2\dot{9}$.
(14) $193\cdot\dot{0}9\dot{0}$, $\cdot4\dot{0}7\dot{1}$, $35\cdot13$, $76\cdot\dot{5}$.

EXERCISE XIV.

Write down the result of adding the following decimals across:—

(1) 5, 3·22, 2·333, 1·4444.
(2) 28·8, 2·5625, 3·125, 7·45.
(3) ·012, 4·21, ·0012, ·00045, 1·720004.
(4) 75·012, 7·50123, ·7501234, ·075012345.
(5) 1·2345, 23·4567, 345·6789, 4567·8901.
(6) ·096, 4·007, ·00769, 960, 875·9.
(7) 10·007, ·006, ·0307, 2·3, ·006.
(8) 3·0008, 5·037, 2·3, 1·01 1·001.
(9) ·00185, 6700, 52·64, 324·5, 1876·14, ·817.
(10) 365·414, 86512·3, 715·02, 28·194, ·0012, 46125·7.

EXERCISE XV.

Find to within one-hundredth the difference between

(1) 4·7538 and 5·29654.
(2) 7·83754 and 10·3268.
(3) 10·62575 and 12·75.
(4) 13·275856 and 14·178759.
(5) 15·218625 and 19·5.
(6) 21·87562 and 32·726251.
(7) 43·575 and 50·01625.
(8) 59·31525 and 60·8625.
(9) 71·87568 and 83·75825.
(10) 85·375654 and 100·0025.
(11) $143\cdot0\dot{2}\dot{9}$ and $174\cdot\dot{1}$.
(12) $25\cdot2\dot{0}8\dot{6}$ and $32\cdot1\dot{8}\dot{1}$.
(13) $1\cdot6\dot{3}\dot{4}$ and $3\cdot1\dot{8}5\dot{3}$.
(14) $23\cdot6\dot{9}\dot{1}$ and $36\cdot6\dot{5}71428$.

MULTIPLICATION OF DECIMALS.

51. When one or both of two numbers whose product is required have a large number of decimal places, the complete multiplication might be very laborious, and if only a few decimal places are needed in the product, the greater part of the result and of the labour involved would be useless. Under such circumstances it is important to be able to modify the ordinary process of multiplication, so that as much as possible of the profitless labour may be avoided.

52. Suppose, for example, that we require the product of 123·4567891 and 12·3456 as far as the 3rd figure after the decimal point. For the reasons stated in Art. 47 we will proceed as if we needed 5 decimal places.

53. It has been noticed that it is quite immaterial whether, in multiplication, we begin with the right-hand figure of the multiplier, as has been usual, or with the left-hand figure. We may, in fact, use the figures in any order we please, provided we take due account of each figure's *place value*. For our present purpose it is especially convenient to begin with the left-hand figure. To multiply by 12·3456 is, in fact, equivalent to multiplying by

10, 2, ·3, ·04, ·005, and ·0006,

and adding the results together. The calculation then appears as below :—

$$
\begin{array}{r|l}
123{\cdot}4567891 & \\
12{\cdot}3456 & \\
\hline
1234{\cdot}56789 & 1 \\
246{\cdot}91357 & 82 \\
37{\cdot}03703 & 673 \\
4{\cdot}93827 & 1564 \\
{\cdot}61728 & 39455 \\
{\cdot}07407 & 407346 \\
\hline
1524{\cdot}14813 & 551296
\end{array}
$$

54. It will hence be seen that as far as the required decimal places are concerned none of the figures to the right of the vertical line are of any use. We have therefore to consider

how to avoid that part of the calculation which produces these unused figures. With this object in view, attention must be paid to the following facts :—

(i) In the first line, which is obtained by multiplying by 10, there is *one* figure (1) unused ; in the lines which follow, there are successively *two* figures (82), *three* figures (673), *four* figures (1564), &c., unused.

(ii) These unused figures will not be introduced if, when we multiply by the left-hand figure 1 of the multiplier, we begin on the figure 9 of the multiplicand and ignore the 1 to the right of it ; and if, when we multiply by the 2, we begin upon the figure 8 of the multiplicand, and ignore the 91 to the right of it ; and if, when we multiply by the 3, we begin upon the 7 and ignore the 891 to the right of it ; and so on.

Thus for every step from left to right, as we use the successive digits of the multiplier, the place at which we should begin in the multiplicand moves one step from right to left.

It will be found convenient to write down at length the product obtained by using the left-hand digit of the multiplier. The first useful figure of the multiplicand can then be ascertained by inspection, and the remainder of the process will follow the course we have laid down above.

As an additional safeguard the last *useful* digits of the multiplicand may be struck out in turn as they are done with. For example, the digit 1 of the multiplier begins upon the digit 9 of the multiplicand ; the digits 1 and 9 may, on the completion of this step, be struck out. Then the digit 2 of the multiplier begins upon the digit 8 of the multiplicand ; on the completion of this step the digits 2 and 8 may be struck out ; and so on.

123·4567891
12·3456
1234·56789
246·91356
37·03701
4·93824
·61725
·07404
1524·14799

The figure in the *fourth* decimal place of the result being 9, it follows that the required result as far as *three* decimal places is 1524·148.

55. When we arrive at a product which has only 0's in the places retained, we terminate the operation, for neither this nor any subsequent product can affect the result within the required limits.

56. We will now apply the method we have described.

Example i. *Multiply* 2·04491 *by itself, so that the result may be correct to* 4 *decimal places.*

2·044910
2·04491
4·089820
·081796
·008176
·001836
·000020
4·1816

We here affix 0 to the multiplicand, in order that the product obtained on multiplying by 2 may have *six* decimal places.
The result is 4·1816

Example ii. *Multiply* ·00040635 *by* 241·6358, *so that the result may be correct to within* ·000001.

·0004063500
241·6358
·08127000
·01625400
·00040635
·00024378
·00001218
·00000200
·00000032
·098189

In this case we affix two 0's at the end of the multiplicand, in order that there may still be *eight* decimal places after multiplication by 200.
The result is ·098189.

Example iii. *Find the value of* 56·876 × ·00144 *as far as* 4 *decimal places.*

56·876
·00144
·056876
·022748
·002272
·0819

The result required is ·0819.

Example iv. *Multiply* $32{\cdot}12\dot{4}$ *by* $502{\cdot}0\dot{8}0\dot{6}$ *to 4 decimal places.*

We write down the multiplicand as far as the 8th decimal place, so that there may be 6 decimal places after multiplication by 500.

We use the figures of the multiplier in succession until we arrive at a product which has its first significant figure in the 6th place. The result required is 16128·9615.

32·12424242
502·08068068
16062·121210
64·248484
2·569936
·019272
·002568
·000018
·000003
16128·9615

EXERCISE XVI.

Multiply

(1) 32·1598 by ·562089 to 3 decimal places
(2) ·23045 by 15·8476 to 3 .....
(3) 3·72 by ·0005962 to 4 .....
(4) ·005762 by 3·20876 to 4 .....
(5) ·156208 by ·36208 to 4 .....
(6) ·0006284 by 840·32105 to 5 .....
(7) 26·378042 by 7204·56 to 5 .....
(8) 71·0324567 by 371·8920 to 5 .....
(9) 28·5631 by ·37642567 to 5 .....
(10) 3·208943621 by 8·4230156201 to 5 .....
(11) 52·687640812 by 18·703216231 to 6 .....
(12) 4·320972418 by 5·630142197 to 6 .....

(13) $7{\cdot}36\dot{7}\dot{4}$ by 560. (14) $\cdot00\dot{7}0\dot{4}$ by 834.
(15) $\cdot52\dot{3}80\dot{9}$ by 7·62. (16) $3{\cdot}0\dot{9}\dot{7}$ by ·00061.
(17) $\cdot2\dot{3}$ by $\cdot3\dot{6}$. (18) $6{\cdot}7\dot{6}$ by $\cdot0\dot{5}$.
(19) $\cdot00792$ by $5{\cdot}6\dot{0}$. (20) $11{\cdot}3\dot{9}$ by $\cdot02\dot{4}\dot{7}$.

DIVISION OF DECIMALS.

57. If we divide 16·80437921 by 3·142 as far as 5 places, the work will appear as in the margin, but it may be shortened by omitting the digits to the right of the vertical line.

```
Quot. = 5·348306
3142 ) 16804·37921
       15710
       10943
        9426
        15177
        12568
         2609|9
         2513|6
           96|32
           94|26
            2|0610
```

The following rule, by which we may do so, will be easily understood, after what has been said about multiplication, with the help of the examples that follow:—

(i) *Find the first digit of the quotient in the ordinary way, or by inspection, so that we may know how many digits there will be in the whole of the quotient wanted—integral part as well as decimal.*

(ii) *Proceed in the ordinary way until the number of quotient-digits still to be obtained is less by one than the number of digits in the divisor.*

(iii) *As soon as this happens, instead of bringing down and annexing the next digit of the dividend, strike off a digit from the end of the divisor, and proceed one step with this curtailed divisor; remembering, however, in the multiplication, to add in the nearest multiple of ten from the multiple of the digit just struck off. Repeat this process, striking off the digits of the divisor in succession, until none are left.*

(iv) *If there are more digits in the divisor than there are to be in the quotient, retain one more, and strike out the rest.*

Example i. *Divide* 673·1489 *by* ·41432 *to* 2 *decimal places.*

After determining the quotient-digits 1, 6, there will be only four more to be found. This is one less than the number of digits in the divisor; so we begin to curtail the divisor now.

```
41432 ) 67314890 (1624·71      41432 | 10237
        41432                          | 8286
        258828                          1951
        248592                          1657
         10236                           294
                                         290
                                           4
                                           4
```

Since the first rejected digit of the dividend is 9, we change the final digit of the second remainder from 6 to 7.

Example ii. *Divide* 2·7182818 *by* 3·1415927 *to* 8 *places.*

Making the divisor a whole number it becomes 31415927, and the dividend becomes 27182818.

The first digit of the quotient is that which follows the decimal point. We have therefore 8 digits to determine, and we have 8 digits in our divisor. Consequently we find *one* digit in the ordinary way, and there will then remain 7 digits to be determined, which is one less than the number of digits in the divisor. We may therefore begin now to curtail the divisor.

```
31,4,1,5,9,2,7 ) 27182818·0 ( ·86525595
                 251327416
                  20500764
                  18849556
                   1651208
                   1570796
                     80412
                     62832
                     17580
                     15708
                      1872
                      1571
                       301
                       283
                        18
                        16
                         2
```

NOTE.—In the shortened form these examples appear thus :—

Example i.

```
41,4,3,2, ) 67314890 ( 1624·71
             258828
              10237
               1951
                294
                 4
```

Example ii.

```
31,4,1,5,9,2,7 ) 271828180 ( ·86525595
                  20500764
                   1651208
                     80412
                     17580
                      1872
                       301
                        18
                         2
```

58. To divide a recurring decimal by a whole number, or by a terminating decimal, we proceed as in ordinary division, bringing down the digits of the period in rotation. The quotient also will be a recurring decimal with a period containing the same number of digits.

When the divisor is a recurring decimal, and the *exact* quotient is required, we must reduce the decimals to vulgar fractions.

EXERCISE XVII.

Divide

(1) 3476·321 by 1248·4176 to 2 decimal places.
(2) 436·3205 by 279·152 to 3 .....
(3) 56·9812 by 47·87631 to 3 .....
(4) 37·20518 by ·1462304 to 3 .....
(5) 185·37612 by ·08764032 to 4 .....
(6) ·13751432 by 1·8763015 to 4 .....
(7) ·5361085 by 23·450842 to 4 .....
(8) 17·1890865 by ·13249672 to 5 .....
(9) 23·2843612 by ·03205617 to 6 .....
(10) 154·362904 by ·000541398 to 7 .....

(11) ·332$\dot{5}$ by 25. (12) 3·$\dot{6}$ by 2·4.
(13) ·$\dot{8}\dot{1}$ by ·027. (14) 20·13$\dot{9}\dot{7}\dot{2}$ by 42·1.
(15) 3·3$\dot{8}$ by 10·$\dot{0}\dot{9}$. (16) ·$07\dot{9}$ by 7·3$\dot{9}$.

DECIMALIZATION OF MONEY.

59. We may mentally reduce any amount expressed in shillings and pence to an approximate decimal of £1, true to 3 places, if we notice that $2s. = £·1$; $1s. = £·05$; $6d. = £·025$; and if we remember too that $\frac{1}{4}d. = \frac{1}{960}$ of £1 $= 1\frac{1}{24}$ of £·001.

Thus $3s = £·15$, $5s. = £·25$, $7s. = £·35$, etc.; and $3s.\ 6d. = £·175$, $5s.\ 6d. = £·275$, $7s.\ 6d. = £·375$, etc.

In dealing with any amount, we first decimalize the shillings, and then the sixpence, if there is one. The remainder cannot exceed $5\frac{3}{4}d.$, *i.e.*, 23 farthings. We have now two cases to consider: (i) when this remainder does not exceed 11 farthings, and (ii) when it does. In the first case, we multiply ·001 by the number of farthings; and in the second case, by the number of farthings increased by 1.

Thus $2\frac{3}{4}d = 11f. = £·011$; $3d. = 12f. = £·013$.

The decimals from the various parts are then added mentally, thus

£4. 13*s.* $7\frac{3}{4}d. = £(4·675 + ·007) = £4·682$;
£4. 17*s.* $4\frac{1}{2}d. = £(4·85 + ·019) = £4·869$.

Conversely,

£·36742 = £·367 app. = £·35 + £·017 = 7*s.* + (17 − 1)*f.* = 7*s.* 4*d.*;
£·78871 = £·789 app. = £·775 + £·014 = 15*s.* 6*d.* + 13*f.* = 15*s.* $9\frac{1}{4}d.$

NOTE.—We may get the *exact* decimal due to the surplus pence and farthings by multiplying ·001 by the *exact* number of farthings, and adding to the result $\frac{1}{24}$ of itself.

60. In the following method of approximation we treat $\frac{1}{4}d.$ as exactly $\frac{1}{1000}$ of £1, and no attempt, such as that of Art. 59, is made to correct the error. The result is consequently less exact than the former, but is sufficiently correct for most purposes. The error is less than 1*d.*

Example i. *Express* 13*s.* 5$\frac{3}{4}$*d. as the decimal of a pound to three places.*

Multiplying 5$\frac{3}{4}$ by **4** we get 23; we put down 3 as the *third* figure of the decimal, and carry 2.

Multiplying 13 by **5**, and adding the 2 carried, we get 67. These are the *first two* figures of the decimal.

We thus get 13*s.* 5$\frac{3}{4}$*d.* = £·673.

Treating 5*s.* 3$\frac{1}{4}$*d.* in a similar way, we get £·263.

Example ii. *Express* £·263 *in shillings and pence.*

We divide the first two figures of the decimal by **5**, and we then *change the divisor* to **4**.

Thus dividing 26 by **5** the quotient is 5, and the remainder is 1. The quotient represents shillings.

Next dividing 13 by **4** the quotient is 3$\frac{1}{4}$. This quotient represents pence.

We thus get £·263 = 5*s.* 3$\frac{1}{4}$*d.*

Similarly we get £·673 = 13*s.* 5$\frac{3}{4}$*d.*

Note.—In *Example* i. we multiply the number of pence by 4, to reduce to farthings, a farthing being taken as ·001 of £1; and we multiply the number of shillings by 5, because 1*s.* is $\frac{5}{1000}$ of £1. The divisors 4 and 5 are used in *Example* ii. for similar reasons.

EXERCISE XVIII.

Reduce to pounds and the decimal of a pound to 3 places

(1)	£7. 4*s.* 1*d.*	(2)	£5. 6*s.* 1$\frac{1}{4}$*d.*	(3)	£2. 8*s.* 1$\frac{3}{4}$*d.*
(4)	£8. 14*s.* 2$\frac{1}{4}$*d.*	(5)	£9. 16*s.* 2$\frac{1}{4}$*d.*	(6)	£1. 18*s.* 2$\frac{3}{4}$*d.*
(7)	£9. 6*s.* 3$\frac{1}{4}$*d.*	(8)	£4. 10*s.* 4$\frac{1}{2}$*d.*	(9)	£3. 12*s.* 5$\frac{3}{4}$*d.*
(10)	£2. 3*s.* 7$\frac{1}{4}$*d.*	(11)	£6. 5*s.* 8$\frac{1}{4}$*d.*	(12)	£5. 7*s.* 10$\frac{1}{2}$*d.*
(13)	£4. 17*s.* 9$\frac{3}{4}$*d.*	(14)	£7. 15*s.* 11$\frac{1}{2}$*d.*	(15)	£3. 13*s.* 7$\frac{3}{4}$*d.*

Express in £. *s.* *d.* to the nearest farthing

(16)	£·156321.	(17)	£·359462.	(18)	£·258295.
(19)	£·654846.	(20)	£·752983.	(21)	£·556537.
(22)	£3·368244.	(23)	£2·417376.	(24)	£4·672468.
(25)	£8·734729.	(26)	£6·543564.	(27)	£2·827541.
(28)	£2·983472.	(29)	£3·662538.	(30)	£5·509569.

COMPOUND ADDITION.

61. Rapid casting of long columns of pounds, shillings, and pence such as the following is very essential in commercial work.

EXERCISE XIX.

A			B			C		
£	s.	d.	£	s.	d.	£	s.	d.
245	6	2	664	13	2	8385	13	6
163	9	11	485	2	2	8019	3	8
684	5	2	214	3	2	4321	18	9
9065	13	8	6409	12	8	8139	7	4
1067	13	8	207	4	6	2935	8	1
612	3	2	4561	10	8	239	2	5
46	3	6	492	13	7	6904	7	3
1130	1	4	752	15	3	752	15	3
8279	12	9	6218	12	8	3241	11	6
287	6	8	9532	8	7	485	2	2
115	7	6	46	3	6	109	19	10
2784	9	11	3276	5	9	56	4	10
3805	16	4	904	7	3	210	14	8
6127	9	10	2015	1	2	6218	12	8
4201	6	8	8019	3	8	783	8	7

D			E			F		
£	s.	d.	£	s.	d.	£	s.	d.
56	4	10	512	7	8	8279	12	9
6408	13	10	94	3	4	4123	17	6
8139	7	4	211	6	8	9035	2	10
869	6	7	9035	2	10	1067	13	8
8385	13	6	6409	12	8	631	7	7
45	6	2	4561	10	8	239	2	5
2935	8	1	492	13	7	9065	13	8
612	3	2	2015	1	2	360	5	5
109	19	10	4287	9	11	2307	4	2
1528	3	6	9603	2	5	163	9	11
1130	1	4	6408	13	10	9603	2	5
94	3	4	8043	7	9	362	3	4
563	4	1	4123	17	6	691	13	9
1287	14	7	563	4	1	5976	2	5
4201	6	8	1528	3	6	287	6	8

NOTE.—The horizontal lines divide the columns into 24 additions: by taking (1) the last 7 lines in each column; (2) the last 10 in each column; (3) the last 13 in each column; (4) the whole of the 15 lines in each column.

EXERCISE XX.

Add together (i) the vertical columns, (ii) the horizontal rows, of the subjoined sums, and verify the gross total.

	A			B			C			D			E			F			G			H			Total
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
(1)	1	13	6	2	8	9	4	16	3	12	16	4	79	13	8	163	3	7	537	14	8	7189	13	6	
(2)	2	14	7	6	13	8	3	18	10	11	13	8	38	16	4	207	16	8	639	15	9	2165	14	10	
(3)	3	5	8	3	10	7	2	9	9	13	6	10	96	13	7	372	13	2	314	16	10	3987	15	11	
(4)	4	16	9	5	16	4	9	10	8	16	3	2	63	8	7	409	9	6	629	17	7	4652	6	2	
(5)	9	17	10	9	13	7	8	16	7	29	19	8	28	3	8	376	18	3	876	8	6	3167	17	3	
(6)	8	19	11	4	18	9	5	17	6	38	17	6	57	16	7	267	10	7	109	19	8	4295	18	4	
(7)	6	8	3	7	3	10	4	15	7	57	13	2	48	14	3	309	12	6	836	7	9	3189	19	10	
(8)	7	6	6	8	16	9	7	16	8	16	17	4	76	16	2	427	17	4	724	16	10	5273	4	11	
(9)	2	10	9	9	17	6	8	2	9	21	6	3	32	19	7	617	18	6	618	12	11	6347	12	9	
(10)	9	18	2	2	12	9	9	17	5	32	12	7	76	13	6	387	7	9	317	3	2	4769	17	8	
(11)	6	4	4	3	18	4	3	8	6	64	16	3	39	7	2	209	8	3	208	14	3	8254	5	10	
(12)	3	13	6	7	13	6	2	19	10	75	9	8	87	8	4	360	16	9	306	15	10	3178	18	7	
Total																									Gross Total.

COMPOUND SUBTRACTION.

62. The following example is a good test for readiness and accuracy.

Example. In one operation take the sum of £17. 16*s.* 4½*d.* *and* £15. 8*s.* 3¾*d.* *from* £35. 16*s.* 7*d.*

We add each column of the subtrahends, and subtract each sum from the corresponding term in the top line by the process of Art. 5. Thus:—

£	*s.*	*d.*
35 .	16 .	7
17 .	16 .	4½
15 .	8 .	3¾
£2 .	11 .	10¾

(i) 3*f.* and 2*f.* make 1¼*d.*; and ¾*f.* makes 2*d.*; write down ¾*f.* and carry 2*d.*

(ii) 2*d.*, 3*d.* and 4*d.* make 9*d.*; and **10***d.* makes 1*s.* 7*d.*; write down **10***d.* and carry 1*s.*

(iii) 1*s.*, 8*s.* and 16*s.* make 25*s.*; and **11***s.* makes £1. 16*s.*; write down **11***s.* and carry £1.

(iv) £1, £15, and £17 make £33; and £**2** makes £35; write down £**2**.

Hence the result is £2. 11*s.* 10¾*d.*

EXERCISE XXI.

In one operation take the sum of

(1) £3. 7*s.* 6*d.*, £1. 5*s.* 9*d.*, £6. 18*s.* 7*d.*, and £4. 3*s.* 4*d.* from £20.

(2) £5. 10*s.* 9*d.*, £6. 14*s.* 7½*d.*, £11. 15*s.* 8*d.*, and 6*s.* 3¾*d.* from £26.

(3) £16. 18*s.* 3½*d.*, £15. 10*s.* 10*d.*, £3. 19*s.* 11¾*d.*, and £1. 10*s.* 6*d.* from £40.

(4) £17. 14*s.* 11½*d.*, £19. 15*s.* 9¾*d.*, £16. 16*s.* 10½*d.*, and £15. 14*s.* 6½*d.* from £95.

(5) £13. 13*s.* 6¼*d.*, £16. 14*s.* 11½*d.*, £23. 16*s.* 9¼*d.*, and 16*s.* 4¾*d.* from £100.

(6) £1. 11*s.* 4½*d.*, £19. 12*s.* 10¾*d.*, 3*s.* 11*d.*, and £12. 13*s.* 2*d.* from £91. 10*s.* 4*d.*

(7) £17. 17*s.* 10*d.*, £13. 14*s.* 11¼*d.*, £29. 15*s.* 6½*d.*, and £34. 10*s.* 3*d.* from £96. 17*s.* 6*d.*

(8) £12. 15*s.* 4½*d.*, £15. 18*s.* 10½*d.*, £31. 14*s.* 6¼*d.*, and £42. 15*s.* 2½*d.* from £105. 2*s.* 9*d.*

(9) £13. 19*s.* 11¼*d.*, £17. 17*s.* 3¾*d.*, £42. 15*s.* 7½*d.*, and £53. 9*s.* 4¼*d.* from £132. 1*s.* 1¼*d.*

(10) £15. 14*s.* 6½*d.*, £19. 13*s.* 4½*d.*, £57. 18*s.* 9¼*d.*, and £69. 8*s.* 3½*d.* from £175. 13*s.* 11½*d.*

COMPOUND MULTIPLICATION.

63. The following process of compound multiplication is a method sometimes used in business.

Example. Multiply £47. 18*s.* 7¾*d.* *by* 253.

Multiply 253 by 3, the number of farthings; the result is 759 farthings. Dividing this by 4, we get 189¾ pence.

Next multiply 253 by 7, the number of pence; the result is 1771 pence, which, added to the 189 pence, makes 1960 pence altogether. Dividing this by 12, we get 163 shillings and 4 pence.

Next, multiplying 253 by 18, the number of shillings, and adding the result to the 163 shillings, we get 4717 shillings, which reduces to £235. 17*s.*

Lastly, multiplying 253 by 47, the number of pounds, and adding the result to the £235, we get £12126.

The complete result is therefore £12126. 17*s.* 4¾*d.*

```
                253
                  3
          4 |  759  farthings
               189¾  pence
               1771
         12 |  1960
              163s. 4d.
               2024
                253
         20 |  4717
             £235. 17s.
               1771
               1012
            £12126. 17s. 4¾d.
```

The correctness of the result may be tested by working the sum again in some different way.

EXERCISE XXII.

Multiply

(1) £5. 16*s.* 7¼*d.* by 59, 61, 89.

(2) £23. 4*s.* 6½*d.* by 37, 97, 83.

(3) £56. 16*s.* 9¾*d.* by 79, 71, 87.

(4) £15. 13*s.* 2¾*d.* by 101, 103, 107.

(5) £17. 12*s.* 10½*d.* by 109, 113, 119.

(6) £23. 7*s.* 5¼*d.* by 89, 109, 127.

(7) £43. 13*s.* 7¾*d.* by 141, 163, 196.

(8) £41. 7*s.* 8¾*d.* by 726, 841, 963.

(9) £450. 4*s.* 10¼*d.* by 40, 56, 110.

(10) £936. 19*s.* 5½*d.* by 81, 121, 75.

(11) £4012. 8*s.* 2¼*d.* by 120, 112, 128.

(12) £3056. 0*s.* 5½*d.* by 112, 270, 750.

COMPOUND DIVISION.

64. To divide a sum of money by any number, we factorize if possible, or we apply the principle explained in Art. 32.

Example i. *Divide* £12903. 3*s*. 9*d*. *by* 756.

We find that $756 = 12 \times 7 \times 9$.

	£.	s.	d.
12)	12903	. 3	. 9
7)	1075	. 5	. $3\frac{3}{4}$
9)	153	. 12	. $2\frac{1}{4}$
	£17	. 1	. $4\frac{1}{4}$

The quotient = £17. 1*s*. $4\frac{1}{4}d$.

Example ii. *Divide* £36104. 9*s*. 6*d*. *by* 9416.

	£	s.	d.	£	s.	d.
9416)	36104	. 9	. 6	(3	. 16	. $8\frac{1}{2}$
	7856					
	157129 shillings					
	6473					
	77682 pence					
	2354					
	9416 farthings.					

The quotient = £3. 16*s*. $8\frac{1}{2}d$.

EXERCISE XXIII.

Divide, by short division,

(1) £144281. 3*s*. $1\frac{1}{2}d$. by 121 and 210.
(2) £98049. 17*s*. 6*d*. by 660 and 231.
(3) £48131. 0*s*. $3\frac{3}{4}d$. by 1575 and 1925.
(4) £530866. 17*s*. 6*d*. by 2772 and 1925.

Divide, by long division,

(5) £46680. 6*s*. $6\frac{1}{4}d$. by 719.
(6) £44415. 8*s*. $11\frac{1}{2}d$. by 761.
(7) £19282. 7*s*. $4\frac{3}{4}d$. by 811.
(8) £121701. 5*s*. $6\frac{1}{2}d$. by 821.
(9) £144438. 1*s*. $7\frac{1}{2}d$. by 919.
(10) £170767. 10*s*. $10\frac{3}{4}d$. by 971.
(11) £283812. 15*s*. $7\frac{1}{2}d$. by 1009.
(12) £4420895. 0*s*. $3\frac{3}{4}d$. by 3001.

CALCULATION OF PRICES.

(Mental Rules.)

65. The following rules should be known for the mental calculation of certain prices :—

(1) *To find the cost of a dozen.*

Reckon the given cost of one article in pence, and call the pence shillings.

Thus the cost of 1 dozen @ 1*s*. 4*d*. = 16*s*.;
and the cost of 1 dozen @ 2*s*. $6\frac{1}{2}d$. = 30*s*. 6*d*. = £1. 10*s*. 6*d*.

(2) *To find the cost of a gross.*

NOTE.—A gross = a dozen times a dozen.

Thus the cost of 1 gross @ 1*s.* 4*d.* = cost of 12 at 16*s.*
= £9. 12*s.*

(3) *To find the cost of* 48.

Reckon as many shillings as there are farthings in the cost of one article.

NOTE.—This method can be applied to numbers such as 384, 432, and 528, and other large multiples of 12.

(4) *To find the cost of a score.*

Reckon the given cost of one article in shillings, and call the shillings pounds.

Thus the cost of 1 score @ 7*s.* 9*d.* = £7. 15*s.*;
and the cost of 1 score @ £1. 2*s.* 7*d.* = £22$\frac{7}{12}$
= £22. 11*s.* 8*d.*

(5) *To find the cost of* 240.

Reckon as many sovereigns as there are pence in the cost of one article.

Thus the cost of 240 @ 1*s.* 5*d.* = £17.

(6) *To find the cost of* 100.

Reckon as many pence and twice as many shillings as there are farthings in the cost of one article.

Thus the cost of 100 @ 8$\frac{1}{2}$*d.* = 34*d.* + 68*s.* = £3. 10*s.* 10*d.*

(7) *To find the cost of* 365.

Reckon as many sovereigns, as many half-sovereigns, and five times as many pence as there are pence in the cost of one article.

Thus the cost of 365 @ 1*s.* 5*d.* = £17 + £8. 10*s.* + 85*d.*
= £25. 17*s.* 1*d.*

(8) *To find the cost of* 252.

Reckon as many guineas as there are pence in the cost of one article.

Thus the cost of 252 @ 1*s.* 5*d.* = £17. 17*s.*

(9) *To find the cost of* 313.

Reckon as many sovereigns, crowns, shillings, and pence as there are pence in the cost of one article.

Thus the cost of 313 @ 6*s.* 9*d.*
= £81 + (5*s.* × 81) + 81*s.* + 81*d.*
= £81 + £20. 5*s.* + £4. 1*s.* + 6*s.* 9*d*
= £105. 12*s.* 9*d.*

(10) *To find the cost of any number of articles.*

Find the cost of the nearest multiple of any of the above numbers, and add or subtract the cost of the remainder.

Thus (by Rule 5), the cost of 239 articles @ 1*s.* 5*d.*
$= £17 - 17d. = £16.\ 18s.\ 7d.$;
and the cost of 241 articles @ 1*s.* 5*d.*
$= £17 + 17d. = £17.\ 1s.\ 5d.$

(11) *To find the cost of a hundredweight, when the price per lb. is given.*

Multiply 9*s.* 4*d.* by the number of pence in the price.

(12) *To find the cost of any number of articles at* 6*s.* 8*d.* *and other aliquot parts of* £1.

Multiply the number of articles by the fraction which expresses the relation that the cost of one article bears to £1.

At 6*s.* 8*d.* the number of £'s in the cost is one-third of the number of articles.

We proceed in a similar manner when the prices are such as 4*s.*, 5*s.*, 1*s.* 8*d.*, 1*s.* 4*d.*, 1*s.* 3*d.*, 13*s.* 4*d.*, 15*s.*, etc.

(13) The above rules must be modified for such prices as the following :—

(*a*) At 4*s.* 2*d.* Since $4s.\ 2d. = £\frac{5}{24} = £\frac{10}{48}$,
add 0 to the number of articles and divide by 48.

(*b*) At 6*s.* 3*d.* Since $6s.\ 3d. = £\frac{5}{16} = £\frac{10}{32}$,
add 0 and divide by 32.

(*c*) At 16*s.* 8*d.* Since $16s.\ 8d. = £\frac{5}{6} = £\frac{10}{12}$,
add 0 and divide by 12.

(*d*) At £1. 13*s.* 4*d.* Since $£1.\ 13s.\ 4d. = £\frac{5}{3} = £\frac{10}{6}$,
add 0 and divide by 6.

(*e*) At £16. 13*s.* 4*d.* Since $£16.\ 13s.\ 4d. = £\frac{50}{3} = £\frac{100}{6}$,
add 00 and divide by 6.

(*f*) At 12*s.* 6*d.* Since $12s.\ 6d. = £\frac{5}{8} = £\frac{10}{16}$,
add 0 and divide by 16.

(*g*) At 8*s.* 4*d.* Since $8s.\ 4d. = £\frac{10}{24}$,
add 0 and divide by 24.

(*h*) At 2*s.* 7½*d.* Since $2s.\ 7\tfrac{1}{2}d. = 2s.\ 6d. + \frac{1}{20}$ of 2*s.* 6*d.*, find the cost at 2*s.* 6*d.*, and add a shilling for every sovereign in that cost.

We proceed in a similar manner when the prices are such as 3*s.* 6*d.*, 10*s.* 6*d.*, and 7*s.*

(*i*) At 2*s*. 4½*d*. Since 2*s*. 4½*d*. = 2*s*. 6*d*. $-\frac{1}{20}$ of 2*s*. 6*d*., find the cost at 2*s*. 6*d*., and subtract a shilling for every sovereign in that cost.

We proceed in a similar manner for such prices as 3*s*. 2*d*., 9*s*. 6*d*., 10½*d*.

NOTE.—In connection with the calculation of prices, the following weights and measures should be known :—

Ale, barrel of	36 gall.	
Butter, firkin of	56 lb.	
„ , tub of	84 lb.	
Calico, piece of	28 yds.	
Candles, barrel of	120 lb.	
Coal, sack of	224 lb.	
Corn, imperial quarter of	504 lb.	(*Mark Lane* basis).
„ , bushel of	63 lb.	
Flour, sack of	280 lb.	
„ , barrel of	196 lb.	
„ , peck of	14 lb.	
Meat, stone of	8 lb.	
Port (Lisbon), pipe of	115 gall.	(56 dozen to the pipe).
Potatoes, ton of	49 bushels.	
„ , sack of	168 lb.	
Salt, bushel of	56 lb.	
Sherry, butt of	108 gall.	(52 dozen to the butt).
Soap, firkin of	64 lb.	
„ , barrel of	256 lb.	
Wool, sack of	364 lb.	

SIMPLE PRACTICE.

66. An **aliquot** part of a quantity is such that we may make up the quantity by taking the part a certain *integral* number of times. Its relation to the whole is therefore expressed by a fraction which has unity for numerator and an integer for denominator.

For example : 2*s*. 6*d*. is an *aliquot* part, namely $\frac{1}{8}$, of £1 ; and 6*d*. is an *aliquot* part, namely $\frac{1}{5}$, of 2*s*. 6*d*.

67. **Practice** is a method of calculating *by addition of aliquot parts* the value of a simple or compound quantity, when the value of a unit of one denomination is given. The method is *simple* or *compound*, according as the quantity considered is simple or compound.

EXERCISE XXIV.

Write down the easiest aliquot parts to take for

(1)	14*s.* 6*d.*	(2)	17*s.* 2*d.*	(3)	19*s.* $2\frac{1}{2}$*d.*
(4)	14*s.* $8\frac{1}{4}$*d.*	(5)	15*s.* $7\frac{3}{4}$*d.*	(6)	13*s.* $4\frac{1}{2}$*d.*
(7)	2*s.* $8\frac{1}{2}$*d.*	(8)	6*s.* $9\frac{3}{4}$*d.*	(9)	18*s.* $11\frac{1}{2}$*d.*
(10)	16*s.* $10\frac{3}{4}$*d.*	(11)	17*s.* $4\frac{3}{4}$*d.*	(12)	12*s.* $5\frac{1}{2}$*d.*

68. The method of Practice will be understood from the following examples :—

Example i. *Find the value of* 1243 *things at* £4. 17*s.* 6*d. each.*

The value at £1 each is clearly £1243. The value at £4 each is found by multiplying the value at £1 by 4.

Again, 10*s.* is an aliquot part, namely $\frac{1}{2}$, of £1; hence the value at 10*s.* each is found by dividing the value at £1 by 2.

Similarly, 5*s.* is an aliquot part of 10*s.*, and we find the value at 5*s.* by dividing the value at 10*s.* by 2; and, finally, we find the value at 2*s.* 6*d.* each by dividing the value at 5*s.* by 2. The sum of these results will be the value at £4. 17*s.* 6*d.* each.

	£	*s.*	*d.*		
	1243	0	0	= value at £1 each.	
			4		
	4972	0	0	=	£4 .....
10*s.* = $\frac{1}{2}$ of £1	621	10	0	=	10*s.* .....
5*s.* = $\frac{1}{2}$ of 10*s.*	310	15	0	=	5*s.* ...
2*s.* 6*d.* = $\frac{1}{2}$ of 5*s.*	155	7	6	=	2*s.* 6*d.*
	£6059	12	6	= value at £4. 17*s.* 6*d.* each.	

It is often a quicker method, to find the value at a higher price in a "round" sum, and then subtract from the amount thus found the value at the difference between this "round" sum and the given price.

Thus, the above example may be worked as below.

	£	*s.*	*d.*		
	1243	0	0	= value at £1 each.	
			5		
	6215	0	0	=	£5 .....
2*s.* 6*d.* = $\frac{1}{8}$ of £1	155	7	6	=	2*s.* 6*d.* each.
By subtraction,	£6059	12	6	=	£4. 17*s.* 6*d.* each.

It is for the student to select the shortest and quickest way of calculating values by this method.

Example ii. *Find the value of* 4886 *things at* £1. 1*s.* 3*d. each*

	£	*s.*	*d.*	
	4886	0	0	= value at £1 each.
1*s.* 3*d.* = $\frac{1}{16}$ of £1	305	7	6	= ..... 1*s.* 3*d.* each.
	£5191	7	6	= ..... £1. 1*s.* 3*d.* each.

Since $16 = 4 \times 4$, we may divide in succession by these factors, and then strike out the first quotient.

In the next example we use decimals of a penny, and carry the decimals to *two* places.

Example iii. *Find the value of* $2512\frac{2}{3}$ *things at* 16*s.* $4\frac{3}{4}$*d. each.*

	£	*s.*	*d.*	
	2512	13	4	= value at £1 each.
10*s.* = $\frac{1}{2}$ of £1.	1256	6	8	= ..... 10*s.* .....
5*s.* = $\frac{1}{2}$ of 10*s.*	628	3	4	= ..... 5*s.* .....
1*s.* = $\frac{1}{5}$ of 5*s.*	125	12	8	= ..... 1*s.* .....
4*d.* = $\frac{1}{3}$ of 1*s.*	41	17	6·66	= ..... 4*d.* .....
$\frac{1}{2}$*d.* = $\frac{1}{8}$ of 4*d.*	5	4	8·33	= ..... $\frac{1}{2}$*d.* .....
$\frac{1}{4}$*d.* = $\frac{1}{2}$ of $\frac{1}{2}$*d.*	2	12	4·16	= ..... $\frac{1}{4}$*d.* .....
	£2059	17	3·15	= ..... 16*s.* $4\frac{3}{4}$*d.* each.

In the addition, the student must be careful here not to include the first line, which is the value at £1 each.

The foregoing example may be worked by two other methods.

I. We can use decimal working throughout, and since the number of pence in a pound lies between 100 and 1000, in taking aliquot parts it will be sufficient to ensure the accuracy of the first *three* decimal places, but to do this we must retain *four* places.

$2512\frac{2}{3} = 2512{\cdot}6666\ldots$

	£
	2512·6666
10*s.* = $\frac{1}{2}$ of £1	1256·3333
5*s.* = $\frac{1}{2}$ of 10*s.*	628·1666
1*s.* = $\frac{1}{5}$ of 5*s.*	125·6333
4*d.* = $\frac{1}{3}$ of 1*s.*	41·8777
$\frac{1}{2}$*d.* = $\frac{1}{8}$ of 4*d.*	5·2347
$\frac{1}{4}$*d.* = $\frac{1}{2}$ of $\frac{1}{2}$*d.*	2·6173
Value =	£2059·8629
=	£2059. 17*s.* 3*d.* (Art. 59.)

II. We can also decimalize the price, as below, and multiply to 3 places by the method of contracted multiplication (Art. 53).

$16s.\ 4\frac{3}{4}d. = \cdot 8197916$ of £1;

25126666
6197918
20101333
251267
226139
17588
2261
25
15

Value = £2059·8628
= £2059. 17*s*. 3*d*.

The foregoing methods can be applied to finding the value of a number of things quoted at so much per dozen, gross, or score.

Example. Find the value of 32475 *things at* £1. 13*s*. 6*d*. *per dozen.*

	£	*s*.	*d*.	
12	32475 .	0 .	0	
	2706 .	5 .	0	= value at £1 per dozen.
10*s*. = $\frac{1}{2}$ of £1	1353 .	2 .	6	= ..... 10*s*. .....
3*s*.4*d*. = $\frac{1}{8}$ of 10*s*.	451 .	0 .	10	= ..... 3*s*.4*d*. .....
2*d*. = $\frac{1}{20}$ of 3*s*.4*d*.	22 .	11 .	$0\frac{1}{2}$	= ..... 2*d*. .....
	£4532 .	19 .	$4\frac{1}{2}$	= value at £1.13*s*.6*d*. per dozen.

EXERCISE XXVa.

Find, by Practice, the value of the following numbers of *things*; the given price being the price of *one*. Use the shortest method in each case.

(1)	6729 at 13*s*. 4*d*.	(2)	7214 at 15*s*.
(3)	8307 at 16*s*.	(4)	3452 at 16*s*. 8*d*.
(5)	4527 at 17*s*. 6*d*.	(6)	5708 at 18*s*.
(7)	6932 at 18*s*. 4*d*.	(8)	4576 at 18*s*. 8*d*.
(9)	7419 at 18*s*. 9*d*.	(10)	3729 at 19*s*.
(11)	4638 at 19*s*. 2*d*.	(12)	9347 at 19*s*. 4*d*.
(13)	3826 at 19*s*. $4\frac{1}{2}d$.	(14)	4769 at 19*s*. 6*d*.
(15)	3728 at 19*s*. 7*d*.	(16)	4657 at 19*s*. 8*d*.
(17)	5384 at 19*s*. $8\frac{1}{4}d$.	(18)	4768 at 19*s*. 9*d*.
(19)	1233 at 19*s*. $9\frac{1}{2}d$.	(20)	1394 at 19*s*. 10*d*.
(21)	1115 at 19*s*. $10\frac{1}{2}d$.	(22)	1428 at 19*s*. 11*d*.
(23)	1550 at 19*s*. $11\frac{1}{4}d$.	(24)	1743 at 19*s*. $11\frac{1}{2}d$.

EXERCISE XXVb.

Find, by Practice, the value of the following numbers of *things*; the given price being the price of *one*.

(1) $369\frac{1}{2}$ at 1*s*. 8*d*.
(2) $428\frac{1}{4}$ at 2*s*. 6*d*.
(3) $537\frac{3}{4}$ at 3*s*. 9*d*.
(4) $247\frac{1}{3}$ at 4*s*.
(5) $369\frac{2}{3}$ at 5*s*. 3*d*.
(6) $213\frac{1}{8}$ at 2*s*. 10*d*.
(7) $425\frac{3}{8}$ at 3*s*. 4*d*.
(8) $636\frac{5}{8}$ at 4*s*. 6*d*.
(9) $345\frac{7}{8}$ at 5*s*. 8*d*.
(10) $524\frac{1}{6}$ at 2*s*. $10\frac{1}{2}d$.
(11) $741\frac{5}{6}$ at 3*s*. $4\frac{1}{2}d$.
(12) $346\frac{5}{12}$ at 9*s*. 6*d*.
(13) $423\frac{7}{16}$ at 16*s*. 4*d*.
(14) $826\frac{11}{16}$ at 18*s*. 8*d*.
(15) $218\frac{2}{5}$ at 2*s*. 9*d*.
(16) $415\frac{3}{5}$ at 3*s*. $6\frac{1}{2}d$.
(17) $435\frac{7}{12}$ at 10*s*. $7\frac{1}{2}d$.
(18) $529\frac{11}{12}$ at 11*s*. 10*d*.
(19) $278\frac{3}{16}$ at 14*s*. $10\frac{1}{2}d$.
(20) $618\frac{9}{16}$ at 17*s*. 5*d*.
(21) $345\frac{13}{16}$ at 19*s*. 10*d*.
(22) $252\frac{1}{10}$ at 4*s*. 5*d*.
(23) $363\frac{3}{10}$ at 5*s*. 7*d*.
(24) $510\frac{9}{10}$ at 7*s*. 2*d*.
(25) $273\frac{1}{12}$ at 8*s*. $4\frac{1}{2}d$.
(26) $452\frac{1}{16}$ at 13*s*. 2*d*.
(27) $615\frac{5}{16}$ at 15*s*. $3\frac{1}{2}d$.
(28) $814\frac{15}{16}$ at 19*s*. $11\frac{1}{2}d$.
(29) $635\frac{4}{5}$ at 4*s*. $8\frac{1}{2}d$.
(30) $482\frac{7}{10}$ at 6*s*. $9\frac{1}{2}d$.

EXERCISE XXVc.

Find, to the nearest penny, by Practice, the value of the following numbers of *things*; the given price being the price of *one*.

(1) 6420 at 4*s*. $3\frac{1}{2}d$.
(2) 1296 at 16*s*. $10\frac{1}{2}d$.
(3) 5362 at £1. 5*s*. $3\frac{1}{4}d$.
(4) 7257 at £5. 16*s*. $9\frac{3}{4}d$.
(5) 6724 at £6. 14*s*. $5\frac{1}{4}d$.
(6) 6327 at £10. 19*s*. $2\frac{1}{4}d$.
(7) 9634 at £12. 8*s*. 5*d*.
(8) 2386 at £7. 17*s*. 6*d*.
(9) 8514 at £4. 16*s*. 8*d*.
(10) 5966 at £10. 18*s*. 4*d*.
(11) 8763 at £12. 19*s*.
(12) 7861 at £12. 10*s*. $9\frac{1}{2}d$.
(13) 6248 at £17. 11*s*. $5\frac{3}{4}d$.
(14) 4989 at £19. 18*s*. 10*d*.
(15) $4118\frac{1}{4}$ at £1. 17*s*. 6*d*.
(16) $2037\frac{1}{2}$ at £1. 19*s*. $4\frac{1}{2}d$.
(17) $2178\frac{3}{4}$ at £3. 18*s*. 4*d*.
(18) $2395\frac{7}{8}$ at £2. 17*s*. 10*d*.
(19) $7395\frac{11}{12}$ at £12. 19*s*. $10\frac{1}{2}d$.
(20) $3523\frac{11}{24}$ at £20. 10*s*. $10\frac{1}{4}d$.

EXERCISE XXVI.

Find to the nearest penny, by Practice, the value of the following numbers of things at the given prices.

(1) 87642 at 16*s.* 3*d.* per dozen.
(2) 416472 at £1. 2*s.* 9*d.* per dozen.
(3) 57625 at £1. 11*s.* 7*d.* per score.
(4) 37945 at £1. 7*s.* 11*d.* per score.
(5) 32562 at £3. 9*s.* per gross.
(6) 489816 at £4. 14*s.* 6*d.* per gross.
(7) 24000 at 5*s.* 9*d.* per hundred.
(8) 26475 at £8. 12*s.* 11*d.* per hundred.
(9) 63000 at 3*s.* 9*d.* per thousand.
(10) 939500 at £1. 5*s.* 6*d.* per thousand.

COMPOUND PRACTICE.

69. In **Compound** Practice we find the value of a compound quantity when the value of a unit of *one* denomination is given.

Example. Find, to the nearest penny, the value of 11 *tons* 17 *cwt.* 3 *qrs.* 21 *lb. at* £4. 17*s.* 6*d. per ton.*

The value of 11 tons is obtained by multiplying £4. 17*s.* 6*d.* by 11. To the product we add the value of 17 cwt. 3 qrs. 21 lb., obtained by taking aliquot parts. Thus :—

	£	*s.*	*d.*	
	4	17	6	= value of 1 ton.
			11	
	53	12	6	= ...11 tons.
10 cwt. = $\frac{1}{2}$ of 1 ton	2	8	9	= ...10 cwt.
5 cwt. = $\frac{1}{2}$ of 10 cwt.	1	4	4·5	= ... 5 cwt.
2 cwt. 2 qrs. = $\frac{1}{2}$ of 5 cwt.		12	2·25	= ... 2 cwt. 2 qrs.
1 qr. = $\frac{1}{10}$ of 2 cwt. 2 qrs.		1	2·625	= ... 1 qr.
14 lb. = $\frac{1}{2}$ of 1 qr.			7·312	= ...14 lb.
7 lb. = $\frac{1}{2}$ of 14 lb.			3·656	= ... 7 lb.
	£57.	19	11·343	= value required.

We have here used decimals of a penny, and with this method it will generally be sufficient to carry the decimals to *two* places.

The foregoing example may be worked by three other methods.

I. We can decimalize the price and use aliquot parts of the *compound quantity*.

Example. Find, to the nearest penny, the value of 11 *tons* 17 *cwt.* 3 *qrs.* 21 *lb. at* £4. 17*s.* 6*d. per ton.*

£4. 17*s.* 6*d.* = £4·875.

	£
	4·875
	11
	53·625
10 cwt. = ½ of 1 ton	2·4375
5 cwt. = ½ of 10 cwt.	1·2187
2 cwt. 2 qrs. = ½ of 5 cwt.	·6093
1 qr. = $\frac{1}{10}$ of 2 cwt. 2 qrs.	·0609
14 lb. = ½ of 1 qr.	·0304
7 lb. = ½ of 14 lb.	·0152

Value = £57·9970
= £57. 19*s.* 11*d.* (Art. 59.)

II. We can express the given weight or measure in decimal form and take aliquot parts of the *money*, ensuring the correctness of the first three places.

11 tons 17 cwt. 3 qrs. 21 lb. = 11·896875 tons.

	11·896875
	4
	47·587500
10*s.* = ½ of £1	5·9484
5*s.* = ½ of 10*s.*	2·9742
2*s.* 6*d.* = ½ of 5*s.*	1·4871

Value = £57·9972
= £57. 19*s.* 11*d.*

III. We can express both quantities in decimal form, and multiply to 3 places by contracted multiplication (Art. 53).

11 tons 17 cwt. 3 qrs. 21 lb. = 11·896875 tons,
and £4. 17*s.* 6*d.* = £4·875.

11896875
5784
475875
95174
8327
595

Value = £57·9971
= £57. 19*s.* 11*d.*

EXERCISE XXVII.

Find, to the nearest penny, the value of

(1) 8 lb. 12 oz. at 3*s.* 9*d.* per lb.
(2) 15 lb. 6 oz. 12 drs. at 2*s.* 8*d.* per lb.
(3) 1 ton 6 cwt. 3 qrs. 9 lb. at £2. 6*s.* 8*d.* per ton.
(4) 3 cwt. 2 qrs. 15 lb. 8 oz. at £9. 6*s.* 8*d.* per cwt.
(5) 9 tons 19 cwt. 3 qrs. 14 lb. at £28 per ton.
(6) 6 tons 13 cwt. 49 lb. at £7. 6*s.* 8*d.* per ton.
(7) 12 tons 8 cwt. 76 lb. at 7*s.* per cwt.
(8) 3 tons 11 cwt. 3 qrs. 16 lb. at £4. 13*s.* 4*d.* per ton.
(9) 1 ton 16 cwt. 3 qrs. 21 lb. 8 oz. at £4. 13*s.* 4*d.* per ton.
(10) 7 cwt. 73 lb. $8\frac{1}{2}$ oz. at 5*s.* 4*d.* per lb.
(11) 3 cwt. 1 qr. $10\frac{1}{2}$ lb. at £8 per ton.
(12) 2 tons 7 cwt. 22 lb. 5 oz. at £32 per ton.
(13) 7 tons 13 cwt. 3 qrs. $26\frac{1}{4}$ lb. at £9. 6*s.* 8*d.* per ton.
(14) 2 qrs. 9 lb. 13 oz. at 15*s.* 9*d.* per qr.
(15) 5 cwt. 3 qrs. 19 lb. at £2. 15*s.* per cwt.
(16) 24 tons 15 cwt. 2 qrs. 14 lb. at 16*s.* $7\frac{3}{4}$*d.* per cwt.
(17) 9 tons 4 cwt. 3 qrs. 21 lb. at £14. 15*s.* 9*d.* per cwt.
(18) 6 tons 1 cwt. 2 qrs. 24 lb. at £3. 16*s.* 9*d.* per cwt.
(19) 137 tons 8 cwt. 60 lb. at £1. 2*s.* 8*d.* per ton.
(20) 3 tons 17 cwt. 2 qrs. 13 lb. 14 oz. at £3. 17*s.* 6*d.* per cwt.

EXERCISE XXVIII.

Find, to the nearest penny, the value of

(1) 27 yds. 2 ft. 7 in. at 15*s.* 9*d.* per yard.
(2) 3 miles 5 fur. 119 yds. at £5. 10*s.* per mile.
(3) 19 miles 7 fur. 176 yds. at £12. 13*s.* 4*d.* per mile.
(4) 80 miles 5 fur. 5 chains at £1. 3*s.* 4*d.* per mile.
(5) 3 miles 8 chains $5\frac{1}{2}$ yds. at £8. 13*s.* 4*d.* per mile.
(6) 13 ac. 1 ro. 16 per. at £2. 3*s.* 4*d.* per acre.
(7) 1 ro. 24 per. at £70. 11*s.* 8*d.* per acre.
(8) 55 ac. 3 ro. 10 per. at 13*s.* 4*d.* per rood.
(9) 25 sq. yds. 7 sq. ft. 126 sq. in. at £1. 10*s.* per sq. yard.
(10) 18 cu. yds. 9 cu. ft. 486 cu. in. at £1. 16*s.* per cu. yard.
(11) 3 cu. yds. 1 cu. ft. 216 cu. in. at 5*s.* 6*d.* per cu. foot.
(12) 3 pks. 1 gall. 3 qts. at 2*s.* 6*d.* per peck.
(13) 14 bush. 1 pk. 1 gall. 1 qt. at 9*s.* 4*d.* per bushel.
(14) 12 qrs. 3 bush. 1 pk. at £1. 14*s.* 8*d.* per quarter.

(15) 3 gall. 2 qts. 1 pt. at 18*s*. 6*d* per gallon.
(16) 12 barrels 27 gall. at £1. 10*s*. 8*d*. per barrel.
(17) 18 gall. 3 qts. 1½ pt. at 17*s*. 10½*d*. per gallon.
(18) 13 weeks 2 days 6 hours at £2. 3*s*. 9*d*. per week.
(8 hrs. = 1 day; 5 days = 1 week.)
(19) 3 months 5 days 7 hours at £1. 17*s*. 6*d*. per week.
(10 hrs. = 1 day; 6 days = 1 week; 4 wks. = 1 month.)
(20) 6 months 3 weeks 4 days 7 hours at £1. 13*s*. per week.
(8 hrs. = 1 day; 6 days = 1 week; 4 wks. = 1 month.)

70. Questions like the following can be dealt with by any of the various methods of Compound Practice.

Example. If a man's debts amount to £57201. 16*s*., *and he can pay only* 5*s*. 7¾*d*. *for each pound, how much do his creditors receive, to the nearest penny?*

	£	*s*.	*d*.	
	57201	16	0	= amount of debts in full.
5*s*. = ¼ of £1	14300	9	0	= amt. obtained at 5*s*. for each £1.
6*d*. = 1/20 of 5*s*.	1430	0	10·8	= ..... 6*d*. .....
1*d*. = ⅙ of 6*d*.	238	6	9·80	= ... . 1*d*. .....
½*d*. = ½ of 1*d*.	119	3	4·90	= ..... ½*d*. .....
¼*d*. = ½ of ½*d*.	59	11	8·45	= ..... ¼*d*. .....
	£16147	11	9·95	= amt. obtained at 5*s*. 7¾*d*. in the £.

This example can also be worked by decimalizing the shillings, etc., and adopting contracted multiplication (Art. 53).

£57201. 16*s*. = £57201·8,
and 5*s*. 7¾*d*. = £·2822916.

572018
661922820
114403600
45761440
1144036
114403
51481
572
343
34

Amount = £16147·5909
= £16147. 11*s*. 10*d*. (Art. 59).

EXERCISE XXIX.

Find, by Practice, the dividend on

(1) £1875 at 9*s.* 8½*d.* in the £.
(2) £274. 10*s.* at 8*s.* 3*d.* in the £.
(3) £560. 10*s.* at 6*s.* 5½*d.* in the £.
(4) £8350. 12*s.* at 12*s.* 11*d.* in the £.
(5) £4736. 5*s.* at 2*s.* 8*d.* in the £.
(6) £3678. 16*s.* at 3*s.* 4*d.* in the £.
(7) £5806. 10*s.* at 18*s.* 10½*d.* in the £.
(8) £2933. 6*s.* 8*d.* at 13*s.* 6*d.* in the £.
(9) £7352. 13*s.* 4*d.* at 12*s.* 6*d.* in the £.
(10) £12712. 8*s.* 4*d.* at 15*s.* 9*d.* in the £.

PERCENTAGES.

71. In many cases we express the fraction which one quantity is of another by a symbol which has 100 for its denominator.

The first quantity is thus estimated *in hundredths*, or as a **percentage**, of the other.

For example, $3s. = \frac{3}{240}$ of £1 $= \frac{15}{100}$ of £1;

i.e., 3*s.* is 15 *hundredths*, or 15 **per cent.**, of £1.

Also 6 pence $= \frac{1}{40}$ of £1 $= \frac{2\frac{1}{2}}{100}$ of £1;

i.e., 6 pence $= 2\frac{1}{2}$ *per cent.* of £1.

72. The *numerator*, which expresses the number of hundredths, is called the **rate per cent.** For example, in the cases just considered the *rates per cent.* are 15 and $2\frac{1}{2}$.

The words *per cent.* are briefly expressed by the symbol %, or by the letters p.c. Thus $3\frac{1}{4}$ *per cent.* may be written $3\frac{1}{4}$ %, or $3\frac{1}{4}$ p.c.

73. 100 is also the standard of comparison adopted in calculating statistical tables treating of the population, revenue, and general resources of a country.

For example, if in a town of 5000 inhabitants the number of deaths per annum is 50, the death rate is $\frac{50}{5000}$, or $\frac{1}{100}$ of the population, *i.e.*, 1 per cent. of the population.

74. Percentages are widely used in all kinds of calculations relating to commercial transactions.

EXERCISE XXX.

What fractions are denoted by the following rates per cent.?

(1) 2. (2) 3. (3) 5. (4) 8. (5) 10.
(6) 20. (7) 25. (8) 40. (9) 50. (10) 75.
(11) $2\frac{1}{2}$. (12) $12\frac{1}{2}$. (13) $33\frac{1}{3}$. (14) $66\frac{2}{3}$. (15) $\frac{1}{2}$.
(16) $\frac{1}{4}$. (17) $\frac{1}{3}$. (18) $\frac{1}{5}$. (19) $\frac{3}{5}$. (20) $\frac{3}{10}$.

EXERCISE XXXI.

What rates per cent. do the following fractions denote?

(1) $\frac{1}{2}$. (2) $\frac{1}{4}$. (3) $\frac{1}{5}$. (4) $\frac{1}{10}$. (5) $\frac{1}{20}$.
(6) $\frac{3}{20}$. (7) $\frac{1}{25}$. (8) $\frac{7}{25}$. (9) $\frac{1}{50}$. (10) $\frac{13}{50}$.
(11) $\frac{1}{11}$. (12) $\frac{1}{12}$. (13) $\frac{1}{30}$. (14) $\frac{1}{40}$. (15) $\frac{3}{40}$.
(16) $\frac{3}{8}$. (17) $\frac{7}{10}$. (18) $\frac{3}{4}$. (19) $\frac{5}{4}$. (20) $\frac{101}{100}$.

75. Case I.—To find the value of a percentage of a number or quantity.

To do this we find the corresponding vulgar or decimal fraction of it.

Example i. *Find the value of* $12\frac{1}{2}$ *per cent. of* £1.

$$12\tfrac{1}{2} \text{ per cent. of £1} = \frac{12\frac{1}{2}}{100} \text{ of £1}$$
$$= \tfrac{1}{8} \text{ of £1}$$
$$= 2s.\ 6d.$$

Example ii. *In* 1881 *the population of a town was* 17125*; in the next ten years it increased* 16 *per cent. What was its population in* 1891?

$$16 \text{ per cent. of } 17125 = \tfrac{16}{100} \text{ of } 17125$$
$$= \tfrac{4}{25} \text{ of } 17125$$
$$= 2740;$$
$$\therefore \text{ population in } 1891 = 17125 + 2740$$
$$= 19865.$$

EXERCISE XXXII.

Find the value of the following percentages:—

(1) 5 per cent. of £20, £40, £100, £70, £150.
(2) 3 per cent. of £50, £100, £150, £200.
(3) $2\frac{1}{2}$ per cent. of £10, £40, £60, £100.
(4) 8 per cent. of £50, £100, £75.
(5) 10 per cent. of £20, £50, £80, £25, £175.
(6) 20 per cent. of £10, £50, £100, £150.
(7) $12\frac{1}{2}$ per cent. of £460, £750.
(8) 3 per cent. of £123. 6*s*. 8*d*.
(9) 3 per cent. of 7 lb. 13 oz.
(10) 5 per cent. of 3 tons 17 cwt. 21 lb.

76. Case II.—To find how much per cent. one number or quantity is of another.

We express the former as a vulgar fraction of the latter, and we convert this fraction to another having 100 for its denominator.

Example i. *What per cent. of* £62. 10*s*. *is* £16?

$$\text{The fraction} = \frac{16}{62\frac{1}{2}} = \frac{32}{125} = \frac{256}{1000} = \frac{25\frac{3}{5}}{100};$$

$$\therefore \text{rate per cent.} = 25\tfrac{3}{5} = \underline{25{\cdot}6}.$$

Example ii. *If out of every* 10000 *children born in one year* 3540 *die in their first year, what percentage of the number survive?*

$$\text{The number of survivors} = \tfrac{6460}{10000} \text{ of number born}$$
$$= \tfrac{64{\cdot}6}{100} \text{ of number born};$$
$$\therefore \text{rate per cent.} = \underline{64{\cdot}6}.$$

EXERCISE XXXIII.

What per cent.

(1) of £28 is £14?
(2) of £35 is £7?
(3) of £75 is £45?
(4) of £7. 10*s*. is £5?
(5) of £6260 is £313?
(6) of £180 is £6. 15*s*.?
(7) of £450 is £50. 12*s*. 6*d*.?
(8) of £637. 10*s*. is £19. 2*s*. 6*d*.?
(9) of £2784 is £92. 16*s*.?
(10) of £402 is £2. 10*s*. 3*d*.?
(11) of £13750 is £103. 2*s*. 6*d*.?
(12) of £1040. 17*s*. 6*d*. is £312. 5*s*. 3*d*.?

77. Case III.—Given the *percentage* and also the *rate per cent.*, to find the original quantity.

We multiply the percentage by the inverse of the vulgar fraction which the rate per cent. denotes.

Note.—Notice the distinction between *percentage* and *rate per cent.* If, for example, £20 be the given quantity and 10 be the *rate per cent.*, the *percentage* is £2.

Example. Find the amount of which £18 *is* 24 *per cent.*

£18 = $\frac{24}{100}$ × the required amount;

∴ the required amount = £$\frac{100}{24}$ × 18 = £$\frac{300}{4}$ = £75.

EXERCISE XXXIV.

(1) £96 is 6 per cent. of what amount?
(2) £960 is $33\frac{1}{3}$ per cent. of what amount?
(3) £114 is $4\frac{3}{4}$ per cent. of what amount?
(4) £1·04 is 4 per cent. of what amount?
(5) £7. 16*s.* is $\frac{3}{4}$ per cent. of what amount?
(6) £235. 10*s.* is $15\frac{7}{10}$ per cent. of what amount?
(7) £117. 15*s.* is $3\frac{1}{3}$ per cent. of what amount?
(8) £103. 2*s.* 6*d.* is $\frac{3}{4}$ per cent. of what amount?
(9) £2. 10*s.* 3*d.* is $\frac{5}{8}$ per cent. of what amount?
(10) £45. 7*s.* $3\frac{1}{2}$*d.* is $\frac{1}{8}$ per cent. of what amount?

78. The following is an example of a percentage calculation very necessary in business.

Example. An engineer's plant depreciates at the rate of 10 *per cent. per annum. If it costs* £2000, *at what price ought it to be valued at the end of three years?*

Original value = £2000,
1st year's depreciation = £200,
∴ value at end of 1st year = £1800;
2nd year's depreciation = £180,
∴ value at end of 2nd year = £1620;
3rd year's depreciation = £162,
∴ value at end of 3rd year = £1458.

EXERCISE XXXV.

(1) A man died and bequeathed £4275 to his wife and two children. His wife received $33\frac{1}{3}$ per cent.; what amount did each child get?

(2) A house is bought for £2425. Twenty per cent. of the money is paid at once: how much remained?

(3) How many persons are engaged in agriculture, when they constitute 24 per cent. of a population of 30725?

(4) A cask contained 42 gallons, of which 14 gallons escaped; what per cent. was this loss?

(5) A car proprietor had 160 horses in 1884, and 172 in 1885; what was the increase per cent.?

(6) The population of a town was 5600 in 1861, and 4802 in 1871; what per cent. was the decrease?

(7) A man owes £67. 10*s.*, which is 75 per cent. of what he possesses; how much does he possess?

(8) How much did a barrel contain, if 10·08 gallons were 16 per cent. of it?

(9) A man spends 65 per cent. of his income, and saves £434; what is the amount of his income?

(10) The population of a certain town is 12175, and is 25 per cent. more than it was five years ago; what was it then?

(11) If $8\frac{1}{4}$ per cent. of the workpeople in a factory is 792, find the number employed.

(12) The receipts of a certain railway for last week amounted to £16286 against £15991 in the corresponding week of last year; find the increase per cent.

COMMISSION, BROKERAGE, PREMIUM, DISCOUNT FOR CASH.

79. When a person employs an agent to buy or sell property of any kind, or to collect rents, he pays the agent for his trouble by giving him a *percentage* on the money paid or received, or upon the amount of property bought or sold. The money thus paid to the agent is called his **commission.**

The agent employed in buying or selling stocks and shares, or who negotiates business for merchants with regard to ships' cargoes, is sometimes called a **broker,** and the commission **brokerage.**

In the case of an *insurance company*, which, in return for certain payments, undertakes to make good a loss incurred through fire (*fire insurance*), or shipwreck (*marine insur-*

ance), or to pay a certain sum of money after a man's death to his relatives or friends (*life insurance*), the person who insures his property or his life pays annually to the company a certain percentage of the sum which he or his heirs are to receive. The percentage is in this case called a **premium.** The agreement is called a **policy.**

A percentage allowed for ready-money payment for goods bought is called **discount for cash.**

Commission, Brokerage, Premium, and Discount for Cash are therefore names given to a percentage in particular cases.

80. In connection with Discount for Cash it should be remembered that

$1\frac{1}{4}$ per cent. is 3*d.* per £1,	5 per cent. is 1*s.* per £1,
$2\frac{1}{2}$ ..... 6*d.* .....,	$7\frac{1}{2}$ ..... 1*s.* 6*d.* .....,
$3\frac{3}{4}$ ..... 9*d.* .....,	$12\frac{1}{2}$ ..... 2*s.* 6*d.* .....

Example i. *An estate agent buys a house for* £5250, *and receives a commission of* 5*s. per cent.; what does he get?*

$$\text{Commission} = \frac{\frac{1}{4}}{100} \times £5250 = £\frac{5250}{400} = £13\tfrac{1}{8}$$
$$= \underline{£13.\ 2s.\ 6d.}$$

Example ii. *A gentleman pays an annual premium of* £2. 14*s. per cent. on a life insurance policy for* £12550*; what does his annual payment amount to?*

$$\text{Annual payment} = £\frac{2\frac{7}{10}}{100} \times 12550$$
$$= £\frac{27 \times 251}{20} = 6777 \text{ shillings}$$
$$= \underline{£338.\ 17s.}$$

Example iii. *Goods worth* £85. 19*s. are to be insured so that in case of loss both the value of the goods and the premium may be recovered. The premium is at the rate of* £4. 10*s. per cent. For how much must the goods be insured?*

If £100—£4. 10*s.* worth of goods, or £95. 10*s.*, be insured for £100, then, in case of loss, for every £100 received from the insurance company, £4. 10*s.* would be for premium and £95. 10*s.* for value of goods;

$$\therefore \text{Amount of insurance} = £85.\ 19s. \times \frac{100}{95\frac{1}{2}}$$
$$= \underline{£90.}$$

EXERCISE XXXVI.

(1) A broker purchases £20000 worth of goods; what is his commission at $\frac{1}{4}$ per cent.?

(2) An agent collects debts to the amount of £481. 10*s.*; what is his commission at $2\frac{1}{2}$ per cent.?

(3) A broker buys £11000 worth of goods; what is his commission at $\frac{7}{8}$ per cent.?

(4) What is the cost of insuring a cargo valued at £6750, the premium being 3 per cent.?

(5) Find the commission on the sale of 736 barrels of flour at £1. 5*s.* per barrel, the rate of commission being $2\frac{1}{2}$ per cent.

(6) An agent buys land for £10500, and receives a commission of $\frac{1}{4}$ per cent.; what has his employer to pay altogether?

(7) An agent received 5 per cent. for buying cotton. His commission amounted to £432. 10*s.*; what was the value of the cotton bought?

(8) Find the premium on a policy of insurance for £1258. 5*s.*, at $6\frac{1}{4}$ per cent.

(9) A ship is worth £34000, and is insured for $\frac{3}{4}$ of its value; what does the premium amount to at 3 per cent.?

(10) A house is insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}$ per cent., and the premium is £18; what is the value of the house?

(11) For what must I insure goods worth £530. 6*s.* 8*d.*, at $3\frac{1}{4}$ per cent., so that in case of loss I may recover the value of both goods and premium?

(12) For what sum must goods worth £427. 15*s.* 3*d.* be insured, at $4\frac{3}{8}$ per cent., so that in case of loss both their value and the premium paid may be recovered?

(13) At $7\frac{1}{2}$ per cent., for what sum must property worth 1200 guineas be insured, so that, in the event of total loss, both the value of the property and the amount paid for insurance may be recovered?

(14) A tradesman allows a discount of 15 per cent. for cash. Find the cash price of articles marked respectively (i). £2. 15*s.*, (ii). £5. 17*s.* 6*d.*, (iii). £10. 12*s.* 6*d.*

(15) Instead of allowing 4 per cent. discount, a tradesman gives $\frac{1}{2}d.$ in the shilling. Find the amount of his error on a bill of £30. 11*s.* 3*d.*

AVERAGES.

81. If at the end of an ordinary year a person reckons his expenditure during the year, and divides the amount by 365, the quotient gives what is called his **average** daily expenditure.

Again, if a man buys 3 horses for £43, £56 and £38 respectively, he obtains 3 horses for £137, at an *average* price of £$1\frac{37}{3}$, or £45. 13*s*. 4*d*. each.

Again, if a train travels 200 miles in 5 hours, including stoppages, the speed will vary considerably from time to time, but the *average* rate is 40 miles per hour.

Similarly, if a school consists of 340 boys in 17 classes, it is somewhat unlikely that all the classes will be of exactly the same size, but we may say that the *average* number of boys in a class is $\frac{340}{17}$, or 20.

Thus the **average** of any number of quantities is the result of dividing the sum of the quantities by the number of them.

The average is also called the **mean** of the quantities.

Example i. *The rainfall at a certain place in the months of April, May, and June was* 1·9, 0·94, *and* 1·27 *inches respectively. Find the average for these three months, to two places of decimals.*

There are *three* numbers, therefore the average
$= \frac{1}{3}[1{\cdot}9 + 0{\cdot}94 + 1{\cdot}27]$ inches
$= \frac{1}{3}[4{\cdot}11]$ inches
$= 1{\cdot}37$ inches.

Example ii. *In four selected weeks of a year, the deaths in a town of* 140000 *inhabitants are* 125, 85, 110, *and* 100 *respectively. Find the annual number of deaths at the same rate per* 1000 *of the inhabitants.*

The average number of deaths in 1 week $= \frac{1}{4}[125 + 85 + 110 + 100]$
$= \frac{1}{4}[420]$
$= 105$;
$\therefore$ ..... 1 year $= 105 \times 52 = 5460$;
$\therefore$ the annual death rate per 1000 $= 5460 \div 140$
$= 39$

EXERCISE XXXVIIA.

Find the average of

(1) 10, 27, 35, 36.
(2) 7, 9, 12, 25, 17, 34, 50.
(3) 967, 846, 538, 261, 1003.
(4) 102, 104, 46, 206, 308, 410.
(5) 50·413, 2·001, 15·02.
(6) 17·05, 15, 5·03.
(7) ·25, 6·12, 89·4, ·0516.
(8) 349·76, 31·36, 365·005, 2·07.
(9) 35·4, 784·15, 2·05, 91·6.
(10) 31, 35·25, 0, 73·5, 66, 20·75, 147·4, 57·5.

EXERCISE XXXVIIB.

(1) The populations of 3 towns are respectively 16501, 18926, and 30543. What is the average population?

(2) The population of England in 1841 was 14995132, and in 1851 it was 16854142. Find the average annual increase between the two dates.

(3) In a school the numbers for the week were :—Mon. morn., 66; Mon. aft., 64; Tues. morn., 60; Tues. aft., 59; Wed. morn., 75; Wed. aft., 59; Thurs. morn., 68; Thurs. aft., 63; Fri. morn., 62; Fri. aft., 60. Find the average attendance for the week.

(4) During seven weeks the following rainfall occurred: 2·5 in., 1·625 in., 1 in., ·025 in., 1·25 in., ·075 in., ·875 in. Find the average rainfall per week.

(5) The average attendance of a school for a week is 723. The average attendance for the first four days is 731. How many were present on Friday?

(6) In a school of 350 children, one-half are 7 years old, one-fifth are 9 years, one thirty-fifth are 8 years, and the remainder are 11 years. Find the average age.

(7) A class of 25 boys wrote an extract from dictation. 3 boys had 5 mistakes each, 2 had 4 mistakes each, 3 had 2 mistakes each, and 17 boys had no mistake. What was the average number of mistakes for the whole class?

(8) A cyclist travels for $4\frac{1}{2}$ hours at the rate of 7 miles an hour, then for $2\frac{1}{4}$ hours at 10 miles an hour, and finishes his ride at a speed of 12 miles an hour for 15 minutes. Find his average speed per hour.

(9) A train travelled for 15 hours at an average rate of 24 miles per hour; for $3\frac{1}{2}$ hours the rate was 12 miles per hour, and during the next $7\frac{1}{2}$ hours 215 miles were covered. What was the average speed, in miles per hour, for the remainder of the journey?

(10) The weights of a Cambridge crew are 10 st. $5\frac{1}{2}$ lb., 11 st. 2 lb., 12 st. $5\frac{1}{4}$ lb., 13 st. $6\frac{1}{4}$ lb., 14 st. 1 lb., 12 st. 3 lb., 11 st. $3\frac{1}{2}$ lb., 10 st. 13 lb. What is the average weight per man?

(11) The weights of Bow Bells are respectively: 2 tons 13 cwt. 25 lb., 1 ton 14 cwt. 2 qrs. 6 lb., 1 ton 6 cwt. 13 lb., 1 ton 1 cwt. 23 lb., 16 cwt. 4 lb., 13 cwt. 2 qrs. 22 lb., 12 cwt. 7 lb., 10 cwt., 9 cwt. 1 qr. 5 lb., 8 cwt. 3 qrs. 7 lb. Find the average weight.

(12) A school with 78 boys and 72 girls on the books meets 432 times in the year. If each boy loses one meeting in 9, and each girl one in 8, find the average attendance of each sex for the year.

(13) The average dividend paid during 8 years by a railway company is 3·8125 per cent. The dividends paid for the first 7 years were respectively $3\frac{1}{2}$, $3\frac{1}{4}$, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, $4\frac{1}{4}$, 4 per cent. What was the dividend for the 8th year?

(14) The populations of 3 towns in 1861 were respectively 22350, 43425 and 10630. In 1891 the two former had increased 6 per cent. and 8 per cent. respectively, and the latter had decreased 10 per cent. Find the average population in 1891.

STATISTICS.

82. The calculation of averages and percentages enters largely into the compilation of tables of statistics, as will be seen from the examples in the following exercise.

EXERCISE XXXVIII.

(1) In the following table supply the totals required in lines (A) and (B); and state the percentages required in lines (C), (D) and (E), to 2 places of decimals.

Intermediate Education Board for Ireland.

Particulars.	1892.	1893.	1894.	1895.	1896.
No. of students presented for examination:—					
Boys	4294	5265	5816	6267	6503
Girls	1465	1709	1866	2056	2208
(A) Total					
No. of students who passed the examination:—					
Boys	2539	3041	3516	3783	3753
Girls	784	955	1104	1190	1213
(B) Total					
Percentage of candidates who were successful:—					
(C) Boys					
(D) Girls					
(E) Total					

(2) In the following table state the average speed in miles per hour to 2 places of decimals.

Railway Speed.

The longest runs without stoppage are made as under:—

From	To	Time.	Distance.	Average Speed.
		H. M.	Miles.	
Paddington	Exeter	3 43	194	
Euston	Crewe	2 55	158¼	
Liverpool Street	North Walsham	2 40	131	
Newcastle	Edinburgh	2 23	124¼	
St. Pancras	Nottingham	2 23	123¾	
King's Cross	Newark	2 20	120	
Carlisle	Stirling	2 23	117¾	
Bournemouth	Vauxhall	2 12	106¼	
Carlisle	Edinburgh	2 20	98¼	
Carlisle	Kilmarnock	1 49	91½	

(3) In the following table supply the calculations required in the blank columns. State the rates per cent. in the last column to 2 places of decimals.

POPULATION ENUMERATED IN ENGLAND AND WALES AT EACH DECENNIAL CENSUS, 1801–91.

	Population.			Increase.	
	Males.	Females.	Total.	Decennial.	Per cent.
1801	4254735	4637801			
1811	4873605	5290651			
1821	5850319	6149917			
1831	6771196	7125601			
1841	7777586	8136562			
1851	8781225	9146384			
1861	9776259	10289965			
1871	11058934	11643332			
1881	12639902	13334537			
1891	14052901	14949624			

NOTE.—In the calculations required for the last column apply the method of Art. 57.

(4) In the following table calculate the numbers required for the blank columns to the nearest integer.

TOTAL LENGTH, RECEIPTS, AND WORKING EXPENSES OF RAILWAYS IN THE UNITED KINGDOM.

Year.	Length of Lines open on Dec. 31st.	Traffic Receipts.		Working Expenses.	
		Total.	Per Mile.	Total.	Per cent. of Gross Receipts.
		£		£	
1880	17933	62961767		33601124	
1881	18175	63908237		37602616	
1882	18457	66537128		36170436	
1883	18681	68210052		37368562	
1884	18864	67701042		37317197	
1885	19169	66644967		36787957	
1886	19332	66615377		36518247	
1887	19578	67914586		37063266	
1888	19812	69739870		37762107	
1889	19943	73717057		40094116	
1890	20073	76548347		43188556	
1891	20191	78361633		45144778	
1892	20325	78529314		45717965	
1893	20646	76844086		45695119	
1894	20908	79874566		47208313	
1895	21174	81396047		47876637	
1896	21277	85296200		50192424	

(5) In the following table calculate the numbers required in the blank columns; for the 4th and 6th columns find them to the nearest integer.

STATEMENT OF REVENUE AND WORKING EXPENSES OF THE PRINCIPAL RAILWAY COMPANIES OF ENGLAND AND WALES, FOR THE YEAR ENDED 31ST DECEMBER, 1896.

Companies.	Miles in Work.	Revenue.		Working Expenses.		Net Receipts.
		Gross Receipts.	Per Mile.	Amount.	Per cent. on Gross Receipts.	
		£		£		
Cambrian	252	285461		167811		
Furness	134	454185		224704		
Great Central..	383	2563894		1420466		
Great Eastern.	1105	4772509		2698715		
Great Northern	804	4550338		2808968		
Great Western	2542	9610400		5310712		
Lancashire and Yorkshire ...	528	4947766		2771952		
London and North Western	1909	12333655		6754187		
London and South Western	875	4201854		2390885		
London, Brighton, and South Coast	433	2399487		1575877		
London, Chatham, and Dover	188	1591030		855410		
London, Tilbury, and Southend ...	79	307766		169842		
Maryport and Carlisle	41	101576		50399		
Metropolitan ..	66	811770		355779		
Metropolitan District	19	459259		212000		
Midland.....	1387	9662617		5321739		
North Eastern	1630	7751908		4424495		
North London	12	518433		263407		
North Staffordshire	193	806344		429905		
South Eastern	414	2503085		1301366		
Taff Vale	121	770807		416738		
Total of above Railways ...						

INVOICES.

83. Tradesmen's bills may be classed under two or three heads. When a buyer has completed his purchases he may be presented with a bill in the form of a written list of the articles bought, with a statement of the cost of them attached. When goods are sent to a buyer's residence there is sent with them a similar list, which is in this case called an **invoice.** Each separate entry in an invoice or a bill is called an **item.**

When payment is not required at the time of purchase, a list of the amounts owing is sent to the purchaser at certain periods, sometimes without any specification of the nature of the goods. Each line on such a list appears thus :—

		£	s.	d.
June 30.	*To goods, as per invoice*	15	16	8

A periodical statement of this kind is called an **account :** and when, as in many cases may be necessary, the full particulars of the items are given, it is called a **detailed account.** When an account is sent to the purchaser it is said to be **rendered.**

Example. Make out an invoice of the following articles, supplied by Messrs. Parker Brothers, Stutchbury & Co., of St. Paul's Churchyard, to Messrs. Stainer, Despard & Marshall, of Constantinople, on the 1st of June, 1897 *: – 5 yds. of muslin at* 1*s.* 2½*d. per yd.,* 39 *yds. of merino at* 4*s.* 3¼*d. per yd.,* 3 *dozen buttons at* 4¾*d. per dozen,* 198 *yds. of braid at* 1*s.* 3½*d. per dozen yds.*

St. Paul's Churchyard,
London, 1*st June*, 1897.

Messrs. Stainer, Despard & Marshall,
Bought of Parker Brothers, Stutchbury & Co.,
Wholesale and Retail Drapers.

		£.	s.	d.
June 1	5 yds. muslin ..... @1*s.* 2½*d.*		6	0½
	39 yds. merino ..... @4*s.* 3¼*d.*	8	6	6¾
	3 doz. buttons.....@4¾*d.*		1	2¼
	198 yds. braid... @1*s.* 3½*d* per 12 yds.	1	1	3¾
		£9	15	1¼

In an *account*, the above transaction would appear thus:—

St. Paul's Churchyard,
London, *Midsummer*, 1897.

Messrs. Stainer, Despard & Marshall,
Dr. to Parker Brothers, Stutchbury & Co.,
Wholesale and Retail Drapers.

1897		£	*s.*	*d.*
June 1	To goods as per invoice	9	15	1¼

In a *detailed account*, the various items would be stated separately just as in an invoice.

When the account is paid, the person receiving the money writes at the bottom *Received*, with his name and the date of receipt. If the sum received is £2 or upwards, a penny stamp must be affixed to the bill by the receiver, who must write his name and the date across it, thus:—

Received July 4th, 1897.
Parker Bros., Stutchbury & Co.

The following is a specimen of a **shipping invoice**:—

Leeds, March 22nd, 1898.

To Messrs. Stainer & Co.,
From Parker Brothers.

Invoice of 7 packages forwarded to Southampton for shipment per s.s. *Tamar* to Cape Town, for your account and risk.
Net c. f. i.

S.			Per bale.	£	*s.*	*d.*	£	*s.*	*d.*
Cape Town.	3	cases containing 45 bales of cloth, 40 yds.	95/-	213	15	0			
1/3 4/7	4	cases as above, 60 bales	85/-	255	0	0			
							468	15	0
		E. & O. E.[1]					468	15	0

[1] The letters c. f. i. stand respectively for the words "cost," "freight," "insurance." E. & O. E. stand for the words "errors and omissions excepted." The marks in the extreme left-hand column are shipping marks on the packages.

84. A statement relating to goods sold on commission, showing the expenses entailed and the amount realised by the sale, is called an **account sales.**

Example. Make out the account sales of 35 *hogsheads of sugar received by Messrs. Allen & Richards, of Mincing Lane, from Messrs. Reynolds & Co., of Kingston, Jamaica, per the 'Launceston,' to be sold on their account, and sold on the* 31*st of January,* 1898, *as follows:—*4 *hhd., net* 84¼ *cwt., at* 20*s. per cwt.;* 13 *hhd., net* 208 *cwt., at* 20*s.* 6*d. per cwt.;* 11 *hhd., net* 176¾ *cwt., at* 21*s. per cwt.;* 7 *hhd., net* 112½ *cwt., at* 21*s.* 6*d. per cwt.; freight and other charges,* £17. 8*s.; commission on sales,* 2½ *per cent.*

Account Sales of 35 hhd. Sugar *ex* s.s. *Launceston,* on account of Messrs Reynolds & Co., Kingston, Jamaica.

R. & C. H 1/35				£	s.	d.	£	s.	d.
35 hhd.	4	hhd. sugar, net 84¼ cwt....	20/-				84	5	0
	13	„ „ „ 208 „ ...	20/6				213	4	0
	11	„ „ „ 176¾ „ ...	21/-				185	11	9
	7	„ „ „ 112½ „ ...	21/6				120	18	9
		Charges:					603	19	6
		Insurance		2	10	0			
		Freight		42	3	0			
		Cartage		1	10	0			
		Storage		1	5	0			
		Commission, 2½% on £603. 19*s.* 6*d.*		15	2	0			
							62	10	0
		Net proceeds due to-day					541	9	6

E. & O. E.

London, 31st January, 1898.
Allen & Richards.

85. The science of recording business transactions in a regular and systematic manner is called **Book-keeping.** The books necessarily required for the purpose are three :—

(i) The **Waste Book,** in which rough entries of transactions are made previous to their being carried into the *Journal.*

(ii) The **Journal** or **Day Book,** in which a condensed statement of the daily transactions is entered and so arranged that they may be subsequently transferred to the *Ledger.*

(iii) The **Ledger**, in which each entry in the *Journal* is arranged systematically under the name of the person with whom the business was transacted: amounts due *from* him being placed in one column, and amounts due *to* him in another.

EXERCISE XXXIX.

Make out the following invoices, with names and dates, and calculate each amount to the nearest farthing.

(1) 3 doz. whiting at $2\frac{3}{4}d.$ each; 7 score oysters at 2*s.* 6*d.* per doz.; 71 lb. cod at $2\frac{1}{2}d.$ per lb.; 1600 herrings at 2*s.* 9*d.* per 100; 13 chickens at 3*s.* 6*d.* per pair.

(2) 2 chests tea, each $3\frac{1}{2}$ cwt., at 2*s.* 9*d.* per lb.; 2 barrels sugar, each 4 cwt. 56 lb., at £3. 6*s.* 8*d.* per cwt.; 7 bars soap, each $4\frac{1}{2}$ lb., at 3*d.* per lb.; 8 cheeses, each 56 lb., at $8\frac{1}{2}d.$ per lb.; 6 boxes starch, each 14 lb., at $5\frac{3}{4}d.$ per lb.

(3) 2 pieces silk, each $19\frac{3}{4}$ yds., at 4*s.* 3*d.* per yd.; 3 pieces flannel, each 57 yds., at 4*s.* 6*d.* per yd.; 22 sealskin jackets at 11 guineas each; 400 yds. calico at 2*s.* per doz. yds.; $84\frac{1}{4}$ yds. lace at 4*s.* per doz. yds.

(4) 12 hams, 19 lb. each, at $10\frac{1}{4}d.$ per lb.; 3 firkins butter, each $\frac{1}{2}$ cwt., at 1*s.* 7*d.* per lb.; $3\frac{1}{2}$ dozen eggs at 14 for 1*s.*; 39 gall. milk at $1\frac{1}{2}d.$ per pint; $17\frac{3}{4}$ lb. bacon at 10*d.* per lb.

(5) 5 gall. strawberries at $5\frac{1}{2}d.$ per quart; 10 casks apples, each containing 250, at $2\frac{1}{4}d$ per score; 4 boxes oranges, each containing 160, at 6*d.* per dozen; 100 cabbages at $\frac{3}{4}d.$ each; $1\frac{1}{2}$ cwt. potatoes at $1\frac{1}{2}d.$ per lb.

(6) 1 cwt. 23 lb. soap at $2\frac{3}{4}d.$ per lb.; 3 qrs. bacon at £3. 13*s.* per cwt.; $13\frac{1}{2}$ cwt. of sugar at £2. 3*s.* $6\frac{1}{2}d.$ per cwt.; 1 cwt. 2 qrs. rice at 3*d.* per lb.; $29\frac{1}{2}$ lb. cheese at $7\frac{1}{2}d.$ per lb.

(7) 75000 bricks at 17*s.* 6*d.* per 1000; 696 tiles at 1*s.* 3*d.* per doz.; 66 loads of sand at 2*s.* 3*d.* per load; 5 men, 9 hours each for $3\frac{1}{2}$ days, at $7\frac{1}{2}d.$ per hour; 72 ft. of wood at $3\frac{1}{2}d.$ per ft.

(8) $2\frac{1}{2}$ reams blotting paper at $10\frac{1}{2}d.$ per quire; 400 envelopes at 11*d.* per 100; 500 bottles ink at $1\frac{1}{4}d.$ each; 69 rulers at 3 for $2\frac{1}{2}d.$; 1 gross exercise books at 3*s.* 9*d.* per doz.

(9) 8 score eggs at 1*s.* 3*d.* per doz.; 49 lb. rice at 3¼*d.* per lb.; 84 lb. butter at £6. 10*s.* 8*d.* per cwt.; 19 lb. starch at 6¾*d.* per lb.; ¾ cwt. of soda at 1½*d.* per lb.

(10) 156½ yds. flannel at 2*s.* 1½*d.* per yd.; 258 buttons at 3 for 1*d.*; 2 doz. pairs gloves at 2*s.* 7½*d.* per pair; 19 cotton handkerchiefs at 5¾*d.* each; 2 gross tape at 1*s.* 2¼*d.* per doz.

(11) 2 cwt. beans at 1*s.* 3*d.* per stone (14 lb.); 3 quarts peas at 3*s.* per peck; 3¾ qrs. barley at 3*s.* 11½*d.* per bush.; 10 bush. bran at 11*d.* per bush.; 5½ stone of flour at 2*s.* 8*d.* per stone (14 lb.).

(12) 15 dozen sherry at 3*s.* 9*d.* per bottle; 126 bottles port at £3. 12*s.* per doz.; 7 gall. whisky at 4*s.* 6*d.* per qt.; 13 doz. claret at 1*s.* 3*d.* per bottle; 9 bottles brandy at £2. 16*s.* per doz.

(13) 19 dozen buttons at ¾*d.* each; 4 gross balls cotton at 7½*d.* per dozen; 3 pieces calico, each 45 yds., at 5¾*d.* per yd.; 7 pieces linen, each 18 yds., at 2*s.* 5½*d.* per yd.; 4 pieces silk, each 27 yds., at 7*s.* 8½*d.* per yd.

(14) 7 bars soap, each 11½ lb., at 2½*d.* per lb.; 3 firkins butter, each ½ cwt., at 1*s.* 7*d.* per lb.; 1 stone 9 lb. butter at 1*s.* 5½*d.* per lb.; 3 pails lard, each 28 lb., at 6½*d.* per lb.; leg of pork, 13 lb., at 7½*d.* per lb.

(15) 10 lb. 8 oz. bacon at 11*d.* per lb.; 12 oz. suet at 10*d.* per lb.; 19¾ lb. pork at 9*d.* per lb.; 104 eggs at 1*s.* 1½*d.* per dozen; 17½ lb. butter at 1*s.* 3*d.* per lb.

(16) 1000 envelopes at 7½*d.* per 100; 29 order books at 11¾*d.* each; 7½ doz. pencils at 1½*d.* each; 1 gross pen nibs at 6 for 1*d.*; 17 reams foolscap at 6*d.* per quire.

(17) ½ cwt. treacle at 1¾*d.* per lb.; 29 lb. starch at 8½*d.* per lb.; 12½ pints vinegar at 1*s.* 8*d.* per gall.; 5 cwt. rice at 2½*d.* per lb.; 5½ lb. spice at 4*d.* per oz.

(18) 3 Stilton cheese, each 12 lb., at 1*s.* 3*d.* per lb.; 4 hams, each 10½ lb., at 11½*d.* per lb.; ¾ cwt. butter at 1*s.* 1*d.* per lb.; 2 sides bacon, each 45¼ lb., at 8½*d.* per lb.; 100 eggs at 1*s.* 3*d.* per dozen.

(19) 3 salmon, each 21 lb., at 1*s.* 3*d.* per lb.; 500 oysters at 2*s.* 6*d.* per score; 2½ dozen crabs at 9½*d.* each; 3 gall. shrimps at 5½*d.* per pint; 150 herrings at 11½*d.* per dozen.

(20) Ribs of beef, 10 lb. 10 oz., at 10*d.* per lb.; steak, 4 lb. 2 oz., at 1*s.* 2*d.* per lb.; neck of mutton, 4 lb. 8 oz., at 8½*d.* per lb.; veal, 7 lb. 4 oz., at 11*d.* per lb.; bacon, 5 lb. 7 oz., at 1*s.* per lb.

(21) 56 lb. 9 oz. of beef at 10½*d.* per lb.; 75 lb. 6 oz. of mutton at 11½*d.* per lb.; 18 lb. 10 oz. of lamb at 1*s.* 2*d.* per lb.; 45 lb. 8 oz. of veal at 1*s.* 1½*d.* per lb.; 14 lb. 12 oz. of suet at 8½*d.* per lb.

(22) 1500 tons of wrought-iron at £16. 5*s.* per ton; 75 tons of wrought-iron at £26. 10*s.* per ton; 640 tons of iron castings at 14*s.* per cwt.; 45 tons of gun-metal at 11*d.* per lb.; labour, 120 men for 52 weeks at 27*s.* per week; coal, 1200 tons at 13*s.* per ton.

Deduct 5 *per cent. for cash.*

(23) 22500 bricks at £1. 12*s.* per 1000; 135 bushels of lime at 1*s.* 4½*d.* per bushel; 16½ loads of sand at 3*s.* 6*d.* per load; the labour at 9*s.* 6*d.* per 1000 bricks laid; 300 coping stones at 1*s.* 7½*d.* each, including cost of laying.

Deduct 7½ *per cent. for cash.*

(24) 300000 bricks at £1. 15*s.* per 1000; 240 tons of lime at £1. 5*s.* per ton; 670 yds. of gravel at 12*s.* 6*d.* per yd.; 250 yds. of sand at 17*s.* 6*d.* per yd.; cartage of lime at 1*s.* 6*d.* per ton; cartage of sand and gravel at 9*d.* per yd.

Deduct 2½ *per cent. for cash.*

(25) 1 cwt. of indigo at 14*s.* 6*d.* per lb.; 1 ton of cloves at 1*s.* 2*d.* per lb.; 5 cwt. 3 qrs. 18 lb. of spelter at 4½*d.* per lb.; 7 cwt. 1 qr. 14 lb. of block tin at £64 per ton.

Deduct 10 *per cent. for cash.*

(26) 14 tons 3 cwt. of copper at £92. 15*s.* per ton; 12 tons 12 cwt. of spelter at £1. 15*s.* 10*d.* per cwt.; 3 tons 5 cwt. of tin at £3. 18*s.* 9*d.* per cwt.; coal consumed, 14 tons at 14*s.* 3*d.* per ton; loss of metal in casting, $\frac{1}{36}$; labour equal to one man for 31 days at 4*s.* 9*d.*; other expenses reckoned at £125.

Deduct 2½ *per cent. for cash.*

(27) 1 st. 9 lb. of butter at 1*s.* 5½*d.* per lb.; 1 qr. 26 lb. of cheese at £3. 10*s.* per cwt.; 5 chests of tea, each 3½ cwt., at 1*s.* 4½*d.* per lb.; 3 barrels of sugar, each 4½ cwt., at £3. 6*s.* 8*d.* per cwt.; 113 hams, each 19 lb., at 10¼*d.* per lb.; 1 qr. 23 lb. of soap at 2¾*d.* per lb.

Deduct 2½ *per cent. for cash.*

(28) 10 gall. 3 qts. of oil at 1*s.* 2*d.* per gall.; 57 cu. yds. 13½ ft. of granite at £3. 7*s.* 9*d.* per cu. yd.; 3 cwt. 2 qrs. 16 lb. of sugar at £3. 7*s.* 8*d.* per cwt.; 72 yds. 2 ft. 6 in. of wire at 2*s.* 3¼*d.* per yd.; 36 qrs. 4 bush. of corn at £1. 10*s.* 9*d.* per qr.

Deduct 5 *per cent. for cash.*

(29) 3 pieces of West of England cloth, each 20½ yds., at 5*s.* 6*d.* per yd.; 9 rolls Scotch tweed, each 19½ yds., at 4*s.* per yd.; 30 yds. Irish frieze at 8*s.* 6*d.* per yd.; 12 pieces Melton cloth, each 30 yds., at 4*s.* 3*d.* per yd.; 152 yds. Scotch mixture at 2*s.* 6*d.* per yd.

Deduct 7½ *per cent. for cash.*

(30) 27½ yds. of red flannel at 1*s.* 6*d.* per yd.; 4½ doz. yds. of dress material at 4*s.* 11*d.* per yd.; 13¼ yds. of linen at 3*s.* 10*d.* per yd.; 13¾ yds. of velvet at 7*s.* 6*d.* per yd.; 1½ gross reels of cotton at 6½*d.* per doz.; 3 dozen pairs of gloves at 2*s.* 11½*d.* per pair; 198 yds. of braid at 1*s.* 3½*d.* per doz.

Deduct 2½ *per cent. for cash.*

Make out account sales on the following transactions:—

(31) 2606 qrs. of maize at 32*s.* per qr.; dock charges, £15. 9*s.* 3*d.*; freight, £110. 7*s.* 4*d.*; commission, 2½ per cent.

(32) 9000 bags of maize. 2000 at 24*s.* 6*d.*; 2500 at 27*s.* 9*d.*; 3000 at 31*s.* 4*d.*; 1500 at 32*s.* 8*d.*; charges, 2½ per cent.; commission, 5 per cent.

(33) 8 pipes of port and 10 butts of sherry. 5 pipes of port at £60; 2 butts of sherry at £40; 3 pipes of port at £80; 8 butts of sherry at £55; duty, £67. 10*s.*; freight, £80 6*s.* 8*d.*; other charges, 1 per cent.; commission, 5 per cent.

(34) 12 bales of Irish linen containing 320 pieces. 60 pieces at 9*s.* 6*d.*; 120 pieces at 9*s.*; 60 pieces at 10*s.* 6*d.*; 80 pieces at 10*s.* 9*d.*; freight and other charges, £18; commission, 5 per cent.

(35) 59 casks of Russian tallow. 12 casks at 47*s.*; 14 casks at 48*s.*; 15 casks at 46*s.*; 18 casks at 45*s.*; freight, carriage and other charges, £15 10*s.*; commission, 2½ per cent.

(36) 40 hhd. of sugar. 6 hhd., net 92½ cwt., at 20*s.* per cwt.; 12 hhd., net 113¼ cwt., at 20*s.* 4*d.* per cwt.; 15 hhd., net 130¾ cwt., at 21*s.* 3*d.* per cwt.; 7 hhd., net 147 cwt., at 21*s.* 4*d.* per cwt.; charges, ¼ per cent.; commission, 2½ per cent.

SIMPLE INTEREST.

86. Since there are some people with *more* money than they are able to use in their own affairs, and others with *less* money than they need to carry on their business, it is natural for the latter to borrow from the former, and for the former to lend to the latter: and it is fair for the former to demand, and the latter to pay, a *proportionate* charge for the money. This charge is called **Interest**, and should clearly be proportional to the *amount* of the money, and also to the *time* for which it is lent.

It is customary to reckon the charge at so much per £100 for a year, *i.e.*, at a certain **Rate per cent. per annum.**

If, for example, the rate be 5 per cent. per annum, it follows that

the interest on £100 for 1 year $= £5$;
..... £50 ... 1 year $= £2\frac{1}{2}$;
..... £50 ... 3 years $= £3 \times 2\frac{1}{2}$;
..... £50 ... $\frac{3}{4}$ year $= £\frac{3}{4} \times 2\frac{1}{2}$;
..... £150 ... $\frac{3}{4}$ year $= £3 \times \frac{3}{4} \times 2\frac{1}{2}$, etc.

87. The sum of money upon which interest is reckoned is called the **Principal.**

When we add the interest to the principal we get the **Amount**, *i.e.*, the sum to which the principal will amount at the end of a given time. When it is agreed between the lender and the borrower that the interest shall be paid over periodically as soon as it becomes due, the money is said to be lent at **Simple Interest.**

Short Methods (Mental).

88. It should be noticed and remembered that, at 5 per cent., the interest upon £1 for 1 year is *one shilling*; upon 15*s.* it is 9*d.*; upon 10*s.* it is 6*d.*; upon 5*s.*, 3*d.*; and so on; also that the interest upon £1 for 1 month is *one penny.*

At $2\frac{1}{2}$ per cent. the interest is at the rate of *sixpence* per £1 for 1 year.

To find the simple interest on any sum of money at 5 per cent. for one year, we may *mentally* reckon the pounds in the principal as shillings, and the shillings and pence in the principal as the same fraction of a shilling as they are of £1.

Also, to find the simple interest for one month on any sum of money at 5 per cent. per annum, we may *mentally* reckon the pounds in the principal as pence.

To find Simple Interest at the following rates, we may calculate it at 5 (or $2\frac{1}{2}$) per cent., and then increase or decrease it. Thus:—

6 per cent. = 5% + one-fifth;
4 per cent. = 5% − one-fifth;
$5\frac{1}{2}$ per cent. = 5% + one-tenth;
$4\frac{1}{2}$ per cent. = 5% − one-tenth;
3 per cent. = $2\frac{1}{2}$% + one-fifth;
2 per cent. = $2\frac{1}{2}$% − one-fifth.

Example i. *Write down the simple interest on* £562. 17*s*. 6*d*. *for* 1 *year at* 5 *per cent.*

Interest = $562\frac{7}{8}$ shillings
= £28. 2*s*. $10\frac{1}{2}$*d*.

Example ii. *Write down the simple interest on* £500. 10*s*. *for* 1 *month at* 5 *per cent.*

Interest = $500\frac{1}{2}$ pence
= £2. 1*s*. $8\frac{1}{2}$*d*.

EXERCISE XL.

Write down the simple interest for one year on

(1) £617 at 5%. (2) £843 at 5%. (3) £937 at 5%.
(4) £741 at 5%. (5) £586 at 5%. (6) £672 at 5%.
(7) £9500 at $2\frac{1}{2}$%. (8) £350 at $2\frac{1}{2}$%. (9) £650 at $2\frac{1}{2}$%.
(10) £450 at 3%. (11) £575 at 2%. (12) £430 at $5\frac{1}{2}$%.
(13) £200 at 3%. (14) £500 at 6%. (15) £600 at 2%.
(16) £800 at $5\frac{1}{2}$%. (17) £900 at 4%. (18) £1000 at $4\frac{1}{2}$%.
(19) £578. 10*s*. at 5%. (20) £1673. 10*s*. at 5%.
(21) £941. 15*s*. at 5%. (22) £637. 15*s*. at 5%.
(23) £984. 15*s*. at 5%. (24) £1762. 5*s*. at 5%.
(25) £3197. 5*s*. at 5%. (26) £5184. 2*s*. 6*d*. at 5%.
(27) £6287. 2*s*. 6*d*. at 5%. (28) £2814. 7*s*. 6*d*. at 5%.
(29) £5623. 17*s*. 6*d*. at 5%. (30) £5861. 12*s*. 6*d*. at 5%.

Write down the simple interest for one month on

(31) £400 at 5%. (32) £576 at 5%. (33) £600 at 5%.
(34) £360 at 6%. (35) £720 at $5\frac{1}{2}$%. (36) £480 at $4\frac{1}{2}$%.

General Methods.

89. Questions involving interest can easily be worked out by the Unitary Method.

There are four quantities involved in such questions: (1) the *Principal*; (2) the *Time*; (3) the *Rate per cent.*; and (4) the *Interest*.

If any three of these be known, it is always possible to determine the fourth; but we cannot do so with *less* than three.

90. Case I.—Given the Principal, the Time, and the Rate per cent.; **to find the Simple Interest.**

Example i. *Find the simple interest on* £3265. 12*s.* 6*d.* *for* $5\frac{1}{2}$ *years at* 4 *per cent. per annum.*

Interest on £100 for $5\frac{1}{2}$ years $= £5\frac{1}{2} \times 4$,

$\therefore$ ..... £$3265\frac{5}{8}$ for $5\frac{1}{2}$ years $= £3265\frac{5}{8} \times \frac{1}{100} \times 5\frac{1}{2} \times 4$

$= £\frac{26125}{8} \times \frac{1}{25} \times \frac{11}{2}$

$= £\frac{11 \times 1045}{16} = £\frac{11495}{16}$

$= £718.\ 8s.\ 9d.$

Hence it will be seen that *to find the simple interest we multiply the principal by the rate per cent. and then by the number of years, and we divide the product by* 100. These operations may be performed in either of the following ways, but the second method is generally the more convenient:

(i)

£	s.	d.	
3265	12	6	
		4	= rate p.c.
13062	10	0	
		$5\frac{1}{2}$	= no. of yrs.
65312	10	0	
6531	5	0	
10)71843	15	0	
10)7184	7	6	
£718	8	9	

(ii) Since $\frac{4 \times 5\frac{1}{2}}{100} = \frac{11}{50}$, we may proceed thus:

£.	s.	d.
3265	12	6
		11
5)35921	17	6
10)7184	7	6
£718	8	9

The interest therefore $= £718.\ 8s.\ 9d.$

91. If we denote the principal by P, the amount by A, the interest by I, the rate per cent. by r, and the number of years for which interest is calculated by n, the method of finding the simple interest and amount can be represented by the following formulæ:

$$I = \frac{P \times r \times n}{100},$$

and $$A = P + I.$$

EXERCISE XLI.

Find the simple interest on

(1) £245 for 2 years at 3 per cent.
(2) £356 for 3 years at $2\frac{1}{2}$ per cent.
(3) £525 for 5 years at $3\frac{1}{2}$ per cent.
(4) £650 for 8 years at $4\frac{1}{4}$ per cent.
(5) £960 for 12 years at $2\frac{1}{4}$ per cent.
(6) £975 for 4 years at $2\frac{1}{3}$ per cent.
(7) £325. 10*s.* for $2\frac{1}{2}$ years at 4 per cent.
(8) £333. 10*s.* for 20 years at $3\frac{3}{4}$ per cent.
(9) £211. 5*s.* for $13\frac{1}{2}$ years at 4 per cent.
(10) £142. 10*s.* for $4\frac{1}{4}$ years at $3\frac{1}{2}$ per cent.
(11) £774. 11*s.* 3*d.* for 4 years at 5 per cent.
(12) £101. 5*s.* 6*d.* for 20 years at $2\frac{1}{2}$ per cent.
(13) £886. 10*s.* 9*d.* for $4\frac{1}{6}$ years at $5\frac{1}{3}$ per cent.
(14) £936. 13*s.* 4*d.* for $4\frac{7}{8}$ years at $4\frac{5}{6}$ per cent.
(15) £1545 for $5\frac{1}{2}$ years at 4 per cent.
(16) £2763 for $4\frac{1}{2}$ years at 5 per cent.
(17) £5321 for $3\frac{3}{4}$ years at $4\frac{1}{2}$ per cent.
(18) £1465 for $7\frac{1}{4}$ years at $3\frac{1}{2}$ per cent.
(19) £2333. 15*s.* for 3 years at $1\frac{1}{2}$ per cent.
(20) £2367. 10*s.* for 3 years at $3\frac{1}{2}$ per cent.
(21) £1250. 12*s.* 6*d.* for $3\frac{3}{4}$ years at 4 per cent.
(22) £2987. 10*s.* 6*d.* for $3\frac{1}{3}$ years at $2\frac{1}{2}$ per cent.
(23) £1715. 1*s.* 8*d.* for 5 years at 3 per cent.
(24) £5982. 7*s.* 6*d.* for 4 years at $3\frac{3}{4}$ per cent.
(25) £670. 15*s.* for 8 months at 5 per cent.
(26) £3460 for 8 months at $3\frac{1}{2}$ per cent.
(27) £850 for 3 years 3 months at $4\frac{1}{2}$ per cent.
(28) £8240 for 14 months at 3 per cent.
(29) £656. 6*s.* 8*d.* for $2\frac{1}{2}$ months at $7\frac{1}{2}$ per cent.
(30) £106. 13*s.* 4*d.* for 15 months at $4\frac{1}{2}$ per cent.
(31) £1238. 14*s.* $4\frac{1}{2}$*d.* for 6 years 8 months at $1\frac{1}{2}$ per cent.
(32) £7455 for 3 months at $2\frac{1}{2}$ per cent.
(33) £8245 for 4 months at 3 per cent.
(34) £41. 13*s.* 4*d.* for 8 months at $4\frac{1}{2}$ per cent.

Approximation.

92. When the time is a number of days, the interest may be found by what is called the **Third, tenth, and tenth rule.**

Example. Find the simple interest on £543. 17*s.* 6*d. for* 43 *days at* 3½%.

$$\text{Interest} = £543{\cdot}875 \times \frac{43 \times 3\frac{1}{2}}{365 \times 100}$$
$$= £543{\cdot}875 \times \frac{301}{73000}$$
$$= £\frac{163706{\cdot}375}{73000}$$
$$= £2{\cdot}243$$
$$= £2.\ 4s.\ 10\tfrac{1}{4}d.$$

£543·875
301
543875
16316250
163706375
54568791
5456879
545687
2·24277732
·00022128
2·24255304

The divisor can always be made 73000; to obtain the quotient on dividing by 73000, the rule is

To the numerator add $\frac{1}{3}$ *of itself, then* $\frac{1}{10}$ *of this, then* $\frac{1}{10}$ *of this again, and in the result move the point* 5 *places to the left.* This must then be corrected as explained below.

The work is stated above in full, and the rule gives the interest to be £2·24277732. But the rule is only true if

$$\frac{1}{100000}\left(1+\frac{1}{3}+\frac{1}{30}+\frac{1}{300}\right) = \frac{1}{73000},$$

i.e., if $\frac{137}{10000} = \frac{1}{73}$,

i.e., if $73 \times 137 = 10000$,

i.e., if $10001 = 10000$.

The process therefore gives a result too great by $\frac{1}{10000}$ of itself; hence, from the £2·24277732, obtained by the rule, we subtract £·00022123; the remainder is the true interest.

Note.—When the period is given from one date to another, in calculating the number of days we reckon the day on which the money is lent and that on which it is paid as **one** day.

93. When the aggregate interest on different amounts for different numbers of days is required, we multiply each amount by twice the rate and by the number of days, and then divide the sum of these products by 73000.

Example. How much interest is due on the following deposit account: £270 *for* 36 *days*, £500 *for* 33 *days*, £210 *for* 30 *days, and* £390 *for* 18 *days, interest being at the rate of* 3½ *per cent. per annum?*

$270 \times 36 = 9720$
$500 \times 33 = 16500$
$210 \times 30 = 6300$
$390 \times 18 = 7020$
39540
7
$73000 \overline{)\ 276780}$

Interest = £3. 15*s.* 9*d.* by the method of Art. 92.

NOTE.—Time is saved in offices and banks by the use of books containing interest tables. Farthings are never entered in accounts.

The difference between two sides of an account is called a **balance**, and when the interest due is included it is called a **cash balance.**

Sums of money paid into the bank are placed in the *Cr.* (*Creditor*) column, and sums drawn from the bank in the *Dr.* (*Debtor*) column of the Bank Ledger.

Example. Find the cash balance on April 27*th*, 1898, *with interest at the rate of* 3½ *per cent. per annum, on the following account:—Deposited Jan.* 1*st*, £270; *Feb.* 5*th*, £230; *withdrew March* 10*th*, £290; *deposited April* 7*th*, £180.

The following shows the form of the account in the *Deposit Ledger* of the bank.

Date.		Dr.			Cr.			Balance.			Days	Products.
1898		£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*		
Jan. 1	Cash.....	—	—	-	270	0	0	270	0	0	36	9720
Feb. 5	.....	—	—	-	230	0	0	500	0	0	33	16500
Mar. 10	.....	290	0	0	—	—	-	210	0	0	30	6300
Ap. 9	.....	—	—	-	180	0	0	390	0	0	18	7020
„ 27	Interest..	—	—	-	3	15	9	—	—	-		
	Balance..	393	15	9				393	15	9		
		683	15	9	683	15	9					39540

The interest will be found, by the method used above, to be £3. 15*s.* 9*d.*

NOTE.—If the amounts paid in or withdrawn are expressed in pounds, shillings, and pence, decimalize the amounts before finding the products.

EXERCISE XLII.

Find the simple interest, to the nearest penny, on

(1) £6870 for 73 days at $3\frac{1}{2}$ per cent.

(2) £2160. 12*s*. 6*d*. for 1 year 73 days at 5 per cent.

(3) £533. 6*s*. 8*d*. for 146 days at 3 per cent.

(4) £183. 6*s*. 8*d*. for 1 year 146 days at $4\frac{1}{4}$ per cent.

(5) £2294. 18*s*. 9*d*. for 2 years 115 days at 5 per cent.

(6) £535 for 117 days at $4\frac{3}{4}$ per cent.

(7) £232. 7*s*. $8\frac{1}{2}$*d*. for 1 year 214 days at $3\frac{1}{6}$ per cent.

(8) £1271. 4*s*. $5\frac{1}{4}$*d*. for 327 days at $7\frac{3}{8}$ per cent.

(9) £568. 6*s*. 3*d*. for 3 years 143 days at $5\frac{3}{5}$ per cent.

(10) £1472. 15*s*. $6\frac{1}{2}$*d*. for 3 yrs. 281 days at £3. 1*s*. 9*d*. p.c.

(11) £300. 8*s*. for 2 years 321 days at £4. 11*s*. 8*d*. p.c.

(12) £1236. 18*s*. 4*d*. for 3 years 183 days at $3\frac{1}{4}$ per cent.

(13) £5609. 10*s*. $9\frac{1}{2}$*d*. for 1 year 301 days at 4 per cent.

(14) £5076. 9*s*. $10\frac{1}{4}$*d*. for 3 years 247 days at $4\frac{3}{4}$ p.c.

(15) £364. 3*s*. 9*d*. from March 5th to July 20th at 4 p.c.

(16) £2105. 12*s*. 7*d*. from January 1st, 1895, to August 14th, 1897, at $3\frac{1}{2}$ per cent.

(17) £329. 13*s*. 3*d*. from February 2nd, 1896, to July 14th, 1897, at $2\frac{1}{4}$ per cent.

(18) £1325. 7*s*. 10*d*. from May 12th, 1895, to January 26th, 1897, at $3\frac{3}{4}$ per cent.

(19) £2503. 18*s*. 5*d*. from January 20th, 1896, to March 30th, 1899, at 4 per cent.

(20) £850. 12*s*. 6*d*. from March 12th, 1896, to December 29th, 1897, at 4 per cent.

(21) Find the cash balance on December 31st, 1897, of the following account; interest 4 per cent.:—

	£		£
1 May, deposited	250	8 June, withdrew.....	124
28 June, „	140	19 July, „	230
11 Aug., „	340	20 Oct., „	150
12 Nov., „	221	12 Dec., „	100

(22) Find the cash balance on December 31st, 1897, of the following account; interest 5 per cent.:—

	£		£
7 Jan., deposited	210	14 April, withdrew ...	130
7 Mar., „	150	27 June, „ ...	215
8 May, „	240	13 Aug., „ ...	167
21 July, „	300	12 Oct., „ ...	280
18 Sept., „	250	18 Nov., „ ...	120
24 Dec., „	160		

94. CASE II.—Given the Interest (or the Amount), the Time, and the Rate per cent.; **to find the Principal.**

Example i. *What sum will amount to* £300. 16*s.* *in* 4 *years at* 5 *per cent.?*

Amount of £100 in 4 years at 5 per cent. $=$£120,
$\therefore$ principal which in 4 yrs. will amount to £120 $=$£100,
$\therefore$ ..... £1 $=$£$\frac{100}{120}=$£$\frac{5}{6}$,
$\therefore$ ..... £300$\frac{1}{5}$ $=\frac{5}{6}\times$£300. 16*s.*
$=\frac{1}{6}\times$£1504
$=$£250. 13*s.* 4*d.*

Example ii. *What sum will produce* £113. 8*s.* *interest in* $4\frac{1}{2}$ *years at* $4\frac{1}{2}$ *per cent.?*

Principal which produces £$4\frac{1}{2}$ in 1 yr. $=$£100,
$\therefore$ ..... £$113\frac{2}{5}$ in $4\frac{1}{2}$ yrs. $=$£$\frac{113\frac{2}{5}}{4\frac{1}{2}}\times\frac{1}{4\frac{1}{2}}\times 100$
$=$£$\frac{567}{5}\times\frac{2}{9}\times\frac{2}{9}\times 100$
$=$£7×80
$=$£560.

EXERCISE XLIII.

What principal will amount to

(1) £411. 5*s.* in 5 years at $3\frac{1}{2}$ per cent.?
(2) £814. 10*s.* in $5\frac{1}{4}$ years at $2\frac{1}{2}$ per cent.?
(3) £188. 2*s.* 6*d.* in $1\frac{1}{2}$ years at 5 per cent.?
(4) £424. 2*s.* 3*d.* in $3\frac{1}{4}$ years at $4\frac{1}{2}$ per cent.?
(5) £350. 19*s.* $4\frac{1}{2}$*d.* in $6\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.?
(6) £323. 11*s.* $5\frac{1}{2}$*d.* in $7\frac{1}{7}$ years at $3\frac{1}{2}$ per cent.?
(7) £5431. 15*s.* $11\frac{1}{4}$*d.* in 6 years at $4\frac{1}{4}$ per cent.?
(8) £1246. 13*s.* 4*d.* in $7\frac{1}{2}$ years at 5 per cent.?

What principal will produce as interest

(9) £551. 18*s.* $6\frac{3}{4}d.$ in $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.?
(10) £344. 16*s.* $8\frac{1}{4}d.$ in $2\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.?
(11) £908. 11*s.* $6\frac{3}{4}d.$ in $4\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.?
(12) £529. 18*s.* $5\frac{1}{4}d.$ in $3\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.?

95. CASE III.—Given the Principal, Rate per cent., and Interest (or Amount); **to find the Time.**

If we find the interest on the Principal for 1 year, and determine how many times this interest is contained in the given interest, the quotient will be the number of years required.

Example i. *In what time will the interest on* £556. 13*s.* 4*d.* *amount to* £167 *at* 5 *per cent.?*

Interest on £556. 13*s.* 4*d.* for 1 yr. at 5% = £27. 16*s.* 8*d.*;
and ..... req^{d}. time = £167;

$$\therefore \text{ required time} = \frac{167}{27\frac{5}{6}} \text{ years}$$
$$= \underline{6 \text{ years.}}$$

Example ii. *In what time will* £2833. 6*s.* 8*d.* *amount to* £3215. 16*s.* 8*d.* *at* 3 *per cent.?*

Amount of £2833. 6*s.* 8*d.* in req^{d}. time = £3215. 16*s.* 8*d.*;
$\therefore$ interest on ..... = £382. 10*s.* 0*d.*;
but interest on £2833. 6*s.* 8*d.* for 1 yr. at 3% = £85;

$$\therefore \text{ required time} = \frac{382\frac{1}{2}}{85} \text{ years}$$
$$= \tfrac{765}{170} \text{ years}$$
$$= \underline{4\tfrac{1}{2} \text{ years.}}$$

EXERCISE XLIV.

In what time will

(1) £350 amount to £700 at 4 per cent.?
(2) £670 amount to £892. 15*s.* 6*d.* at $4\frac{3}{4}$ per cent.?
(3) £540 amount to £712. 16*s.* at 4 per cent.?
(4) £225 amount to £256. 10*s.* at 4 per cent.?
(5) £2700 amount to £3219. 15*s.* at $2\frac{3}{4}$ per cent.?
(6) £2175 amount to £2338. 2*s.* 6*d.* at $2\frac{1}{2}$ per cent.?
(7) £3745 amount to £3832. 5*s.* at $2\frac{1}{2}$ per cent.?

(8) £1260 amount to £1496. 5*s.* at $3\frac{3}{4}$ per cent.?
(9) £1650 amount to £1891. 6*s.* 3*d.* at $4\frac{1}{2}$ per cent.?
(10) £4500 amount to £5141. 5*s.* at 3 per cent.?
(11) £4550 amount to £6597. 10*s.* at 6 per cent.?
(12) £4550 amount to £5573. 15*s.* at 3 per cent.?
(13) £1300 amount to £1493. 7*s.* 6*d.* at $3\frac{1}{2}$ per cent.?
(14) £1637. 16*s.* 8*d.* amount to £2129. 3*s.* 8*d.* at $3\frac{3}{4}$ p.c.?
(15) £2151. 6*s.* 8*d.* amount to £2366. 9*s.* 4*d.* at $2\frac{1}{2}$ p.c.?
(16) £256. 6*s.* 8*d.* amount to £352. 9*s.* 2*d.* at $7\frac{1}{2}$ p.c.?
(17) £3125 amount to £3681. 12*s.* $9\frac{3}{4}$*d.* at $3\frac{3}{4}$ per cent.?
(18) £2977. 18*s.* 4*d.* amount to £3446. 18*s.* $9\frac{1}{4}$*d.* at $3\frac{3}{4}$ p.c.?

96. Case IV.—Given the Principal, Time, and Interest (or Amount); **to find the Rate per cent.**

If we find the interest on the Principal at the rate of 1 per cent. for the given time, and determine how many times this is contained in the given interest, the quotient will be the rate per cent. required.

Example. At what rate per cent. will £1260 *amount to* £1496. 5*s. in* 3 *years* 9 *months?*

Interest on £1260 for $3\frac{3}{4}$ years at 1 per cent. = £47. 5*s.*;
and..... req^{d}. rate = £236. 5*s.*;

$$\therefore \text{ required rate per cent.} = \frac{\text{£236. 5s.}}{\text{£47. 5s.}} = 5.$$

EXERCISE XLV.

At what rate per cent will

(1) £500 amount to £567. 10*s.* in $1\frac{1}{2}$ years?
(2) £500 amount to £625 in 3 years?
(3) £325 amount to £379. 3*s.* 4*d.* in 5 years?
(4) £500 amount to £755 in 12 years?
(5) £900 amount to £1120. 10*s.* in 7 years?
(6) £250 amount to £302. 10*s.* in 2 years?
(7) £1650 amount to £1784. 1*s.* 3*d.* in $2\frac{1}{2}$ years?
(8) £326 amount to £546. 1*s.* in 15 years?

(9) £1250 amount to £1962. 10*s.* in 6 years?

(10) £3825 amount to £4972. 10*s.* in 3 years?

(11) £3745 amount to £3932. 5*s.* in 2 years?

(12) £345 amount to £515. 15*s.* 6*d.* in 11 years?

(13) £1315 amount to £1545. 2*s.* 6*d.* in $3\frac{1}{2}$ years?

(14) £461. 10*s.* amount to £611. 9*s.* 9*d.* in 13 years?

(15) £2416 amount to £3050. 4*s.* in 5 years?

(16) £325. 16*s.* 8*d.* amount to £331. 18*s.* $10\frac{1}{4}d.$ in 5 months?

(17) £1035 amount to £1071. 4*s.* 6*d.* in $10\frac{1}{2}$ months?

(18) £3695. 15*s.* amount to £3957. 10*s.* $7\frac{3}{4}d.$ in 16 months?

97. Using the symbols given in Art. 91, we may represent the processes explained in Arts. 94–96 by the following formulæ:

$$P=\frac{100\times I}{r\times n};\quad n=\frac{100\times I}{P\times r};\quad r=\frac{100\times I}{P\times n}.$$

SQUARE ROOT.

98. The **square root** of a given number is that number which, when multiplied by itself, produces the given number.

We know, for example, that $3^2=9$. Hence 9 is the *square* of 3, and 3 is the *square root* of 9.

99. The square root of a number is denoted by the symbol $\surd$ placed before the number, as $\surd 9=3$.

The student will find it convenient gradually to make himself acquainted with the following, which he may easily verify.

The square root of

1 is 1	36 is 6	121 is 11	256 is 16	441 is 21
4 ... 2	49 ... 7	144 ... 12	289 ... 17	484 ... 22
9 ... 3	64 ... 8	169 ... 13	324 ... 18	529 ... 23
16 ... 4	81 ... 9	196 ... 14	361 ... 19	576 ... 24
25 ... 5	100 ... 10	225 ... 15	400 ... 20	625 ... 25

THE SQUARE ROOT OF AN INTEGER.

100. When a number can be easily separated into prime factors, its square root may be determined by inspection. Thus

$$7056 = 2^4 \times 3^2 \times 7^2,$$
$$\therefore \sqrt{7056} = 2^2 \times 3 \times 7 = 84.$$

In a number which has an exact square root, *every prime factor which occurs at all must occur raised to an* **even** *power.*

EXERCISE XLVI.

Find, by factors, the square root of

(1)	36.	(2)	900.	(3)	1764.
(4)	7056.	(5)	11025.	(6)	17424.
(7)	53361.	(8)	63504.	(9)	99225.
(10)	122500.	(11)	148225.	(12)	245025.
(13)	480249.	(14)	571536.	(15)	680625.
(16)	2480625.	(17)	12446784.	(18)	18593344.
(19)	22325625.	(20)	300155625.		

101. Again, since $\sqrt{100} = 10$, and $\sqrt{10000} = 100$, and $\sqrt{1000000} = 1000$, etc., it follows that the square roots of all numbers between 100 and 10000 lie between 10 and 100, and consist of *two* digits; that the square roots of all numbers between 10000 and 1000000 lie between 100 and 1000, and consist of *three* digits, and so on.

Hence we see that *two* additional digits in a number lead to *one* additional digit in the square root.

If therefore we mark off the digits of a number in periods of *two*, beginning at the right hand, the number of these periods will be the same as the number of digits in the square root.

The left-hand period will sometimes contain only *one* digit.

102. Let us now consider the number 1444, obtained by multiplying 38 by 38. Its square root is 38.

In extracting the square root we determine first the number of *tens*, and then the number of *units*.

If we write 38 in the form $30+8$, and multiply it by $30+8$, thus,

$$\begin{array}{rl} & 30+8 \\ & 30+8 \\ \hline & 30\times 8+8^2 \\ 30^2+ & 30\times 8 \\ \hline 30^2+2\times 30\times 8+8^2 & \end{array}$$

we see that $1444=30^2+(2\times 30+8)\times 8$.

We can find the number of *tens* in the square root by ascertaining what multiple of 10 has its square *next less than* 1444; this is clearly 30, for $40^2=1600$ and is too great.

Having found the *tens*, we next find the *units*.

Subtracting 30^2 or 900 from 1444, the remainder is 544; hence $544=(2\times 30+8)\times 8$.

If we could divide 544 by $2\times 30+8$, we should get the number of units 8 at once. But, since the divisor itself involves this unascertained number 8, the plan adopted is to use the part 2×30 as a **trial** divisor, to find by means of it a **trial** quotient, and then to see whether

$$(2\times 30+\text{trial quotient})\times \text{trial quotient}=544.$$

The first trial quotient may not prove correct; if too great, we try the number next less, and so on.

In this particular case, if we divide 544 by 2×30, *i.e.*, by 60, we get a trial quotient 9. But $(60+9)\times 9=621$, and is too great; we therefore try 8, and we find that $(60+8)\times 8=544$.

The digits of the square root 38 are thus found in succession.

The above operations may be stated concisely thus:—

$$\begin{array}{r|l} 14\,44 & 30+8, \\ \underline{900} & \\ 544 & \\ \underline{544} & \end{array} \qquad \text{or thus} \qquad \begin{array}{r|l} 14\,44 & 38 \\ \underline{9} & \\ 544 & \\ \underline{544} & \end{array}$$

$$60+8 \qquad\qquad 68$$

In the latter form, which is the customary one, the process is briefly as follows:—

(i) We mark off the digits in twos, beginning at the right hand.

(ii) The greatest square in 14 is 9, and its square root is 3. We place 3 in the root place. We multiply 3 by 3, and subtract the product from 14. The remainder is 5.

(iii) To this remainder we annex the period 44, and our dividend becomes 544. Twice the root digit 3 is 6, so we put 6 in the divisor.

(iv) Instead of 60 we take 6 as a *trial divisor*, and we take 54 as a *trial dividend*. Proceeding as explained above, we find that 8 is the true quotient. We annex 8 to the 3 in the root place, and also to the divisor. We multiply 68 by 8 and, subtracting the product from 544, we get no remainder.

The square root is thus 38.

103. In this way a square root can be obtained, however many digits the number may have.

Example. Find the square root of 56644.

We first mark off the digits of 56644 in periods of *two*, beginning at the right hand.

There are 3 periods, and therefore 3 digits in the square root. The steps are as follows:—

(i) The root nearest to that of the first period 5 is 2. The square of 2 is 4. Subtracting 4 from 5, the remainder is 1. We bring down the second period, and we get 166 for the next dividend.

(ii) The *trial* divisor is 2×2 or 4, and the *trial* dividend is 16. The true quotient is 3. We place 3 as the second digit in the root, and we also annex 3 to the trial divisor.

(iii) In the next step the *trial* divisor is 46, obtained by doubling the 23 in the root; and the *trial* dividend is 374.

The work necessary may be written thus:—

$$\begin{array}{r|l} 2\,|\,5,66,44 & 238 \\ \quad|\,4 & \\ \hline 43\,|\,166 & \\ \quad|\,129 & \\ \hline 468\,|\,3744 & \\ \quad|\,3744 & \\ \hline \end{array}$$

104. If a, b, c, d, e stand for the units, tens, hundreds, etc., respectively in any number, then

$$\begin{aligned}(e+d+c+b+a)^2 = e^2 &+ (2e+d)\times d \\ &+ \{2(e+d)+c\}\times c \\ &+ \{2(e+d+c)+b\}\times b \\ &+ \{2(e+d+c+b)+a\}\times a,\end{aligned}$$

a form which leads directly to the methods of Arts. 102 and 103.

EXERCISE XLVII.

Find the square root of

(1)	361.	(2)	529.	(3)	1681.
(4)	3481.	(5)	6889.	(6)	8649.
(7)	14161.	(8)	22201.	(9)	22801.
(10)	64009.	(11)	146689.	(12)	226576.
(13)	249001.	(14)	253009.	(15)	375769.
(16)	546121.	(17)	1261129.	(18)	1121481.
(19)	14837904.	(20)	39740416.	(21)	22099401.
(22)	64432729.	(23)	72471169.	(24)	10588516.

THE SQUARE ROOT OF A DECIMAL.

105. It follows from the nature of decimals that the method just explained is applicable to pure and to mixed decimals in exactly the same way as to integers. One or two points however must be noticed.

(i) We mark off the periods not from the *end* of the decimal, but *from the decimal point* both ways, right and left, thus—3,21,54,8.

(ii) We put a decimal point in the square root immediately before the digit obtained from using the first period in the decimal part of the number.

(iii) If the given decimal has an *odd* number of decimal places, we annex a zero in order to make an exact number of pairs; and we may then annex as many additional pairs of zeros as we please.

(iv) If the square root of an integer does not come out exactly, we may approximate to it by affixing a decimal point and zeros, and then continuing the process.

Example i. *Find the square root of* 1127·6164.

```
3 | 11,27,·61,64 | 33·58
  |  9
63 | 227
   | 189
665 | 3861
    | 3325
6708 | 53664
     | 53664
```

Before we bring down the period 61, which comes next after the decimal point, we put a point in the root.

The required square root is 33·58.

Example ii. *Find the square root of* ·000625.

```
2 ·00,06,25 | ·025
  |  4
 45 | 225
    | 225
```

The first period ·00 gives ·0 in the root, and the second 06 gives the digit 2.

The square root is ·025.

Example iii. *Find the square root of* 321·73025.

```
1 | 3,21,·73,02,50,00,00 | 17·9368...
  | 1
27 | 221
   | 189
349 | 3273
    | 3141
3583 | 13202
     | 10749
35866 | 245350
      | 215196
358728 | 3015400
       | 2869824
         145576
```

NOTE. The operations may be stated also in the abbreviated form described in Art. 32.

```
1 | 321·73025 | 17·9368...
27 | 221
349 | 3273
3583 | 13202
35866 | 245350
358728 | 3015400
         145576
```

We affix a 0 to make the number of decimal places even.

When we have brought down the last period, we find that there is still a remainder. We may then annex pairs of zeros, and continue the process as long as we please.

We may notice here that if a decimal, in its simplest form, has an *odd* number of digits, its square root can never be obtained exactly.

Example iv. *Find the square root of* $1\cdot0\dot{3}1\dot{2}$.

We write the number thus, 1·0312312312..., and proceed as before.

106. When we have to extract a square root to a large number of decimal places, and we have found it to one more than half this number, we may find the remaining digits by dividing the last remainder by the last divisor as in Art. 57. But since we cannot rely upon the correctness of the *last* digit thus obtained, we should determine two digits more than we want.

Example. Find the square root of 321·73025 *to* 12 *places.*

(i)

```
1 321·73025 17·936840580213...
  1
27 221
   189
349 3273
    3141
3583 13202
     10749
35866 245350
      215196
358728 3015400
       2869824
3587364 14557600
        14349456
358736805 20814400 00
          17936840 25
3587368108 2877559 7500
           2869894 4864
 358736811602 7665 26360000
              7174 73623204
 3587368116041 490 5273679600
               358 7368116041
35873681160423 131 790556355900
                107 621043481269
                  24 169512874631
```

(ii)

```
1 321·73025 17·93684058021
  1
27 221
   189
349 3273
    3141
3583 13202
     10749
35866 245350
      215196
358728 3015400
       2869824
3587364 14557600
        14349456
35,8,7,3,6,8,0 20814400
               17936840
                2877560
                2869894
                   7666
                   7175
                    491
                    359
                    132
                    107
                     25
```

It will be seen that the figures to the left of the vertical line in (i), if in each line we make the usual allowance for the first rejected figure, are nearly what we should have got by an approximate division as in Art. 57. The shortened work appears as in (ii).

The work may be shortened still further by writing down *remainders only* as in the abbreviated method of Art. 32.

EXERCISE XLVIII.

Find the square root of

(1)	1·96.	(2)	86·49.
(3)	1·4161.	(4)	331·24.
(5)	12·0409.	(6)	1015·6969.
(7)	170·3025.	(8)	8·678916.
(9)	100·861849.	(10)	144·600625.
(11)	8264·446281.	(12)	26·04469156.
(13)	·11854249.	(14)	·042849.
(15)	·00000961.	(16)	·00010201.
(17)	·00047961.	(18)	·0000001849.
(19)	·0000021904.	(20)	·0000063001.
(21)	2 to 4 places.	(22)	5 to 4 places.
(23)	12 to 4 places.	(24)	13 to 4 places.
(25)	1·6 to 4 places.	(26)	·4 to 4 places.
(27)	·7 to 4 places.	(28)	·002 to 4 places.

THE SQUARE ROOT OF A VULGAR FRACTION.

107. We know that $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, and generally that the square of any vulgar fraction is also a fraction, whose numerator is the square of the given numerator, and denominator the square of the given denominator.

Conversely, the square root of $\frac{4}{9}$ is $\frac{2}{3}$; and generally, whenever the numerator and denominator are both squares, the root of the fraction is a fraction whose numerator is the root of the original numerator, and denominator the root of the original denominator.

The fraction whose square root is required must first be reduced to its lowest terms.

Example. Find the square root of $\frac{49}{256}$.

$$\sqrt{\frac{49}{256}} = \frac{\sqrt{49}}{\sqrt{256}} = \frac{7}{16}.$$

108. To determine the square root of a mixed number, we express the number in the form of an improper fraction, and, if the denominator is a square number, we proceed as in Art. 107.

Example. Find the square root of $20\frac{1}{4}$.

$$\sqrt{20\frac{1}{4}} = \sqrt{\frac{81}{4}} = \frac{9}{2} = \underline{4\frac{1}{2}}.$$

109. When the denominator is not a square number, we proceed as follows:—

Example. Find the square root of $\frac{7}{2}$.

(i) $\sqrt{\frac{7}{2}} = \sqrt{3{\cdot}5} = \underline{1{\cdot}87\ldots}$ (ii) $\sqrt{\frac{7}{2}} = \sqrt{\frac{14}{4}} = \frac{1}{2}\sqrt{14} = \underline{1{\cdot}87}\ldots$

EXERCISE XLIX.

Find the square root of

(1)	$\frac{121}{169}$.	(2)	$\frac{256}{841}$.	(3)	$\frac{70561}{6}$.	(4)	$\frac{4504}{2048}$.
(5)	$\frac{38642}{81752}$.	(6)	$1\frac{7}{9}$.	(7)	$39\frac{1}{16}$.	(8)	$210\frac{1}{4}$.
(9)	$37\frac{36}{49}$.	(10)	$2352\frac{1}{4}$.	(11)	$65\frac{64}{81}$.	(12)	$269\frac{4}{49}$.
(13)	$4\frac{213}{289}$.	(14)	$4\frac{93}{529}$.	(15)	$9\frac{352}{841}$.	(16)	$37\frac{212}{841}$.

APPLICATIONS OF SQUARE ROOT.

110. The following are examples of problems involving the application of square root.

Example i. *Find the number of yards in the side of a square park containing* 109 *ac.* 3 *ro.* 8 *per.* 9 *sq. yds.*

We first reduce the area to sq. yards.

109 ac. 3 ro. 8 per. 9 sq. yds.

4

439

40

17568

$30\frac{1}{4}$

527040

4392

9

531441 sq. yds.

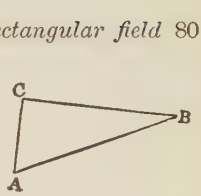
7|53,14,41|729
49
142|414
284
1449|13041
13041

Area of park = 531441 sq. yds.,

∴ length of side = $\sqrt{531441}$ yds.

= $\underline{729 \text{ yards.}}$

Example ii. *Find the diagonal of a rectangular field* 80 *yds. long and* 60 *yds. wide.*



If, in the triangle ACB, the angle ACB be a right angle, we know from Euclid I. 47, that the square described upon AB is equal to the sum of the squares described upon AC and BC.

Hence diagonal $= \sqrt{60^2 + 80^2}$ yds.
$= \sqrt{3600 + 6400}$ yds.
$= \sqrt{10000}$ yds.
$= 100$ yds.

EXERCISE L.

(1) The area of a square garden is 44100 square feet; find the length of a side in yards.

(2) The area of a square field is 40 acres; find the length of a side in yards.

(3) The area of a square field is $122\frac{1}{2}$ acres; find the length of a side in yards.

(4) How many yards of fencing are required to enclose a square park containing 832 ac. 2 ro. 25 per.?

(5) A square field has an area of $2\frac{1}{2}$ acres. What would it cost to fence it round at $6\frac{1}{2}d.$ per foot?

(6) Calculate the cost of enclosing a square bowling-green containing 4 ac. 26 per. $17\frac{1}{2}$ yds., with wire fence at $2s.$ $6d.$ per yd.

MENSURATION OF RECTANGLES.

111. Lengths are measured with reference to a standard unit which is called a **yard**.

112. The principal measure of **area** is a square which has each of its sides equal to the principal measure of length, one yard. It is therefore called a **square yard.**

113. A **rectangle** is a four-sided figure whose opposite sides are parallel, and whose angles are right angles.

The floor, ceiling and walls of a room are generally rectangular.

114. Suppose that a rectangular surface is 4 yds. long and 3 yds. wide; we can divide the sides into 4 equal parts and 3 equal parts respectively, each part being 1 yard. If then through the points of division we draw lines parallel to the sides in the manner shown in the margin, it is clear that we divide the surface into 4 rows of smaller areas, and that there are 3 of the smaller areas in each row. Moreover, each of the smaller areas is a square yard, and the area of the surface is therefore 3×4 square yards.

Hence, to find the number of *square yards* in a rectangular area, we multiply the number of *yards* in the length by the number of *yards* in the breadth. If the length and breadth be given in *feet* or in *inches*, the product will give the number of *square feet* or *square inches*.

We must take care that the length and breadth are expressed in measures of the same name; both in yards, or both in feet, or both in terms of some other measure: the product will then give the area in terms of the *corresponding* square measure.

115. Using the words *length*, *breadth*, and *area* to denote the *number of units* in each respectively, the relation between them may be stated thus:—

$$\text{length} \times \text{breadth} = \text{area};$$

therefore, $$\text{length} = \text{area} \div \text{breadth},$$

and $$\text{breadth} = \text{area} \div \text{length};$$

hence, if any two be given, we can always find the third.

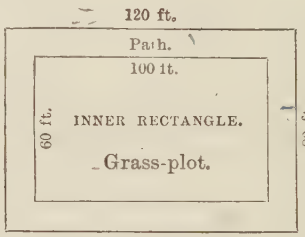
The length and the area, or the breadth and the area, must be expressed respectively in feet and square feet, or in yards and square yards, or in some other measure of length and the square measure that corresponds to it.

NOTE.—Artificers generally use the term *width* in preference to *breadth*.

Example i. *Find the area of the floor of a room* 12 *ft.* 8 *in. long and* 10 *ft.* 9 *in. wide.*

Length of room $= 12\frac{2}{3}$ ft. $= \frac{38}{3}$ ft.,
width ..... $= 10\frac{3}{4}$ ft. $= \frac{43}{4}$ ft.;
$\therefore$ area ..... $= \frac{38}{3} \times \frac{43}{4}$ sq. ft. $= 81\frac{7}{6}$ sq. ft.
$= 136$ sq. ft. 24 sq. in.

Example ii. *A rectangular courtyard measures* 120 *ft. by* 80 *ft. In the centre is a rectangular grass-plot, and outside this is a path* 10 *ft. wide, which is covered with square tiles* 6 *in. long. How many tiles are there?*



To obtain the length and breadth of the inner rectangle, *twice* the width of the path must be taken from the length and breadth of the courtyard.

Area of outer rectangle
$= 120 \times 80$ sq. ft.
$= 9600$ sq. ft.;
area of inner rectangle
$= 100 \times 60$ sq. ft.
$= 6000$ sq. ft.;
$\therefore$ area of path
$= (9600 - 6000)$ sq. ft.
$= 3600$ sq. ft.

But area of one tile $= (\frac{1}{2} \times \frac{1}{2})$ sq. ft.;
$\therefore$ number of tiles $= 3600 \div \frac{1}{4} = 14400.$

116. In measuring **land**, surveyors use a **chain**, called **Gunter's chain**, which is 22 yards long. This chain is divided into 100 equal parts, each of which is called a **link**. Each link therefore measures 7·92 inches.

The area of land is often estimated in **square links** and **square chains**, which are connected with the *square yard* through their linear relations.

Thus 1 *chain* $= 22$ *yards;*
$\therefore$ 1 *square chain* $= 22 \times 22$ *square yards*
$= 484$ *square yards;*
$\therefore$ 10 *sq. chains* $= 4840$ *square yards;*
$\therefore$ 10 sq. chains = 1 acre.

Square chains can therefore be expressed in acres by dividing by 10, or by moving the decimal point one place to the left.

Also 1 *sq. chain* $= 100 \times 100$ *sq. links;*
$\therefore$ 100000 sq. links = 1 acre.

Square links can therefore be expressed in acres by dividing by 100000, or by moving the decimal point five places to the left.

Example. Find the acreage of a rectangular field 6 *chains* 25 *links long, and* 5 *chains* 40 *links broad.*

Area = (6·25 × 5·4) sq. chains
= 33·75 sq. chains
= 3·375 acres
= 3 acres 1 rood 20 perches.

EXERCISE LI.

Find the area of a room whose dimensions are

(1) Length = 13 ft. 3 in., and width = 12 ft. 10 in.
(2) ..... = 24 ft. 5 in., ... = 21 ft. 8 in.
(3) ..... = 14 ft. 6 in., ... = 13 ft. 2 in.
(4) ..... = 16 ft. 8 in., ... = 12 ft. 6 in.
(5) ..... = 18 ft. 10 in., ... = 15 ft. 7 in.
(6) ..... = 8 ft. $7\frac{1}{2}$ in., ... = 11 ft. $4\frac{1}{2}$ in.

Find the area (in acres, roods, and perches) of a rectangular field whose dimensions are

(7) Length = 94 ch. 50 links, and width = 9 ch. 25 links.
(8) ..... = 26 ch. 50 links, ... = 6 ch. 25 links.
(9) ..... = 7 ch. $81\frac{1}{4}$ links, ... = 5 ch. 67 links.
(10) ..... = 11 ch. 25 links, ... = 8 ch. 30 links.
(11) ..... = 18 ch. 75 links, ... = 12 ch. 50 links.
(12) ..... = 15 ch. 10 links, ... = 8 ch. 75 links.

EXERCISE LII.

(1) What will be the cost of varnishing the floor of a room 15 ft. 4 in. long and 14 ft. 6 in. broad, at 6*d.* per sq. yard?

(2) What will it cost to pave a courtyard 266 ft. 3 in. long and 48 ft. 9 in. broad, at $11\frac{1}{4}$*d.* per sq. yard?

(3) How many planks, each 13½ ft. long and 10½ in. wide, will be required for the construction of a platform, 54 yds. long and 21 yds. broad? What will be the cost at 5½*d.* per sq. foot?

(4) Find the cost of laying a plot of ground, 40 yds. long and 100 ft. wide, with turfs, each 1 yard long and 1 foot wide; the turfs when laid costing 6*s.* 9*d.* per hundred.

(5) How many marble slabs, 1 ft. long and 9 in. broad, would pave a hall, 50 yds. long and 50 ft. broad? What will the cost be, if the price of the slabs is 5*s.* per dozen?

(6) A rectangular walled yard is 50 yds. long and 35 yds. wide. All round the yard, next the wall, is a path 6 ft. wide. Find the area of the path in square feet.

(7) A rectangular plot is 17 ft. 8 in. long and 12 ft. 4 in. broad. What would it cost to make a path around it, 2 ft. 6 in. wide, at 1*s.* 10½*d.* per sq. foot?

(8) A room is 22 ft. 6 in. long by 13 ft. 5 in. wide. Find the cost of staining a border on the floor, 18 in. wide, at 2*s.* per sq. yard.

(9) A rectangular piece of land measures 71 ft. 4 in. by 65 ft. 7 in. A path is cut round it, 1½ yards wide. Find (i) the area of the land left after cutting the path, (ii) the area of the path, (iii) the cost of cutting the path at 1*s.* 6*d.* per sq. yard.

(10) Find the total cost of paving, at the rate of 3*s.* 4*d.* per square yard, an open square space whose side is 88 ft. 6 in., and two streets, one 240 yds. long by 48 ft. 4 in. wide, and the other 319 yds. long by 40 ft. wide.

(11) Find the cost of gravelling a 5-foot path running along two consecutive sides of a garden, which, excluding the path, is 172 ft. long and 54 ft. wide; a load of gravel worth 12*s.* 6*d.* being required for every 40 square yards of path, and the work occupying a man and a boy for 4½ days, at 3*s.* 9*d.* and 2*s.* a day respectively.

(12) A parish contains 6 miles of roads of an average breadth of 44 feet. Find the extent of these roads in acres.

CARPETING FLOORS.

Example i. *What length of carpet, 27 inches wide, will cover a floor* $10\frac{1}{2}$ *ft. by* $15\frac{1}{2}$ *ft.?*

Area of room $= 10\frac{1}{2} \times 15\frac{1}{2}$ sq. ft.
$= \frac{21}{2} \times \frac{31}{2}$ sq. ft.;
$\therefore$ area of carpet $= \frac{21}{2} \times \frac{31}{2}$ sq. ft.,
and width of carpet $= \frac{9}{4}$ ft.;
$\therefore$ length of carpet $= \frac{21}{2} \times \frac{31}{2} \div \frac{9}{4}$ ft.
$= \frac{21}{2} \times \frac{31}{2} \times \frac{4}{9}$ ft.
$= \frac{217}{3}$ ft.
$= 72\frac{1}{3}$ ft.

Example ii. *Find the cost of the carpet in Ex.* i. *at* 4*s.* 6*d. a yard.*

Length of carpet $= 72\frac{1}{3} \times \frac{1}{3}$ yards;
$\therefore$ cost of carpet $= \frac{1}{3} \times \frac{217}{3} \times \frac{9}{2}$ shillings
$= 108\frac{1}{2}$ shillings
$=$ £5. 8*s.* 6*d.*

Example iii. *A room is* 28 *ft. long and* 22 *ft. wide. All round the floor there is a border,* 2 *feet wide, which is stained at* 1*s.* $1\frac{1}{2}$*d. per square yard. The rest of the floor is carpeted with a carpet,* 27 *in. wide, which costs* 3*s.* 9*d. per yard. Find the total cost.*

Since the border is 2 ft. wide all round the room,

$\therefore$ area of carpet $= 24 \times 18$ sq. ft.;
and width of carpet $= \frac{9}{4}$ ft.;
$\therefore$ length of carpet $= \frac{24}{1} \times \frac{18}{1} \times \frac{4}{9} \times \frac{1}{3}$ yards
$= 64$ yards;
$\therefore$ cost of carpet $= 3s.\ 9d. \times 64$
$=$ £12.

Area of border $= (28 \times 22 - 24 \times 18)$ sq. ft.
$= (616 - 432)$ sq. ft.
$= 184$ sq. ft.
$= \frac{184}{9}$ sq. yds.;
$\therefore$ cost of staining $= \frac{27}{2} \times \frac{184}{9}$ pence
$=$ £1. 3*s.*

Total cost $=$ £12 + £1. 3*s.*
$=$ £13. 3*s.*

EXERCISE LIII.

(1) How many yards of drugget, $5\frac{1}{4}$ ft. wide, will cover the floor of a room 22 ft. 6 in. long and 16 ft. 4 in. wide?

(2) How many yards of carpet, 2 ft. wide, will cover the floor of a room 8 yds. long and $7\frac{1}{4}$ yds. wide?

(3) How many yards of carpet, 27 in. wide, will cover the floor of a room 27 ft. long and $17\frac{1}{2}$ ft. wide?

(4) How many yards of carpet, 2 ft. 11 in. wide, will cover the floor of a room 15 ft. long and 14 ft. 7 in. wide?

(5) How much carpet will be wanted to cover the floor of a square room 17 ft. 6 in. long, the width of the carpet being 2 ft. 4 in.? Find the cost of the carpet at 3*s.* 9*d.* per yard.

(6) How much carpet will be wanted to cover a floor 21 ft. 8 in. by 16 ft. 6 in., the width of the carpet being 27 in.? Find the cost of the carpet at 3*s.* $4\frac{1}{2}$*d.* per yard.

(7) How many yards of carpet will be required for a room 19 ft. long and 14 ft. 3 in. wide, the carpet being $\frac{3}{4}$ yd. wide? Find also the cost at 4*s.* 6*d.* per yard.

(8) How much will it cost to carpet the floor of a room 12 ft. 9 in. by 16 ft. 6 in., the carpet being 33 in. wide, and the price 3*s.* 8*d.* per yard?

(9) A Turkey carpet 19 ft. 6 in. by 15 ft. 9 in., and costing 16*s.* per sq. yard, is laid down in a room 23 ft. by 17 ft., and the rest of the floor is covered with linoleum at 7*d.* per sq. foot. Find the total cost.

(10) If 242 yards of carpet, $1\frac{1}{4}$ yards wide, can be made for £68. 1*s.* 3*d.*, what will be the cost of carpet for two rooms, each $28\frac{1}{2}$ ft. long by $13\frac{1}{2}$ ft. wide, and a passage between them 35 ft. long and $7\frac{1}{2}$ ft. wide; allowing for a twenty-fifth part of the whole as waste in cutting?

AREA OF WALLS, PAPERING, PAINTING, GLAZING, etc.

117. To find the area of a rectangular wall we multiply the length of the wall by the height (Art. 114). In such

rooms as we can consider there are 4 walls, and opposite walls are equal.

The area of each of two walls = length of room × height, and the area of each of the other two = breadth of room × height; therefore the total area of the four walls

= 2 × length of room × height + 2 × breadth of room × height
= 2 × (length + breadth) × height
= perimeter of room × height,

the **Perimeter** being the distance round the room.

This expression gives the area of the walls, including that occupied by windows, doors, etc. To find how much *paper* would be wanted to cover the walls, we must subtract the area occupied by these from the whole area.

NOTE.—To find the quantity of paper, the student *must not form the product of the length, the breadth, and the height* of the room; for this would give the volume necessary *to fill the room* instead of the quantity required merely to cover the walls.

Wall papers are sold in pieces, each 12 yards long, and the width of English wall paper is generally 21 inches.

Example i. *Find the area of the walls of a room* 12 *ft.* 6 *in. long*, 10 *ft.* 6 *in. wide, and* 10 *ft. high.*

Perimeter $= 2(12\frac{1}{2} + 10\frac{1}{2})$ ft.
$= 46$ ft.;
height $= 10$ ft.;
∴ area of walls $= \underline{460 \text{ sq. ft.}}$

Example ii. *Find the cost of painting the walls of the room in Ex.* i., *allowing* 49 *sq. ft. for the area occupied by doors and windows, at* 3*s.* 9*d. per sq. yard.*

Area of walls $= 460$ sq. ft.,
area to be painted $= 460$ sq. ft. $- 49$ sq. ft.
$= 411$ sq. ft.
$= 45\frac{2}{3}$ sq. yds.;
∴ cost of painting $= 3s.\ 9d. \times 45\frac{2}{3}$
$= \underline{£8.\ 11s.\ 3d.}$

NOTE.—Painting and plastering are charged by the square yard per coat, *i.e.*, one thickness of paint or plaster; and glazing is estimated by the square foot, the price depending on the quality of the glass.

Example iii. *If in the room in Ex.* i. *there is one door,* 7 *ft. by* 4 *ft., and one window,* 6 *ft. by* $3\frac{1}{2}$ *ft., how many pieces of paper (each* 12 *yards),* 21 *in. wide, would be required?*

Area of walls $=460$ sq. ft.,
area of door $=7\times4$ sq. ft. $=28$ sq. ft.,
area of window $=6\times3\frac{1}{2}$ sq. ft. $=21$ sq. ft.;
$\therefore$ area of paper required $=(460-28-21)$ sq. ft. $=411$ sq. ft.:
width of paper $=1\frac{3}{4}$ ft.,
$\therefore$ length of paper $=411\div1\frac{3}{4}$ ft.
$=137\times\frac{4}{7}$ yards;
$=78\frac{2}{7}$ yards;
$\therefore$ No. of pieces required $=78\frac{2}{7}\div12=7$.

NOTE.—Generally one piece in ten must be allowed for waste in fitting the pattern.

Example iv. *Find the cost of the paper in Ex.* iii. *at* 5*s. per piece.*

Cost of paper $=5\times7$ shillings
$=$£1. 15*s.*

The usual method of measurement adopted by paper-hangers to ascertain the number of pieces of paper required is as follows:—The number of feet in the perimeter of the room is multiplied by the number of feet in the height; this result, the number of square feet, is then divided by 9 to reduce it to yards, and again by 7. The result is the number of pieces, each of which has an area of 7 sq. yds.

EXERCISE LIV.

Find the area of the walls of a room whose dimensions are

(1) Length $=24$ ft., width $=18$ ft., and height $=15$ ft.

(2) Length $=18$ ft., width $=15$ ft., and height $=13$ ft.

(3) Length $=21$ ft. 5 in., width $=18$ ft. 7 in., and height $=10$ ft.

(4) Length $=23$ ft. 8 in., width $=15$ ft. 10 in., and height $=11$ ft. 11 in.

(5) Length $=22$ ft. 5 in., width $=18$ ft. 4 in., and height $=10$ ft. 9 in.

(6) Length $=14$ ft. 9 in., width $=9$ ft. 3 in., and height $=10$ ft. 6 in.

EXERCISE LV.

(1) Find the cost of painting the walls of a room 24 ft. 9 in. long, 17 ft. 3 in. wide, and 11 ft. 6 in. high, at 3*s.* 9*d.* per sq. yard.

(2) Find the cost of painting the walls of a room 30 ft. $6\frac{1}{4}$ in. long, 18 ft. $9\frac{3}{4}$ in. wide, and 10 ft. 6 in. high, at 1*s.* $3\frac{1}{2}d.$ per sq. yard.

(3) Find the cost of painting a room 21 ft. long, 15 ft. wide, and 11 ft. 4 in. high; the walls and doors being painted at 1*s.* 3*d.* per sq. yard, and the ceiling at 9*d.* per sq. yard, allowance being made for two windows, 7 ft. high by 3 ft. wide, and a fireplace, 6 ft. wide by $4\frac{1}{2}$ ft. high.

(4) A room is 17 ft. square and 11 ft. 6 in. high. Find the cost, at $2\frac{1}{2}d.$ per sq. yard, of whitewashing the ceiling and walls, after deducting $\frac{1}{10}$ of the area of the latter for windows, etc.

(5) Find the cost of plastering the walls and ceiling of a room 21 ft. long, 15 ft. wide, and 12 ft. high, at 10*d.* per sq. yard, allowing for a door 7 ft. high and 3 ft. wide, 3 windows, each 5 ft. by 3 ft., and a dado 2 ft. 9 in. high around the room.

(6) A room is 45 ft. 6 in. long, 24 ft. 10 in. wide, and 13 ft. 4 in. high. The walls are to be painted at $10\frac{1}{2}d.$ per sq. yard, and the ceiling whitewashed at $1\frac{1}{2}d.$ per sq. yard. Find the cost.

(7) Two rooms, each 16 ft. long, 12 ft. wide, and 8 ft. high, are to have the ceilings plastered at 6*d.* per sq. yard, and the walls whitewashed at $2\frac{1}{4}d.$ per sq. yard. What will be the cost, allowing in each room for two doors, each 7 ft. by 3 ft., and for two windows, each 4 ft. by $2\frac{1}{2}$ ft.?

(8) How many panes of glass, each 2 ft. long and 1 ft. 5 in. wide, will glaze a window 8 ft. high and 5 ft. 8 in. wide?

(9) In a row of 25 houses each house has 17 windows, each window 4 panes, and each pane measures 18 in. by 9 in. What will be the cost of glazing all these windows at 6*d.* per sq. foot?

(10) A certain building has 63 windows, 46 of which contain 12 panes, each measuring 20 in. by 16 in.; the others contain 9 panes, each measuring 16 in. square. Find the cost of glazing at 2*s.* 3*d.* per sq. foot.

EXERCISE LVI.

(1) What length of wall paper, 27 in. wide, will be required for a room 18 ft. long, 12 ft. wide, and 11 ft. high?

(2) A room is 28 ft. 6 in. long, 18 ft. 9 in. wide, and 12 ft. high; what length of paper, 1 ft. 9 in. wide, will be needed to cover the walls?

(3) How much paper, $\frac{3}{4}$ yd. wide, will be sufficient to paper a room 22 ft. 5 in. long, 12 ft. 1 in. wide, and 11 ft. 3 in. high? What will it cost at $4\frac{1}{2}d.$ per yard?

(4) What is the cost of papering a room 6 yds. 1 ft. 1 in. long, 6 yds. 4 in. wide, and 12 ft. high, with paper $\frac{1}{6}$ yd. wide, at $4\frac{1}{2}d.$ per yard?

(5) Find the cost of papering a room 5 yds. 1 ft. $2\frac{1}{3}$ in. long, 5 yds. $3\frac{1}{2}$ in. wide, and 4 yds. high, with paper 9 in. wide, at $2\frac{1}{2}d.$ a yard.

(6) Find the cost of papering a room 25 ft. long, 18 ft. 6 in. wide, and 10 ft. high, with paper 2 ft. wide, at $3d.$ a yard.

(7) A room measures 16 ft. by 21 ft., and is 11 ft. high. There is a door 7 ft. by 3 ft., and 2 windows 8 ft. by 4 ft. Find the cost of papering the room with paper, 2 ft. wide, at $2\frac{1}{2}d.$ a yard.

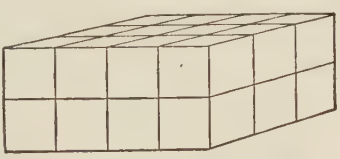
(8) A room 25 ft. 7 in. long, 16 ft. 9 in. wide and 13 ft. 6 in. high, has 3 windows 5 ft. by 3 ft. 6 in., a door 7 ft. by 4 ft., and a fireplace 5 ft. by 6 ft. Find the cost of papering the room with paper, $\frac{5}{8}$ yd. wide, at $4s.$ $6d.$ per piece of 12 yards.

(9) Find the cost of papering a room 24 ft. 10 in. long, 13 ft. 5 in. wide, and 11 ft. 4 in. high, with paper 1 ft. 11 in. wide, at $7\frac{3}{4}d.$ per yd., allowing for an area of 108 sq. ft. occupied by doors and windows.

(10) A room is 21 ft. 4 in. long, 15 ft. 9 in. wide, and 14 ft. high. The door and windows occupy 65 sq. ft. Find the cost of papering the remaining surface with paper, 25 in. wide, at $3s.$ $9d.$ per piece of 12 yards.

MENSURATION OF RECTANGULAR SOLIDS.

118. A **rectangular solid** has six faces, each of which is a rectangle. If such a solid be 4 feet long, 3 feet wide and 2 feet high, we can divide the edges respectively into 4, 3, and 2 equal parts, each of which parts will be 1 foot: and if we draw planes through the points of division as in the diagram, the solid will be separated into a number of smaller blocks each of which is a *cubic foot:* and since there are 2 layers, in each of which there are 3×4 blocks, we see that there are $2 \times 3 \times 4$ blocks altogether, and that the solid therefore contains $2 \times 3 \times 4$ cubic feet.



We can thus find the **volume,** or **cubical content,** of any rectangular solid by expressing the length, breadth and height in terms of a common unit of measurement, and multiplying the three numbers together. The product will give the volume of the solid in terms of the *corresponding* unit in cubic measure.

If we use the words *volume*, *length*, *breadth*, and *height* to denote the *number of units* in each respectively, the relation between them may be stated thus :—

volume = length × breadth × height ;
therefore, length = volume ÷ (breadth × height),
breadth = volume ÷ (length × height),
and height = volume ÷ (length × breadth) ;

hence, if any three be known, the fourth can be determined.

Example i. *Find the cubic content of a trench* 6 *ft. long,* 5 *ft. wide, and* 7 *ft. deep.*

Length = 6 ft.,
breadth = 5 ft.,
depth = 7 ft. ;
∴ cubic content = $6 \times 5 \times 7$ cu. ft.
= 210 cu. ft.

Example ii. *Find the cost of excavating the trench in Ex.* i., *at* 1*s.* $1\frac{1}{2}$*d. per cubic yard.*

Cubic content $= \frac{21}{27}\frac{0}{}$ cu. yards;
∴ cost of excavating $= \frac{27}{2} \times \frac{21}{27}\frac{0}{}$ pence
$= 105$ pence
$= 8s.\ 9d.$

NOTE.—The cost of excavating is estimated per cubic yard.

Example iii. *Find the cubic capacity of a rectangular block of wood* 16 *ft.* 6 *in. long,* 9 *in. wide, and* 4 *in. thick.*

Length $= 16\frac{1}{2}$ ft.,
width $= \frac{3}{4}$ ft.,
thickness $= \frac{1}{3}$ ft.;
∴ cubic capacity $= \frac{33}{2} \times \frac{3}{4} \times \frac{1}{3}$ cu. ft.
$= \frac{33}{8}$ cu. ft.
$= 4\frac{1}{8}$ cu. ft.

Example iv. *A closed vessel is* 8 *ft.* 3 *in. long,* 7 *ft.* 5 *in. wide, and* 4 *ft.* 3 *in. high, measured externally. If the material be* 1 *in. thick, find* (i) *how many cu. feet the vessel will contain, and* (ii) *the volume of the material used.*

Inside length $= 8$ ft. 3 in. $- 2$ in. $= 8\frac{1}{12}$ ft.,
breadth $= 7$ ft. 5 in. $- 2$ in. $= 7\frac{1}{4}$ ft.,
depth $= 4$ ft. 3 in. $- 2$ in. $= 4\frac{1}{12}$ ft.;
∴ cubic content $= 8\frac{1}{12} \times 7\frac{1}{4} \times 4\frac{1}{12}$ cu. ft.
$= \frac{97}{12} \times \frac{29}{4} \times \frac{49}{12}$ cu. ft.
$= 139\frac{837}{576}$ cu. ft.
$= 239\frac{173}{576}$ cu. ft.

The volume of the material used is the difference between the cubic contents estimated from the external and from the internal measurements;

∴ Volume of material $= (8\frac{1}{4} \times 7\frac{5}{12} \times 4\frac{1}{4} - 8\frac{1}{12} \times 7\frac{1}{4} \times 4\frac{1}{12})$ cu. ft.
$= (\frac{1064}{64}\frac{3}{} - 139\frac{837}{576})$ cu. ft.
$= 11\frac{950}{576}$ cu. ft.
$= 20\frac{115}{288}$ cu. ft.

119. The surface of a rectangular solid comprises six rectangles (Art. 118), and the area of its whole surface is equal to the sum of the areas of these six rectangles.

Example. What is the area of the surface of a rectangular solid 4 *ft. long,* 3 *ft. wide, and* 2 *ft. thick?*

Referring to the diagram on page 99 it will be seen that there are (i) two rectangles whose area is the product of the

length and width, (ii) two whose area is the product of the length and thickness, and (iii) two whose area is the product of the width and thickness.

Area of surface (i) $= 4 \times 3 \times 2$ sq. ft. $= 24$ sq. ft.,
..... (ii) $= 4 \times 2 \times 2$ sq. ft. $= 16$ sq. ft.,
..... (iii) $= 3 \times 2 \times 2$ sq. ft. $= 12$ sq. ft.;
$\therefore$ whole area of surface $= (24 + 16 + 12)$ sq. ft.
$= 52$ sq. ft.

EXERCISE LVII.

Find the cubical content of a rectangular solid whose dimensions are

(1) Length, $10\frac{1}{3}$ ft., width, $5\frac{1}{4}$ ft., and depth, 4 ft.

(2) Length, $6\frac{1}{2}$ ft., width, $2\frac{3}{4}$ ft., and depth, $3\frac{1}{4}$ ft.

(3) Length, 3 ft. 5 in., width, 4 ft. 9 in., depth, 8 ft. 7 in.

(4) Length, 3 ft. 4 in., width, 2 ft. 5 in., depth, 1 ft. 7 in.

(5) Length, 7 ft. 6 in., width, 5 ft. 8 in., and depth, 10 in.

(6) Length, 10 ft. 9 in., width, 6 ft. 4 in., and depth, 4 ft. 2 in.

(7) Length, 17 ft. 3 in., width, 14 ft. 7 in., and depth, 11 ft. 2 in.

(8) Length, 45 ft. 8 in., width, 19 ft. $1\frac{1}{2}$ in., and depth, 16 ft. 9 in.

EXERCISE LVIII.

(1) Find the height of a room which is 12 ft. long, 10 ft. wide, and contains 1440 cu. ft. of air.

(2) If 17 cu. ft. 594 cu. in. of oak are required to floor a room 13 ft. 6 in. long, and 12 ft. 4 in. wide, what is the thickness of the boards?

(3) A block of stone is 4 ft. long, $2\frac{1}{2}$ ft. broad, and $1\frac{1}{4}$ ft. thick; how many cu. feet does it contain?

(4) A tank is 30 ft. 9 in. long, 16 ft. 7 in. wide, and 6 ft. 4 in. deep; find how much water it will hold in cu. feet and inches.

(5) Find the cost of covering the top, bottom and sides of a box with metal at $3\frac{1}{2}d.$ per square foot, the dimensions of the box being—length 4 ft., width 3 ft., height 2 ft.

(6) Find the cost of painting, at 1*s.* 6*d.* per square yard, the outside of a box whose edges measure 6 ft., 5 ft., and $4\frac{1}{2}$ ft. respectively.

(7) A room, 42 ft. by 20 ft., accommodates 105 children. What must be the height of the room if each child has $83\frac{1}{3}$ cu. ft. of air?

(8) A rectangular cistern, 2 yds. 2 ft. long and 2 yds. wide, contains a depth of 4 ft. of water. How many cu. ft. of water are there? If a cu. foot of water weighs 1000 oz., find the weight of the water in the cistern.

(9) What is the value of a block 5 ft. 3 in. long, 2 ft. 4 in. wide and 1 ft. 2 in. thick, at 4 guineas per cu. foot?

(10) Find the value of a balk of timber 39 ft. 6 in. long, and 3 ft. 7 in. thick each way, at 2*s.* 6*d.* per cu. foot.

(11) Find the weight of a block of marble 9 ft. 6 in. long, 2 ft. 3 in. broad, and 2 ft. thick, supposing the weight of a cu. foot of marble to be 2716 oz.

(12) A rectangular cistern, $13\frac{3}{5}$ ft. long and 6 ft. broad, contains $294\frac{1}{4}$ cu. ft. of water; what is the depth of the water?

(13) If a pint of water weighs $1\frac{1}{4}$ lb., and a cu. foot weighs 1000 oz., find how many gallons a cistern 3 ft. 4 in. long, 2 ft. 8 in. broad, and 2 ft. 3 in. deep will hold.

(14) Find to the nearest penny the cost of excavating a ditch 1800 ft. long, 7 ft. wide, and 5 ft. deep, at the rate of 1*s.* 6*d.* for every 9 cubic yards.

(15) A closed cubical box has an edge of 2 ft. 10 in. The material is 3 in. thick. Find the weight, if 3 cu. feet of the material weigh 10 lb.

(16) Find the cost of excavating a cellar whose length, breadth and depth are respectively 6 yds., 16 ft., and 7 ft., at 9*d.* per cu. yard.

MENSURATION OF CERTAIN PLANE FIGURES AND SOLIDS.

120. Besides rectangles there are other figures of common occurrence, the area (or volume) of which it may sometimes be necessary to find. We will state the rules, the

words *base*, *height*, *area*, etc., being used to denote the *number of units* of measurement in each respectively.

Area of a **triangle** :—

(i) Given the base and the height; multiply the base by the height, and divide the product by 2.

(ii) Given the three sides; multiply half the perimeter by the product of the remainders obtained on subtracting each side from it, and take the square root of the result.

Area of a **parallelogram** :—

Multiply the base by the perpendicular height.

Area of a **rhombus** :—

(i) Given the base and the perpendicular height; multiply the one by the other.

(ii) Given the diagonals; divide the product of the diagonals by 2.

Area of a **trapezoid** :—

Multiply half the sum of the parallel sides by the perpendicular distance between them.

Area of a **circle** :—

(i) Multiply the square of the diameter by ·7854 (or $\frac{\pi}{4}$);

or (ii) Multiply the square of the radius by 3·1416 (or π);

or (iii) Multiply the square of the circumference by ·08;

or (iv) Multiply the circumference by half the radius.

NOTE.—The circumference of any circle bears to its diameter a *constant* ratio, which is denoted by the symbol π, and may be represented approximately by $\frac{22}{7}$, or by 3·1416.

The volume of a **right prism** = area of base × height.

The volume of a **right cylinder** = area of base × height.

The volume of a **right cone** = $\frac{1}{3}$ × area of base × height.

The volume of a **sphere** = $\frac{4}{3}$ × 3·1416 × (radius)^3.

The area of the curved surface of a **cylinder**
= circumference of base × height.

The area of the curved surface of a **right cone**
= $\frac{1}{2}$ × circumference of base × slant height.

The area of the surface of a sphere
= 4 × 3·1416 × (radius)^2.

NOTE.—To find the quantity of water in a well, multiply half the circumference by half the diameter and by the depth, reckoning $6\frac{1}{8}$ gallons per cubic foot.

DUODECIMALS.

121. In calculations, such as those which builders and engineers have to make, it is found convenient to divide and subdivide the foot **duodecimally**, *i.e.*, into *twelfths*; thus

1 foot = 12 *primes*,
1 prime = 12 *seconds*,
1 second = 12 *thirds*, and so on.

The square foot and the cubic foot are similarly divided; thus

1 sq. foot = 12 *superficial primes*,
1 sup. prime = 12 *superficial seconds*,
1 sup. second = 12 *superficial thirds*, and so on;

and 1 cu. foot = 12 *solid primes*,
1 solid prime = 12 *solid seconds*,
1 solid second = 12 *solid thirds*, etc.

Thus 1 prime $= \frac{1}{1^2}$ of a foot,
1 second $= \frac{1}{1^{2^2}}$ of a foot,
1 third $= \frac{1}{1^{2^3}}$ of a foot, etc.
} whether the foot be lineal, superficial or solid.

Primes, seconds, thirds, etc., are denoted as follows, whether the measure is lineal, superficial, or solid;

5 primes by 5′, 5 fourths by 5^{iv},
5 seconds ... 5″, 5 sevenths ... 5^{vii}.
5 thirds ... 5‴,

The *index* is written in the Roman method to distinguish the expressions from 5, 5^2, 5^3, 5^4, 5^7..., which have other meanings.

It will be convenient also to consider feet, primes, seconds, thirds, etc., as respectively of the order 0, 1, 2, 3, etc.

NOTE.—Instead of this notation, 5 *feet* are sometimes represented by 5′, 5 *primes* by 5″, etc., but the processes of operation will be exactly the same.

122. The following examples will show how the ordinary measures may be converted into feet, primes, seconds, etc.; and conversely, how duodecimal measures may be converted into feet and inches.

Example i. *Express* 17 *ft.* 7½ *in. in duodecimals.*

17 ft. 7½ in. $=(17+\frac{7}{12}+\frac{1}{24})$ ft.
$=(17+\frac{7}{12}+\frac{6}{144})$ ft.
$=17$ ft. 7′ 6″.

Example ii. *Express* 186 *sq. ft.* 87 *sq. in. in duodecimals.*

186 sq. ft. 87 in. $=186\ \frac{87}{144}$ sq. ft.
$=(186+\frac{84}{144}+\frac{3}{144})$ sq. ft.
$=(186+\frac{7}{12}+\frac{3}{144})$ sq. ft.
$=186$ sq. ft. 7′ 3″.

Example iii. *Express* 57 *cu. ft.* 8′ 5″ 4‴ *in feet and inches.*

57 cu. ft. 8′ 5″ 4‴ $=(57+\frac{8}{12}+\frac{5}{144}+\frac{4}{1728})$ cu. ft.

Hence, to reduce 8′ 5″ 4‴ to inches, we multiply 8 by 12 and add the 5; this gives 101; we next multiply 101 by 12 and add the 4; this gives 1216;

∴ 57 cu. ft. 8′ 5″ 4‴ $=$ 57 cu. ft. 1216 cu. in.

Note.—All these reductions should be done mentally.

EXERCISE LIX.

Express the following measures in duodecimals:—

(1) 2 ft. 6 in.
(2) 25 ft. 10½ in.
(3) 14 ft. 5¼ in.
(4) 10 ft. 4⅓ in.
(5) 12 ft. 2⅛ in.
(6) 15 sq. ft. 28 sq. in.
(7) 23 sq. ft. 47 sq. in.
(8) 154 sq. ft. 117 sq. in.
(9) 108 sq. ft. 113½ sq. in.
(10) 121 sq. ft. 132⅓ sq. in.
(11) 276 cu. ft. 183 cu. in.
(12) 42 cu. ft. 334 cu. in.
(13) 47 cu. ft. 892 cu. in.
(14) 39 cu. ft. 1120½ cu. in.
(15) 156 cu. ft. 1714½ cu. in.

123. Before we use this notation in reckoning areas and volumes, we must notice the relation which exists between the *order* of a product and the *orders* of its factors.

The area of a rectangle, 1 foot long and 1 prime wide,
$=1\times1'=1\times\frac{1}{12}$ sq. ft. $=\frac{1}{12}$ sq. ft. $=1'$.
Similarly, 1 ft. $\times\ 1'''=1\times\frac{1}{12^3}$ sq. ft. $=\frac{1}{12^3}$ sq. ft. $=1'''$,
$1'\times1''=\frac{1}{12}\times\frac{1}{12^2}$ sq. ft. $=\frac{1}{12^3}$ sq. ft. $=1'''$,
and $1'''\times1^{iv}=\frac{1}{12^3}\times\frac{1}{12^4}$ sq. ft. $=\frac{1}{12^7}$ sq. ft. $=1^{vii}$.

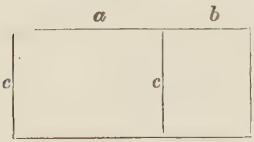
Hence we deduce the following rule, which holds when there are three factors as well as when there are only two, *i.e.*, for volumes as well as for areas :—*The order of a product is the sum of the orders of its factors.*

Example i. *If length of rect.* $=5'$, *and breadth* $=7''$, *find area.*
Area $=5' \times 7'' = 35''' = \underline{2'' \ 11'''}$.

Example ii. *If length* $=3$ *ft.*, *and breadth* $=10''$, *find area.*
Area $=3$ ft. $\times 10'' = 30'' = \underline{2' \ 6''}$.

Example iii. *If length* $=4$ *ft.*, *and breadth* $=3'$, *find area.*
Area $=4$ ft. $\times 3' = 12' = \underline{1}$ sq. ft.

124. If the length of a rectangle be $a+b$ units, and the breadth be c units, its area is equal to the sum of the areas of the rectangles whose sides are a and c, and b and c, respectively. Hence, to find the area, we multiply a by c, and b by c, and add the results. We use this property when the length or breadth involves terms of more than one order.



Example i. *Find the area of a rectangle* 3 *ft.* 4′ *by* 2 *ft.*

Area $=2$ ft. $\times 3$ ft $+2$ ft. $\times 4'$
$=6$ sq. ft. 8′
$=6$ sq. ft. 96 in.

The work may be arranged as in the margin.

ft.	′
3	4
2	0
6	8

Example ii. *Find the area of a rectangle* 15 *ft.* 3′ 5″ *by* 4′.

Multiplying 5″ by 4′ we get 20‴, or 1″ 8‴. We put the 8‴ in a *thirds* column to the right of the *seconds* column, and carry forward the 1″.

ft.	′	″	
15	3	5	
	4	0	
5	1′	1″	8‴

Again, multiplying 3′ by 4′ we get 12″, which becomes 13″, or 1′ 1″, when we add the 1″ brought forward. We set down 1″ and carry 1′.

Lastly, $4' \times 15$ ft. $= 60'$, which becomes 61′, or 5 ft. 1′.

Hence the area $=5$ sq. ft. $13\frac{2}{3}$ in.

125. If the lengths of both sides be compound expressions, we proceed in a similar way. We take the length of

either side for multiplier, and we use the terms of it in succession, beginning with that of highest denomination.

Example i. *Multiply* 2 *ft.* 5′ *by* 2 *ft.* 3′.

Multiplying 2 ft. 5′ by 2 ft., we get 4 ft. 10′
Next, multiplying 2 ft. 5′ by 3′ we get 7′ 3″.

Adding these products together we find that the area = 5 sq. ft. 63 in.

ft.	′	
2	5	
2	3	
4	10	
	7	3′
5	5′	3″

Example ii. *Find the area of a room* 15 *ft.* 4′. 7″ *long and* 13 *ft.* 2′. 10″ *wide.*

We first use 13 ft. as multiplier, then 2′, and lastly 10″; and we add the results together

The area = 203 sq. ft. $85\frac{7}{12}$ in.

ft.	′	″		
15	4	7		
13	2	10		
199	11	7		
2	6	9	2‴	
1	0	9	9‴	10^{iv}
203	7′	1″	11‴	10^{iv}

Example iii. *Find the volume of a rectangular solid* 3 *yds.* 2 *ft.* 1′ *long,* 2 *ft.* 9′ *broad, and* 10 *ft.* 5′ *high.*

	ft.	′	″	
Length =	11	1		
breadth =	2	9		
	22	2		
	8	3	9″	
area of base =	30	5′	9″	
height =	10	5′		
	304	9	6	
	12	8	4	9‴
volume =	317	5′	10″	9‴

Hence the volume = 317 cubic ft. 849 in.

EXERCISE LX.

(*To be worked by Duodecimals.*)

Find, in square feet and inches, the areas of the rectangles whose dimensions are

(1) 2 ft., 3′.
(2) 5 ft., 4″.
(3) 4′, 7″.
(4) 13 ft. 3′, 12 ft. 10′.
(5) 4 ft. 5′, 21 ft. 8′.
(6) 22 ft. 6′, 10 ft. 3′.
(7) 5 ft. 2′ 4″, 8 ft. 1′.
(8) 12 ft. 3′ 5″, 11 ft. 2′ 9″.
(9) 15 ft. 10′ 4″, 15 ft. 4′ 10″
(10) 27 ft. 9′ 4″, 6 ft. 2′ 3″.
(11) 17 ft. 6′ 4″, 19 ft. 7′ 8″
(12) 29 ft. 7′, 17 ft. 9′ 8″.

EXERCISE LXI.

(To be worked by Duodecimals.)

Find, in cubic feet and inches, the volume of a rectangular solid measuring

(1) 3 ft. 5′ by 4 ft. 9′ by 8 ft. 7′.
(2) 17 ft. 3′ by 14 ft. 7′ by 11 ft. 2′.
(3) 3 ft. 4′ 5″ by 4 ft. 3′ 7″ by 1 ft. 1′.
(4) 4 ft. 2′ 7″ by 3 ft. 2′ 3″ by 7 ft. 4′.
(5) 5 ft. 6′ 4″ by 2 ft. 7′ 3″ by 6 ft. 5′.
(6) 7 ft. 3′ 5″ by 5 ft. 7′ 4″ by 4 ft. 2′.
(7) 6 ft. 1′ 3″ by 8 ft. 7′ 2″ by 8 ft. 7′.
(8) 7 ft. 4′ 3″ by 9 ft. 5′ 7″ by 12 ft. 11′.
(9) 11 ft. 4′ 3″ by 8 ft. 9′ 7″ by 10 ft. 11′.
(10) 7 ft. 6′ by 10 ft. 1′ 6″ by 12 ft. 4′ 6″.

126. The results required in the following exercise are to be obtained by Duodecimals.

EXERCISE LXII.

(1) The length of a building is 128 ft. 8′ 6″, and the width is 97 ft. 10′ 9″; find the area of the basement.

(2) Find the cubic content of a rectangular tank, 12 ft. 6′ by 5 ft. 3′ by 3 ft. 9″.

(3) If a pane of glass be 2 ft. 6′ long and 1 ft. 9′ wide, how many square feet does it contain?

(4) What will be the cost of glass for a window that measures 9 ft. 8′ by 4 ft. 3′, at 2*s.* 3*d.* per square foot?

(5) Find the cost of the ceiling of a room 30 ft. 9′ by 20 ft. 6′, at 1*s.* 3*d.* per square yard.

(6) Find the cost of painting a room 19 ft. 10′ 3″ long, 16 ft. 1′ 9″ wide, and 10 ft. 3′ high, at 1*s.* 7*d.* per square yard.

(7) What is the cost of three marble slabs, each measuring 4 ft. 4′ 6″ by 1 ft. 4′, at 7*s.* 6*d.* per square foot?

(8) What will be the expense of digging a cellar, 16 ft. 9′ long, 12 ft. 6′ wide, and 8 ft. 3′ high, at 1*s.* 1½*d.* per cubic yard?

(9) A rectangular tank is 37 ft. 5′ 6″ long, 22 ft. 7′ 8″ wide, and 25 ft. 4′ deep. Find the cost of lining the four sides, at 1*s.* per square foot.

(10) Find the volume of a block 45 ft. 8′ long, 19 ft. 1′ 6″ wide, and 16 ft. 9′ high. Find also the value at £1. 16*s.* per cubic yard.

Pupils should also work out some of the examples in Exercises LI.–LVIII. by Duodecimals for additional practice.

QUANTITIES AND ESTIMATES.

127. The general course in preparing a set of Builder's Quantities is as follows:—

(i) **Taking off**, *i.e.*, taking measurements to scale from the drawings, and putting down the dimensions on ruled paper.

(ii) **Squaring the dimensions.** Dimensions are multiplied together by duodecimals, except, of course, when only one dimension is given, and are either *cubic*, *superficial*, or *lineal*, according as there are 3, 2, or 1 dimensions respectively.

(iii) **Abstracting**, *i.e.*, transferring the results and the descriptions from the dimension sheets to sheets specially ruled in columns headed according to the different trades. These sheets are termed *abstract sheets*. In abstracting, all the dimensions of the same kind are placed in the column pertaining to them, and the deductions (if any) are put into the next column. As a general rule the cubic dimensions are put in the first columns, the superficial in the following, and the lineal in the next columns. In the quantities each trade should be kept separate throughout.

(iv) Casting up the columns of figures, and reducing them to the denominations in common use (such as cubic *yards* in excavator's work, *rods* in bricklayer's work, *squares* in slater's work, etc.), is termed **Reducing the Abstracts.**

(v) **Billing** means transferring in the customary order to paper ruled as follows:—

EXCAVATOR AND BRICKLAYER.

Yds.	Ft.	In.				£	s.	d.
162	9		Cub.	Digging trenches and well ramming bottom of same to receive concrete. .	1/-			
Rods 23	Ft. 50		Sup.	Reduced brickwork built solid in mortar	260/-			
	Ft. 54		Run.	Steps formed of blue bricks on edge in cement	1/-			
				Total . .				

The first three columns are for stating the measurements; the fourth column is for stating whether the figures are cubic, superficial, or "running" (lineal).

The column to the left of the money column is for pricing the several items; thus the first is 1*s.* per cubic yard of excavation, the second item is charged at 260*s.* per rod of brickwork, and the third 1*s.* per foot run. of edging. The prices worked out are of course put in the money columns.

NOTE.—*Cub.* is the abbreviated form used by builders to denote cubic measure; *sup.* for superficial measure; *run.* for "running" or lineal measure.

128. We have already considered questions relating to plastering, painting, papering, and glazing (Ex. LV. and LVI). We will now deal with the methods usually employed in the measurement of brickwork and slating.

THE MEASUREMENT OF BRICKWORK.

129. Bricklayers' work is always reckoned at the rate of a *brick and a half thick*; and walls of any other thickness must be reduced to this standard thickness.

The amount of brickwork is estimated in **standard rods.** A rod of brickwork was originally taken from the rod of $5\frac{1}{2}$ yards, and should therefore contain $272\frac{1}{4}$ square feet. In calculations, however, a rod of brickwork of the standard thickness is taken as 272 feet super. A brick ordinarily

measures 8½ in. by 4 in. by 2½ in., but in estimating the number of bricks ½ in. is added to each dimension to allow for mortar. A rod of brickwork—272 square feet—of the standard thickness will require 4500 bricks, allowing for waste.

To find the number of standard rods of brickwork in a wall, if the wall is not of the standard thickness, we must multiply the number of square feet in the area of the wall by the number of half-bricks it is in thickness, divide by 3, and then by 272.

Example. How many rods of standard brickwork are there in a wall 56 yards long, 6 ft. high, and 2½ bricks thick?

Area of wall $= 168 \times 6$ sq. ft. $= 1008$ sq. ft.

No. of sq. feet of standard thickness $= \frac{1008 \times 5}{3} = 1680$;

$\therefore$ quantity of brickwork $= \frac{1680}{272}$ rods $=$ 6 rods 58 ft. sup.

EXERCISE LXIII.

(1) How many square yards of standard brickwork are there in a wall 2½ bricks thick, the length being 46 ft. 7 in., and the height 8 ft. 4 in.?

(2) How many square yards of standard brickwork are there in a wall 40 ft. 9 in. long, 18 ft. high and 3 bricks thick, deducting 5 windows, each 5 ft. by 2 ft. 10 in., and a door 6 ft. 6 in. by 3 ft. 3 in.?

(3) How many rods of standard brickwork are there in a wall 2½ bricks thick, the length being 68 ft. 4 in., and the height 16 ft. 3 in.?

(4) How many rods of standard brickwork are there in a wall 2½ bricks thick, the length being 84½ ft., and the height 12 ft. 6 in.?

(5) How many rods of standard brickwork are there in the end wall of a house 24 ft. wide, 3 bricks thick to the height of 10 ft., and 2½ bricks for the next 9 ft.; the gable, 8 ft. high, above this being 2 bricks thick, and the chimney stack 9 ft. wide, 5 ft. 10 in. high and 2 bricks thick?

(6) How many rods of standard brickwork are there in a building 150 ft. in perimeter, the cellars and foundation being 15 ft. deep and 2½ bricks thick, the ground floor 12 ft. high and 2 bricks thick, the next floor 10 ft. high and 1½ bricks thick, and the attic floor 9 ft. high and 1 brick thick?

(7) How many bricks, each 9 in. by $4\frac{1}{2}$ in. by 3 in., are there in a wall 36 ft. long, 9 ft. wide, and 12 ft. high?

(8) How many bricks, each 9 in. long, $4\frac{1}{2}$ in. wide, and 3 in. deep, will be needed for a wall 25 yds. long, 15 ft. high, and 1 ft. $10\frac{1}{2}$ in. thick?

(9) How many bricks, 9 in. by $4\frac{1}{2}$ in. by 3 in., will be required to build a wall 27 ft. long, 6 ft. high, and 18 in. thick for $1\frac{1}{2}$ ft. above the ground, and 9 in. thick for the remainder?

(10) Find the cost of the bricks needed for building a wall 30 yds. long, 6 ft. high, and $13\frac{1}{2}$ in. thick, having given that 1000 bricks cost 25*s.*, and that each brick fills up a space 9 in. long, $4\frac{1}{2}$ in. wide, and 3 in. deep.

130. Slating is a superficial measurement, and is always described as so many *squares super.*, a square super. containing 100 square feet.

Thus to find the number of squares super. we divide the number of square feet in the area to be covered by 100.

Example. What will the roofing of a house cost at 27*s.* 9*d.* *per square, each side of the roof being* 40 *ft. by* 15*?*

Area of roof both sides $= (40 \times 15 \times 2)$ sq. ft.
$= 12$ squares of 100 ft.;
$\therefore$ cost $= 27s.\ 9d. \times 12$
$= £16.\ 13s.$

131. Lead laid on a roof is described as 6 lb., 5 lb., etc., according as there are 6 lb. or 5 lb. to the square foot in weight. Sheet lead is always charged per cwt., and lead pipes per foot run.

132. It must be remembered that artificers' work is computed by special measures; for example,

(i) Glazing and joinery by the *square foot.*

(ii) Painting, plastering, paving, etc., by the *square yard.*

(iii) Partitioning, flooring, roofing, tiling, etc., by the *square containing* 100 *square feet.*

(iv) Brickwork by the *rod of* 272 *square feet.*

133. The following are specimens of estimates or bills of quantities.

Yds.	Ft.	In.			£	s.	d.
366	6		Sup.	Taking off turf, rolling, and stacking same	6d.		
82			Cub.	Digging over surface not more than 6″ deep and levelling site	9d.		
162	19		,,	Digging trenches not more than 2′ 6″ deep, and well ramming bottom of same to receive concrete	1/-		
64	21		,,	Blue lias concrete well mixed and rammed solid in trenches	8/-		
264			Sup.	6″ Portland cement, rammed solid, and finished with a smooth and level surface	2/6		
18			Run.	Digging for and laying 6″ glazed, socketted, stoneware pipes, with cement joints, bedded 6″ all round pipes in concrete (as above), average 3′ deep	5/3		
40			,,	Do. 6″ pipes averaging 5′ 6″ deep	5/6		
			No.	13 4″ bends to above	1/3		
Rods	Ft.						
23	50		Sup.	Reduced brickwork of all seconds quality kilns, built solid in mortar	260/-		
11	35		,,	Do., but built hollow in 11″ walls	300/-		
			No.	36 10″ Red flue lining pipes as chimney pots	1/-		
	Ft.						
	277		Sup.	Rough cutting to gables	1/-		
Yds.							
313½			,,	Work fair and point in walls	3d.		
			No.	6 glazed stoneware sinks 2′ 6″ long on ½-brick piers in cement	22/6		
15½			Sup.	Hearths of 6″ red quarries (best quality)	6/-		
			No.	Set 6 ranges (small), using all necessary fire bricks, etc., required	15/-		
117			Sup.	Cement paving 2″ thick finished with smooth, trowelled face	2/-		
	44	6	Run.	9″×8″ Purbeck stone steps	3/-		
	69	6	Run.	12″×4½″ Bath stone heads	1/6		
				Allowed for all minor items of work and materials, and assist all trades to complete		1	10
				Total			

EXERCISE LXV.

CARPENTER AND JOINER.

Sqr.	Ft.	In.			£	s	d.
	563		Cub.	Rough fir in plates, joists, and lintels			2/3
	735		„	Do. framed-in roof			2/9
3			Sup.	4″×2″ stud partitions with 4″×3″ heads and sills			10/-
	120		Run.	2″×1″ cross bridging			3d.
	104		„	7″×1½″ ridge board			4d.
	294		„	5″×1″ wrot. fascia			4½d.
	1062		„	7″×1″ torus skirting plugged to walls			4½d.
	384		„	5″ beaded do.			3d.
	220		„	4½″×3″ rebated and beaded frame			8d.
	102		„	4″×2½ do. do.			6d.
	714		„	6″×1¼″ jamb lining with ½″ stops nailed on			5½d.
	204		„	10×1¼″ do. do.			7½d.
	96		„	12″×1¼ do. do.			9d.
	112		Sup.	1¾″ five-panel doors, top panel prepared for glazing			1/3
	108		„	1¾″ framed ledged and braced doors			1/3
	684		„	1¾″ moulded sashes, double hung, in deal-cased frames, sunk sills, with best white lines, brass axle pulleys, iron weights, etc.			2/5½
	94		„	1½″ moulded sashes in 4½″× 3″ rebated frames and mullions, hung on battens			1/6½
	1620		Run.	3″ Architrave moulding			2¾d.
	711		„	2″×1″ chamfered fillet			1½d.
	84		Sup.	1¼″ panelled and moulded cupboard fronts			1/-
	112½		„	1¼″ panelled and square do.			10d.
	170		„	1½″ treads and 1″ risers			1/3
	72		Run.	3″×2″ moulded handrail			1/6
			No.	6 wrot. wooden chimney-pieces as per sketch			15/-
			„	6 do. do.			20/-
			„	12 do. do.			13/6
				Total			

EXERCISE LXVI.

SLATER.

Sqr.	Ft.	In.				£	s.	d.
45	75		Sup.	Seconds Bangor slating with 3″ lap and fastened with 2 stout zinc nails to each slate	28/6			
	88		Run.	Sawn slate fillets set in cement and screwed with brass screws (hips)	5d.			
	362		„	Cutting to hips and valleys	6d.			
	105	6	„	Red tile ridge set in cement, coloured to match	1/9			
45	75		Sup.	2″ × ¾″ battens spaced for above slating	2/6			
				Allowed for assisting all other trades and doing all minor items of work and providing minor items of materials			10	
				Total				

EXERCISE LXVII.

PLASTERER.

Yds.	Ft.	In.				£	s.	d.
1312			Sup.	Render, float and set walls	11d.			
64			„	Lath and 3 coats plaster partitions	1/6			
564			„	Do. ceilings	1/7			
	228		Run.	6″ cement skirting	5d.			
			No.	6 Cement render top and front of coppers	2/6			
			No.	6 Cement render jambs and head of scullery fireplaces	4/6			
313½			Sup.	Twice lime-white walls	1/-			
				Allowed for minor items of work and materials and assisting all trades to complete			10	
				Total				

EXERCISE LXVIII.

PLUMBER, GLAZIER, AND PAINTER.

Yds.	Ft.	In.				£	s.	d.
	296		Run.	4″ cast-iron eaves gutter with red-lead joints and 3 coats of paint	1/10			
			No.	10 angles to do.	4/-			
				12 stopped ends to do.	2/-			
				12 nozzles	1/6			
	180		Run.	2½″ downpipes with red-lead joints and 3 coats of paint	6d.			
			No.	6 12-gallon galvanized iron boilers	5/-			
	573		Sup.	21-oz. English sheet-glass, well-bedded, sprigged, and back puttied	1/4			
1114			„	Knot, stop, prime, and paint with coats of 3 oil paints, finished in 2 selected shades.	9d.			
			No.	20 doz do. do. sash squares	3/-			
			„	12 doz. do. do. (large)	3/9			
	136		Run.	Wrought-iron railing with bars and standards, and paint same with 3 coats of oil paints	2/6			
			No.	6 gates do. do.	15/-			
Cwt.	qrs.	lb.						
16	1	14		Milled lead, laying and dressing in flashings, soakers, etc.	22/6			
				Allow for stoves and ranges		27		
				Allowed for all minor items of work and materials, and assisting all trades to complete			15	
				Total				

THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

134. A system of *weights* and *measures*, based on the decimal principle, is in common use in France and several other countries; but in England it is little used except for scientific purposes.

By the Weights and Measures (Metric System) Act passed in 1897, however, the use in trade of a weight or measure in the Metric System is made lawful.

The *unit* of lineal measure in this system is the **metre**, whence the system is called the *Metric* System. The metre is equal to 39·37079... inches, and was adopted originally because it was supposed to be $\frac{1}{10000000}$ of the distance between the north pole and the equator, measured on the surface of the ocean and along a circle of longitude. It was supposed to be a natural unit, as distinguished from such an arbitrary unit as the length of a certain rod kept at Westminster and called a yard.

Although, however, an error has since been found in the measurement of the distance between pole and equator, yet the length of the metre has not been corrected, and the metre therefore is no longer the natural unit it was supposed to be; it is not exactly $\frac{1}{10000000}$ of the distance stated, but is merely the length of a certain rod of platinum kept at Paris.

135. Multiples of the metre are distinguished by the **Greek** prefixes *deca-*, *hecto-*, *kilo-*, *myria-*, thus

10 metres make 1 deca-metre,
100 ..... 1 hecto-metre,
1000 ..... 1 kilo-metre,
10000 ..... 1 myria-metre.

Submultiples of the metre are distinguished by the **Latin** prefixes *deci-*, *centi-*, *milli-*, thus

$\frac{1}{10}$ of a metre is 1 deci-metre,
$\frac{1}{100}$ ..... 1 centi-metre,
$\frac{1}{1000}$ ..... 1 milli-metre.

The measures of length and their relation to one another are given in the following table:—

10 millimetres	make	1 centimetre (cm.),
10 centimetres	...	1 decimetre (dm.),
10 decimetres	...	1 *metre* (m.),
10 metres	...	1 decametre (Dm.),
10 decametres	...	1 hectometre (Hm.),
10 hectometres	...	1 kilometre (Km.),
10 kilometres	...	1 myriametre (Mm.).

The abbreviations used to denote the measures are given in brackets.

The decimetre, decametre, hectometre, and myriametre are rarely used. For example, it is usual to speak of 35 centimetres instead of 3 decimetres, 5 centimetres.

The lengths of articles such as cottons and cloths are expressed in metres and centimetres; the heights of mountains in metres; distances by railway or road in kilometres; scientific calculations in centimetres and millimetres.

136. The unit of *area* is a square of which each side is a unit of length. It is called a **square metre.**

The measures of area are arranged in the following table:—

100 square millimetres	make	1 square centimetre,
100 square centimetres	...	1 square decimetre,
100 square decimetres	...	1 *square metre*,
100 square metres	...	1 square decametre,
100 square decametres	...	1 square hectometre,
100 square hectometres	...	1 square kilometre.

Builders and kindred tradesmen measure work by square metres.

In measuring land on a small scale it is customary to take for unit a *square decametre*, or a square whose side is 10 metres. This is called an **are.** Its multiple and submultiple are as follows:—

100 *ares* = 1 *hectare*,
$\frac{1}{100}$ of an *are* = 1 *centiare*.

The following is the table of land measurement:—

100 centiares make 1 are,
100 ares ... 1 hectare.

An *are* is equal to a square whose side is 10 metres.
An *hectare* ..... 100 metres.
A *centiare*..... 1 metre.

The areas of fields are generally expressed in ares, or in hectares and ares; and the areas of countries and departments in square kilometres.

137. The unit of *volume* is a **cubic metre**, that is, a solid the length, breadth and height of which are each *one metre*. The cubic metre is employed in measuring blocks of stone and timber, for building and excavations. In measuring wood this unit is called a **stère.**

The measures relating to solids and their relation to one another are arranged in the following table:—

1 cubic metre makes 1000 cubic decimetres,
1 cubic decimetre ... 1000 cubic centimetres,
1 cubic centimetre... 1000 cubic millimetres.

138. The unit of *capacity*, in liquid and dry measure, is called a **litre**, and is equal to a *cubic decimetre*. Its multiples and submultiples are distinguished by the same prefixes as are the multiples and submultiples of the metre. Thus

10 millilitres make 1 centilitre (cl.),
10 centilitres ... 1 decilitre (dl.),
10 decilitres ... 1 *litre* (l.),
10 litres ... 1 decalitre (Dl.),
10 decalitres ... 1 hectolitre (Hl.),
10 hectolitres ... 1 kilolitre (Kl.).

The litre is used in retailing liquids; the hectolitre in selling liquids wholesale.

139. The unit of *weight* is the weight of a cubic centimetre of distilled water when at its greatest density and in a vacuum. This weight is called a **gramme.** The weight of a *litre* of such water is 1000 *grammes*, or one **kilogramme.** The weight of 1000 litres, *i.e.*, of a **kilolitre**, is called a **tonneau de mer.**

The measures relating to weight are arranged in the following table:—

10 milligrammes	make	1 centigramme (cg.),
10 centigrammes	...	1 decigramme (dg.),
10 decigrammes	...	1 *gramme* (g.),
10 grammes	...	1 decagramme (Dg.),
10 decagrammes	...	1 hectogramme (Hg.),
10 hectogrammes	...	1 kilogramme (Kg.).

The gramme, decigramme, etc., are used in weighing precious metals, and in scientific calculations. The kilogramme is the ordinary commercial weight, and is used in business where we use the pound in England. It is generally called a *kilo*.

NOTE.—Weights are verified practically not by comparison with the weight of a certain quantity of water under the given conditions, but with the platinum standard weight obtained once for all, and now kept in the state archives at Paris.

140. The reduction of weights and measures in the Metric System can be effected very simply by inserting, or omitting, or changing the position of, the decimal point. For example, the length which denotes

6 *kilometres* 5 *hectometres* 2 *decametres* 7 *metres* 8 *decimetres* 3 *centimetres*,

may be expressed as

	6·52783 *kilometres*,	or	6527·83 *metres*,
or	65·2783 *hectometres*,	or	65278·3 *decimetres*,
or	652·783 *decametres*,	or	652783 *centimetres*.

Similarly, 7 sq. metres 5 sq. decimetres 5 sq. centimetres may be expressed as

7·0505 sq. metres.

And 6 cu. metres 530 cu. centimetres may be expressed as

6·00053 cu. metres.

141. We will now give the principal measures in the Metric System with their English equivalents.

Measures of Length.

1 metre = 39·37079 inches = 1·0936331 yds.
1 kilometre = ·6213824 miles = 1093·6330556 yds.

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1 inch = 2·539954 cm. 1 yard = ·91438348 metres.
1 foot = 3·0479449 dm. 1 mile = 1·6093149 kilometres.

Hence, a **kilometre** is about 5 furlongs, or $\frac{5}{8}$ of a mile, and a **metre** is about $1\frac{1}{11}$ yards.

Approximately, English measures of length may be expressed in the Metric System by reducing to yards and deducting 10 per cent., or $\frac{1}{10}$, and calling the remainders metres.

Measures of Area.

1 sq. metre or centiare = 10·764299 sq. ft. = 1·1960333 sq. yds.
1 are = 1076·429934 sq. ft.
1 hectare = 2·4711431 acres.

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1 sq. in. = 6·4513669 sq. cm.
1 sq. ft. = 9·2899683 sq. dm.
1 sq. yd. = ·8360972 sq. metre.
1 acre = ·404671 hectares.

Hence an **hectare** is a little less than $2\frac{1}{2}$ acres, an **are** about $\frac{1}{40}$ of an acre, and a **centiare** about $1\frac{1}{5}$ sq. yards.

Measures of Capacity.

1 litre, or 1 cu. decim. = ·0353166 cu. ft. = 1·760773 pints.
1 kilolitre or 1 cu. metre = 35·3165807 cu. ft. = 220·0966768 galls.

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1 cu. in. = 16·3861759 cu. cm.
1 cu. ft. = 28·3153119 cu. dm.
1 pint = 0·568 of a litre.
1 gallon = 4·54345797 litres.

Hence we see that a **litre** contains a little more than $1\frac{3}{4}$ pints.

MEASURES OF WEIGHT.

1 gram = 15·432349 grains.
1 kilogram = 2·2046213 lb. av.
1 tonneau = 2204·62126 lb.

1 grain = ·06479895 grams.
1 lb. av. = ·45359265 kilogram.
1 cwt. = 50·80238 kilograms.

Hence a **gram** is about $15\frac{1}{2}$ grains, a **kilogram** about $2\frac{1}{5}$ lb., and a **tonneau** rather less than 1 ton.

NOTE.—The word *gramme* is generally shortened into *gram*.

142. It is often necessary in business transactions with the Continent to change English weights and measures into the corresponding equivalent in the Metric System.

Example i. *Express* 8 *tons* 8 *cwt.* 2 *qrs.* 2 *lb. in kilogrammes.*

8 tons 8 cwt. 2 qrs. 2 lb. = 18874 lb.
By Reference Table, 1 kilog. = 2 2046213 lb.;

$$\therefore \quad 18874 \text{ lb.} = \frac{18874}{2{\cdot}2046213} \text{ kilog.}$$

$$= 8561{\cdot}107 \text{ kilog.}$$

Example ii. *How many hectolitres of wine would be required to fill* 100 *dozen bottles, each to contain a pint and a half, if a litre is equivalent to* ·2201 *of a gallon?*

Contents of 100 dozen bottles. $= 1200 \times 1\frac{1}{2}$ pints
$= 225$ gallons
$= \frac{225}{\cdot 2201}$ litres
$= \frac{225}{22{\cdot}01}$ hectolitres;

$\therefore$ No. of hectolitres $= 10{\cdot}2$ (approximately).

EXERCISE LXIX.

(1) Express 1000 decimetres in decametres, and 140 hectometres in centimetres.

(2) How many revolutions will the wheels of a carriage make in travelling 41 kilometres, if the wheels are 125 cm. in diameter? [circumference = 3·1416 × diameter.]

(3) If a man walks 11 metres in 5 sec., how many kilometres can he walk in an hour at the same rate?

(4) Find the time required to travel 31 kilometres 150 metres at the rate of 1 min. 28 secs. per kilometre.

(5) How many miles will be travelled in 1 hr. 28 min. 21 sec. at the rate of 5 kilometres an hour?

(6) How long, to the nearest hour, will a train going at the average rate of 25 miles an hour take in travelling from Paris to Madrid, a distance of 1450 kilometres?

(7) A man rides a bicycle at the rate of 5·75 metres per second. How long, to the nearest second, will it take him to ride 100 kilometres?

(8) Reduce 532 kilometres to miles, furlongs and poles.

(9) How many kilometres can be measured from 393·708 miles?

(10) Given that 1 metre = 39·3708 inches, prove that the difference between 5 miles and 8 kilometres is nearly 51 yards.

(11) If a metre = 39·3708 inches, express 2 miles 5 fur. 32 po. 4 yds in metres.

(12) The velocity of light is 186000 miles per second express this in metres per second, assuming 1 metre = 3·3 ft.

EXERCISE LXX.

(1) How many hectares make a square kilometre?

(2) How many square metres will 240 sheets cover, if each sheet measures 303 mm. by 195 mm.?

(3) How many slabs of marble, 150 sq. cm. in area, will be required to pave a court whose area is 50·7 sq. metres?

(4) How many panes of glass, 40 cm. by 80 cm., will be required for the windows of a house, if the total surface of the glass in the windows is 203·52 sq. metres?

(5) How many ares of land are there in a strip measuring 1 kilometre by 7 metres?

(6) Find the area of a field, 225 metres long and 100 metres wide. Express the result in hectares and ares.

(7) If a square field contain 10·24 hectares, find the length of a side in metres.

(8) How many decimetres are there in the side of a square plot, the area of which is 21904 hectares (or square hectometres)?

(9) If a metre = 39·3708 in., and a hectare = 10000 sq. metres, express a hectare in acres, correct to 3 decimal places.

(10) Assuming that a metre = 39·37 in., find the number of sq. cm. in a sq. foot, neglecting fractions of a centimetre.

(11) If a sq. metre = 1550·031 sq. inches, find to the fifth decimal place the number of sq. cm. in a sq. inch.

(12) Reduce 150 hectares to acres, supposing that 1 sq. metre = 1·196 sq. yards.

(13) If 3 hectares contain 35881 sq. yards, and 1 hectare contains 10000 sq. metres, find the length of a metre in yards.

(14) How many sq. miles are there in 2000 hectares?

EXERCISE LXXI.

(1) Find the volume of a beam, 14·14 metres by 51·6 cm. by 174 mm.

(2) A cistern is 4 metres long, 24 dm. wide, and 80 cm. deep. Find its capacity in cu. metres and in litres.

(3) Express 6453 litres in kilolitres, and 4·15 litres in centilitres.

(4) Find the sum of 1871 cubic metres, 541 litres, and 4·51 hectolitres, and give the result in litres.

(5) Find the number of cu. metres in a box, 2 yds. by 1 yd. by $2\frac{1}{2}$ ft., reckoning 1 cu. yd. = ·76453 cu. metres.

(6) How many cubic inches are there in 25 litres, assuming that 1 linear metre = 39·4 inches?

(7) How many (i) hectolitres, (ii) pints, will be contained in a tank whose capacity is 22 cubic hectometres, assuming that 1 litre = 1 cubic decimetre, and contains 1·76 pints.

(8) Assuming that a litre = ·2201 gallon, find the number of pints in 10 litres, and the number of gallons in 73 litres.

143. In the following examples, the **specific gravity** of a substance means the ratio which the weight of any volume of a substance bears to the weight of an equal volume of water.

EXERCISE LXXII.

(1) Given that a cubic centimetre of water weighs a gramme, how many kilogrammes of water will be wanted to

cover the floor of a skating-rink, 20 metres long and 15 metres broad, to a depth of a decimetre?

(2) How many grammes are there in 2 cwt. 3 qrs. 11 lb., assuming 1 lb = 7000 grains, and 1 gramme = 15·4 grains?

(3) What is the weight in kilograms of 27 hectolitres of water?

(4) What is the weight of 20 cu. cm. of mercury, whose sp. gravity = 13·5?

(5) Find the weight in kilograms of 15 gallons of water.

(6) A farmer lays 2 tons of lime on an acre of land; how many grams is it per sq. metre? [1 metre = $39\frac{3}{8}$ in., 1 kilo = $2\frac{1}{5}$ lb.]

(7) A box measures 3·75 metres by 3·50 metres by 50 cm. Find how many litres of oil it will hold, and the weight of the oil in kilos; the oil being ·914 as heavy as water.

(8) How many litres are there in a vat measuring 2 metres by 75 cm. by 50 cm.? Find also the weight in kilos of the sulphuric acid (sp. gravity 1·840) required to fill it.

(9) Find the weight in kilos of a bar of gold, 10 cm. by 30 mm. by 25 mm., its sp. gravity being 19·36.

(10) Find the weight in kilograms of the air in a room 23 metres long, 16 metres wide, and 10 metres high; air being ·00129206 as heavy as water.

(11) If the sp. gravity of sea water be 1·026, and that of olive oil be ·915, find the weight of a hectolitre of each in kilos.

(12) If the sp. gravity of alcohol be ·8, how many kilograms of alcohol will be required to fill, one-third full, a tank measuring 1·50 metres by 3·20 metres by 80 cm.?

(13) If a cu. foot of oil weighs 55 lb., what will a cubic metre weigh in kilograms? [1 ft. = ·3048 metres, 1 kilo = $2\frac{1}{5}$ lb.]

(14) A rectangular sheet of tin of uniform thickness is 85 cm. wide and 2·7 metres long, and weighs 536 grams. Find its thickness, if the sp. gravity of tin be 7·3.

(15) A plate of iron, 137 cm. by 643 mm. by 43·1 mm., weighs 277·54 kilos. Find its sp. gravity.

THE COINAGE OF FRANCE.

144. In France, Belgium and Switzerland the principal coin is called a **franc**, and this amount is divided into 100 equal parts called **centimes**.

In consequence of the relation between the centime and the franc, all operations with money are much simplified.

For example, 13 centimes = ·13 of a franc,
513 ..... = 5·13 francs,
59609 ..... = 596·09 francs,
2 ..... = ·02 of a franc.

Again, 5 francs 69 centimes = 5·69 francs = 569 centimes,
159 ..... 24 ..... = 159·24 francs = 15924 centimes.

The reduction of centimes to francs is therefore effected by inserting a decimal point to the left of the second digit from the end, and the reduction of francs to centimes by removing the point. The reduction of money in this system is therefore very much simpler than in our own.

A franc is equivalent to about $9\frac{1}{2}d.$

All accounts in France, Belgium and Switzerland, are kept in francs and centimes. The *sou*, a piece of money worth 5 centimes, and the *napoleon*, worth 20 francs, are in circulation.

Notes are issued by the Bank of France for 50 fr., 100 fr., 500 fr., and 1000 fr., and are legal tender.

EXERCISE LXXIII.

Reduce to francs

(1)	35010 c.	(2)	3161 c.	(3)	4035 c.	(4)	6234 c.
(5)	15601 c.	(6)	34001 c.	(7)	15 c.	(8)	57 c.
(9)	4 c.	(10)	3 c.	(11)	1 c.	(12)	2·5 c.

Reduce to centimes

(13)	56 fr. 34 c.	(14)	4 fr. 15 c.	(15)	3 fr. 82 c.
(16)	5 fr. 9 c.	(17)	25 fr. 3 c.	(18)	36 fr.

(19) Add together 120 fr. 65 c., 69 fr. 55 c., 85 c., 150 fr. 45 c., 1500 fr. 75 c.

(20) Add together 69 fr. 12 c., 150 fr. 28 c., 75 c., 190 fr. 50 c., 1006 fr. 80 c.

(21) From 1000 fr. 19 c. take 507 fr. 91 c.

(22) From 2050 fr. 10 c. take 770 fr. 75 c.

(23) Multiply 205 fr. 15 c. by 19.

(24) Multiply 1876 fr. 82 c. by 365.

(25) Divide 2664 fr. 75 c. by 17.

(26) Divide 8680 fr. 66 c. by 113.

145. It is often necessary to express an English price in its equivalent in francs and centimes.

Example. What is the value in French money of £627. 10*s.*, *if the value of a sovereign is* 25·2 *francs?*

Value of £1 = 25·2 francs,
∴ .....£627·5 = 25·2 × 627·5 francs
= 15813 francs.

146. The following is a more complicated example, of a kind which is often met with in business transactions.

Example. Express 15 *francs per kilogramme in pence per lb., assuming that* 25·1 *francs* = £1, *and* 1 *kilog.* = 2·2 *lb.*

The cost of 1 kilog. = 15 fr.,
∴ ..... 2·2 lb. = 15 fr.,
∴ ..... 2·2 lb. $= \frac{15 \times 240}{25 \cdot 1}$ pence,
∴ ..... 1 lb. $= \frac{15 \times 240}{25 \cdot 1 \times 2 \cdot 2}$ pence
$= \frac{1800}{27 \cdot 61}$ pence
= 65 pence (nearly)
= 5*s.* 5*d.*

EXERCISE LXXIV.

Express in English money, to the nearest penny,

(1) 511·175 francs, assuming 25·4 fr. = £1.

(2) 19054·28 francs, assuming 25·25 fr. = £1.

(3) 12687 fr. 50 c., assuming 25 fr. 32½ c. = £1.

Express in francs

(4) £246. 13*s*. 10*d*., assuming £1 = 25 fr. 18 c.

(5) £256. 18*s*. 4*d*., assuming £1 = 25 fr. 30 c.

(6) £316. 10*s*. 3*d*., assuming £1 = 25 fr. 28 c.

(7) A bill of 253 francs is paid partly in French, partly in English, money. If the amount of French money paid is 95 francs, find to the nearest penny how much English money is required, assuming that 25·06 francs = £1.

(8) A Frenchman ordered an 18 guinea watch from England. How many francs must he remit, assuming that £1 = 26·25 francs, and that 10*s*. discount is allowed for ready money?

(9) A merchant in London buys goods in Paris, the price of which is a million francs. What will be their cost in English money, allowing 10 per cent. additional cost for duty and transit, and assuming £1 = 25·25 francs?

(10) A dealer buys 30 lambs for 1420 fr. 75 c., and 25 more for 915 fr. 25 c., and sells them at 43 fr. 5 c. each. What does he gain in francs and centimes?

EXERCISE LXXV.

(1) Find the cost of 82 kilog. 125 gr. of butter at 3 fr. 44 c. per kilog.

(2) A contractor excavates a trench 6 m. 50 cm. long, 4 m. 20 cm. wide, and 7 m. 40 cm. deep, at 2 fr. 50 c. per cubic metre. Find the cost.

(3) A man pays 12000 francs for 5 ares of land, and sells it for 25 fr. 20 c. per sq. metre. How much does he gain?

(4) If olive oil costs 60 centimes a kilogram, find the cost of a hectolitre, if the sp. gravity of the oil be ·914.

(5) Find the cost of a carpet 75 cm. wide at 4 fr. 25 c. a metre, for a room 5·25 metres by 4·75 metres.

(6) A package of candles which weighs 465 grams is sold for 28 centimes; find the price of 10 kilograms.

(7) French silk is quoted at $7\frac{1}{2}$ francs per metre. Express the price of a yard in English money, to the nearest penny, assuming that 1 metre = 39·371 inches, and 1 fr. = 9·38 pence.

(8) What is 1*s.* 6*d.* per gallon equivalent to in francs per litre, assuming that 1 gallon of water weighs 10 lb., 1 kilo = $2\frac{1}{5}$ lb., and £1 = 25 fr.?

(9) If tar-paving costs 2*s.* 6*d.* per sq. yard, what amount in francs will pay for a pavement 1 kilometre long by 15 metres broad? [1 linear foot = 3·05 decim.; 1 franc = 10*d.*]

(10) The annual rent in France of a hectare of land is 130 francs; find the rent of one acre in English money to the nearest penny. [25 francs = £1; 100 hectares = 247 ac.]

(11) A length of 8 m. 60 cm. of Lyons silk was bought for 253 fr. 8 c.; find the cost of 17 m. 64 cm. of the same silk in English money, assuming £1 = 25·15 francs.

(12) Find the exact value of 1·7625 metres of cloth at 3·78 fr. per metre; and give the value of the cloth in English money, to the nearest penny, reckoning 25 fr. to £1.

(13) Find, to the nearest penny, the value of a bar of gold $2\frac{1}{3}$ metres long, at £1 per inch, assuming 1 metre = 39·37079 inches.

(14) Calculate, to the nearest pound, the value of 5122·7 kilog. at 3·43 fr. per kilog., assuming 25·42 fr. = £1.

(15) Which is the cheaper:—French silk at 3 fr. 85 c. per metre, or English silk at 2*s.* $7\frac{1}{2}$*d.* per yard?

(16) The railway fare from Calais to Paris, a distance of 240 kilometres, is 38·16 francs; and from London to Dover, a distance of 70 miles, is 14*s.* 7*d.* Compare the cost of travelling in the two countries, assuming that 1 metre = 39·3708 in., and 25·44 francs = £1.

PART II.

THE COINAGE, WEIGHTS AND MEASURES OF INDIA.

147. In **British India** the common medium of exchange is **silver.** The principal coin is therefore made of silver. It is called a **rupee**, and in size resembles an English *florin*.

The exchange value of the rupee varies to some extent with the price of silver. It is now about **1***s.* 4*d.*, so that £1 in gold is now worth about 15 rupees.

The other silver coins are

(i) a half-rupee, 2 of which make a rupee,
(ii) a four-anna piece, 4 of which make a rupee,
(iii) a two-anna piece, 8 of which make a rupee.

For smaller values there are copper coins, the chief of which is called a **half-anna**; 32 of these represent the same value as one rupee. The other copper coins are

(i) a pice, 4 of which make an anna,
(ii) a half-pice, 8 of which make an anna,
(iii) a pie, 3 of which make a pice, and 12 of which make an anna.

The rupee is the standard coin also in British East Africa.

148. A sum of money, such as 7 rupees, 10 annas and 2 pies, is written thus :—

Rs. 7. 10 as. 2 p.,

where the symbols *Rs.*, *as.* and *p.* are abbreviations respectively of the words rupees, annas and pies.

149. The relations between the rupee, anna, etc., are stated in the following

BRITISH INDIAN MONEY TABLE.

3 pies make 1 pice (or paisa),
4 pice ... 1 anna,
16 annas ... 1 rupee.

NOTE.—There is no coin called an anna. Accounts are kept in rupees and annas.

The number 100000 is called a **lac**, and is expressed in figures thus: 1,00,000.

150. There is also a **paper currency** in use in India, consisting of paper notes representing 5, 10, 20, 50, 100, 500, 1000 and 10000 rupees respectively.

151. In **Ceylon**, too, the principal coin is the silver rupee, but it is there divided into 100 **cents**.

The other coins in use are

(i)	a 50-cent piece,	2 of which make a rupee,
(ii)	a 25-cent piece,	4 of which make a rupee,
(iii)	a 10-cent piece,	10 of which make a rupee.

CEYLON MONEY TABLE.

100 cents make 1 rupee.

152. The basis of the British Indian legal system of weights is a **tola**, which is of exactly the same weight as the rupee, *i.e.*, 180 grains.

THE INDIAN IMPERIAL MEASURE OF WEIGHT.

The weight of one rupee } is { 1 tola or 180 grains,
5 tolas make 1 chatak,
4 chataks ... 1 powa,
4 powas ... 1 seer,
5 seers ... 1 panch-seri,
40 seers ... 1 maund (Bazaar).

The term **maund** is used with different local meanings in different parts of India, being 82 lb. in Bengal, 28 lb. in Bombay City, and $24\frac{2}{3}\frac{4}{5}$ lb. in Madras. The Bazaar maund is equal to 100 lb. troy, or $82\frac{2}{7}$ lb. av.

The metric system of weights and measures was legalised in 1870, the French kilogramme being taken as the standard of weight, and named the **ser**.

NOTE.—1 tola = ·011664 ser.

153. The standard of capacity in the metric system (the litre) is called a **ser** (1·760773 pints).

154. Lineal measures vary throughout India. In Bengal the chief measure is the **hath** (18 inches); the English yard is much used, and the metre also is legal.

155. British weights and measures are used in Ceylon.

EXERCISE LXXVI.

Reduce

(1) Rs. 25. 4 as. to annas.
(2) Rs. 13. 6 as. 4 p. to pies.
(3) Rs. 32 to two-anna pieces.
(4) Rs. 23 to four-anna pieces.
(5) 4314625 pies to rupees.
(6) 18436 half-anna pieces to rupees.
(7) 13756 pice to rupees.
(8) 365688 pies to rupees.

Add together

(9) Rs. 53625. 13 as. 7 p., Rs. 4837. 4 as. 9 p., Rs. 4. 11 as. 6 p.
(10) Rs. 10. 6 as. 8 p., Rs. 54. 9 as. 4 p., Rs. 893. 15 as. 9 p.
(11) Rs. 368. 6 as. 8 p., Rs. 535. 7 as. 1 p., Rs. 97. 3 as. 2 p.

Subtract

(12) Rs. 13. 12 as. 6 p. from Rs. 24. 15 as. 10 p.
(13) Rs. 24. 5 as. 9 p. from Rs. 150. 4 as. 10 p.
(14) Rs. 488. 15 as. 6 p. from Rs. 1000. 8 as. 4 p.

Multiply

(15) Rs. 83. 13 as. 5 p. by 3, 7, 11.
(16) Rs. 4012. 8 as. 2 p. by 120, 112, 128.

Divide

(17) Rs. 944. 8 as. 6 p. by 25 and 75.
(18) Rs. 38407. 6 as. 8 p. by 121 and 143.

Express in English money

(19) Rs. 500 at 1*s.* 3½*d.* per rupee.
(20) Rs. 1786 at 1*s.* 7½*d.* per rupee.
(21) Rs. 1900 at 1*s.* 8*d.* per rupee.

Express in rupees

(22) £198 at Rs. 9. 10 as. per £.
(23) £279 at Rs. 10. 4 as. per £.
(24) £580 at Rs. 10. 8 as. per £.

EXERCISE LXXVII.

(1) Find the value of $425\frac{3}{4}$ yds. of muslin at Rs. 3. 13 as. 4 p. per yard.

(2) A broker received 5 per cent. for buying indigo. His commission amounted to Rs. 432. 8 as.; find the value of the indigo.

(3) Find the commission, at $2\frac{1}{2}$ per cent., on the sale of 736 barrels of flour at R. 1. 4 as. per barrel.

(4) What is the cost of insuring a cargo valued at Rs. 6750, the premium being 3 per cent.?

(5) A broker buys cotton valued at Rs. 5692. 12 as.; what is his commission at $1\frac{1}{2}$ per cent.?

(6) If Rs. 450 were borrowed on Jan. 1st, 1898, at 5 per cent. per annum, what sum would repay the debt on Oct. 20th, 1898?

PROFIT AND LOSS.

156. When a thing is sold for more than it cost, it is said to be sold at a profit: when it is sold for less, it is said to be sold at a loss.

Profit or loss may be reckoned in general terms as a *percentage* of the money originally expended.

For example, a man who buys goods for £100, and sells them for £110, makes a *profit* of £10 on an outlay of £100, *i.e.*, a profit of 10 per cent.

And if he buys them for £100, and sells them for £90, he *loses* £10 on an outlay of £100, *i.e.*, he loses 10 per cent.

The meaning of the expressions "profit per cent." and "loss per cent." must be noticed carefully. **The percentage is reckoned on the original outlay.**

157. Case I.—Given the cost price and the selling price; **to find the gain or loss per cent.**

Example i. *A grocer buys coffee at* 10*d. per lb., and sells it at* 1*s.* 4*d. per lb.; what is the gain per cent. on the cost price?*

Gain on $10d. = 6d.$;
$\therefore$ gain on $100d. = 60d.$;
$\therefore$ gain per cent. $= 60$.

Example ii. *A grocer sells, at* 1*s.* 6*d. per lb.. tea, which cost him* 2*s. per lb.; what is the loss per cent. on the cost price?*

Loss on $24d. = 6d.$;

$\therefore$ loss per cent. $= \frac{6}{24}$ of 100

$= \underline{25}.$

Example iii. *A grocer buys sugar at* £1. 5*s.* 4*d. per cwt., sells it at* 5*d. per lb., and allows a discount of* 5 *per cent. to purchasers for ready money; what is the gain or loss per cent. on the cost price?*

Money obtained per cwt. after allowing 5 per cent. disct. $= \frac{19}{20}$ of $5d. \times 112$

$= £2.\ 4s.\ 4d.$;

$\therefore$ gain on £1. 5*s.* 4*d.* $= 19s.$;

$\therefore$ gain per cent. $= \frac{19}{25\frac{1}{3}}$ of 100

$= \frac{3}{4}$ of 100

$= \underline{75}.$

Example iv. *A dealer buys eggs at* 7*s. a hundred, and sells them at* 13 *for* 1*s.; what is his gain per cent. on the cost price?*

Selling price of 1 egg $= \frac{1}{13}s.$;

$\therefore$ .....100 eggs $= \frac{100}{13}s. = 7\frac{9}{13}s.$;

$\therefore$ gain on $7s. = \frac{9}{13}s.$;

$\therefore$ gain per cent. $= \frac{\frac{9}{13}}{7}$ of 100

$= \frac{9}{91}$ of 100

$= \underline{9\frac{81}{91}}$

EXERCISE LXXVIII.

(1) A watch which cost £4 was sold for £4. 18*s.*; find the gain per cent.

(2) A man buys goods for £15. 6*s.* 3*d.*, and sells them again for £11. 15*s.* $9\frac{3}{4}d.$; how much does he lose per cent.?

(3) Goods costing £35. 0*s.* 5*d.* are sold for £42. 0*s.* 6*d.*; what is the gain per cent.?

(4) Sugar costing £2. 12*s.* per cwt. is sold at $7\frac{1}{2}d.$ per lb.; what is the gain per cent.?

(5) A draper pays 10 guineas for a piece of silk containing 46 yards, and sells it at the rate of £2. 7*s.* 3*d.* for 8 yards; what does he gain per cent.?

(6) Ten tons of goods, bought at 27*s.* 6*d.* per cwt., are sold at £30 per ton; what is the gain per cent.?

(7) If 126 sheep are bought for £409. 10*s.*, and sold for £488. 5*s.*, what is the profit on each, and the profit per cent.?

(8) A house was sold for £5000, at a profit of £500; what would have been the loss per cent. if it had been sold for £4000?

(9) A man buys goods at the rate of £24 per cwt., and sells 3 tons 13 cwt. 1 qr. for £2197. 10*s.*; how much has he gained or lost per cent. on his outlay?

(10) A merchant bought 24 cwt. of sugar at $5\frac{1}{2}d.$ per lb., and sold $\frac{1}{3}$ of it at $6\frac{1}{2}d.$, $\frac{1}{4}$ of it at $6d.$, and the remainder at $5\frac{1}{2}d.$ per lb.; what is his gain per cent.?

(11) If I buy 10 horses for £300, and 12 cows for £120, and sell the former at £35 each, and the latter at £12 each, what profit do I make per cent. on my outlay?

(12) Sugar is bought at $1\frac{1}{4}d.$ a lb., and sold at £18. 13*s.* 4*d.* a ton; find the gain or loss per cent.

(13) *A* and *B* buy cigars at £2. 1*s.* 8*d.* per 100. *A* sells them at 6*d.* each, while *B* sells them in bundles of 25 for 12*s.*; compare their gains per cent.

(14) A publisher sells books to a bookseller at 5*s.* a copy, but allows 25 copies to count as 24; if the bookseller retail the 25 copies at 6*s.* 9*d.* each, what profit per cent. does he make?

(15) Two parts of chicory, costing £1. 9*s.* 2*d.* per cwt., are mixed with five parts of coffee, costing £8. 4*s.* 6*d.* per cwt.; if the mixture be sold at 1*s.* 4*d.* per lb., find the profit per cent.

(16) A mixture of tea is made by mixing one chest containing 2 qrs. 17 lb. at £14 per cwt. with two chests, each containing 3 qrs. 7 lb., at £18. 13*s.* 4*d.* per cwt.; if the mixture be sold at 4*s.* 2*d.* per lb., find the profit per cent.

(17) A grocer buys rice at £1. 2*s.* 2*d.* per cwt., sells it at 4*d.* per lb., and takes off 5 per cent. for cash payment; what profit per cent. does he make?

(18) A grocer buys tea at £8. 6*s.* 8*d.* per cwt., and sells it at 2*s.* 1*d.* per lb., deducting 5 per cent. for cash payment; what profit per cent. does he make?

(19) A woman buys apples at the rate of 3 for 2*d.*, and sells them at 2 for 3*d.*, what rate per cent. profit does she make?

(20) If oranges are bought at the rate of 20 for 1*s.*, and sold at the rate of 7*s.* for a hundred, what is the gain per cent.?

(21) A newspaper boy buys papers at 9*d.* per dozen, getting 13 to the dozen; if he sell them at a penny each, what is his profit per cent.?

(22) A pedlar buys pins 18 in a row, and sells them 11 in a row at the same price; what is the gain per cent. on his outlay?

(23) How much is gained or lost per cent. by buying oranges at 5 for 2*d.*, and selling half of them at 2 a penny and half at 3 a penny?

(24) A man buys eggs at a certain price per score, and sells them at half that price per dozen; what is his gain or loss per cent.?

158. CASE II.—Given the cost price and the gain or loss per cent.; **to find the selling price.**

Example i. *If a house was bought for* £9600, *and is sold at a profit of* $12\frac{1}{2}$ *per cent., what does the profit amount to, and for how much is the house sold?*

$$\text{Profit} = \frac{12\frac{1}{2}}{100} \text{ of } £9600$$
$$= \tfrac{1}{8} \text{ of } £9600$$
$$= £1200;$$
$$\therefore \text{ selling price of the house} = £9600 + £1200$$
$$= \underline{£10800}.$$

Otherwise, more directly:—

$$\text{selling price of the house} = \frac{112\frac{1}{2}}{100} \text{ of } £9600$$
$$= \tfrac{9}{8} \text{ of } £9600$$
$$= £1200 \times 9$$
$$= \underline{£10800}.$$

Example ii. *If a house was bought for* £1350, *and is sold at a loss of* 20 *per cent., what does the loss amount to, and for how much is the house sold?*

$$\text{Loss} = \tfrac{20}{100} \text{ of } £1350$$
$$= £270;$$
$$\therefore \text{ selling price of the house} = £1350 - £270$$
$$= \underline{£1080}.$$

Otherwise, more directly:—

$$\text{selling price of the house} = \tfrac{80}{100} \text{ of } £1350$$
$$= \tfrac{4}{5} \text{ of } £1350$$
$$= £270 \times 4$$
$$= \underline{£1080}.$$

EXERCISE LXXIX.

(1) A house is bought for £4250, and is sold at a profit of 12 per cent.; how much was gained, and for how much was the house sold?

(2) Cloth is bought at 1*s.* 6*d.* a yard; at what price per yard must it be sold to gain 25 per cent.?

(3) A tradesman, selling off, reduces his prices to 10 per cent. below cost price; what did he get for goods for which he gave £65?

(4) The cost price of a book is 8*s.* 4*d.* If the expense of sale be 15 per cent. upon this, and the profit 20 per cent. upon the total expense, what would be the retail price?

(5) A man buys goods for £10. 5*s.* 2½*d.*, and sells them at a profit of 16 per cent.; what did he sell them for?

(6) A man bought a horse for £45, and sold it again at a loss of 7 per cent.; for how much was the horse sold?

(7) A merchant buys a fifty-gallon cask of wine for £62. 10*s.*, and sells it at a profit of 4 per cent.; at what price per gallon does he sell the wine?

(8) A grocer buys 10 cwt. 3 qrs. 21 lb. of sugar for £30, and pays 12*s.* 6*d.* for expenses; at what rate must he sell it per lb. to gain 25 per cent.?

(9) A farmer rents 305½ acres of land at £4. 17*s.* 6*d.* per acre, but the landlord allows an abatement of 20 per cent.; what does the tenant pay?

(10) A plumber bought 4 tons 16 cwt. of lead for £1. 0*s.* 2½*d.* per cwt., and sold the whole so as to clear 12½ per cent.; what did he receive from the sale?

(11) A grocer mixes two kinds of sugar at 4*d.* and 6½*d.* per lb., taking 3 lb. of the first to 2 lb. of the second; at what price per lb. must he sell the mixture to make a profit of 20 per cent. on his outlay?

(12) A spirit merchant mixes 80 gallons of whisky at 15*s.* 6*d.* per gallon with 96 gallons at 17*s.* 1*d.*, and sells the mixture so as to make a profit of 10 per cent. on his outlay; at what price per gallon does he sell it?

(13) A grocer buys coffee at £8. 10*s.* per cwt., and chicory at £2. 10*s.* per cwt.; he mixes them in the proportion of 5 parts chicory to 7 parts coffee; at what price per lb. must he sell the mixture to gain 16⅔ per cent. on his outlay?

159. The following examples of business transactions must be carefully noticed.

Example i. *A shopkeeper, having a watch which cost him* £13. 2*s.* 6*d.*, *prices it so that he may make a profit of* 30 *per cent. upon the cost, and allow a discount of* $2\frac{1}{2}$ *per cent. to the purchaser; at what price is the watch marked?*

Selling price to gain 30 per cent. $= \frac{130}{100}$ of £13. 2*s.* 6*d.*
$= \frac{13}{10}$ of £13. 2*s.* 6*d.*
$=$ £1. 6*s.* 3*d.* $\times 13$
$=$ £17. 1*s.* 3*d.*

Marked price to allow $2\frac{1}{2}$ per cent. discount $= \frac{100}{97\frac{1}{2}}$ of £17. 1*s.* 3*d.*
$= \frac{40}{39}$ of £17. 1*s.* 3*d.*
$=$ 8*s.* 9*d.* $\times 40$
$=$ £17. 10*s.*

Otherwise, more directly:—

Marked price $= \frac{130}{97\frac{1}{2}}$ of £13. 2*s.* 6*d.*
$= \frac{4}{3}$ of £13. 2*s.* 6*d.*
$=$ £4. 7*s.* 6*d.* $\times 4$
$=$ £17. 10*s.*

Example ii. *A tradesman's prices are* 20 *per cent. above cost price; if he allow a customer* 10 *per cent. on his bill, what profit does he make? What profit per cent. would he make if he were to allow the customer a penny in the shilling?*

(i) Tradesman's receipts $= \frac{9}{10}$ of $\frac{120}{100}$ of cost
$= \frac{108}{100}$ of cost;
$\therefore$ profit $=$ 8 per cent.

(ii) Tradesman's receipts $= \frac{11}{12}$ of $\frac{120}{100}$ of cost
$= \frac{110}{100}$ of cost;
$\therefore$ profit $=$ 10 per cent.

Example iii. *How much per cent. must a tradesman add on to the cost price of his goods, that he may make* 10 *per cent. profit after allowing a customer a reduction of* 4 *per cent. on his bill?*

$\frac{96}{100}$ of selling price $= \frac{110}{100}$ of cost;
$\therefore$ selling price $= \frac{55}{48}$ of cost $= 1\frac{7}{48}$ of cost;
$\therefore$ rate of profit $= \frac{700}{48}$ per cent.
$= 14\frac{7}{12}$ per cent.

EXERCISE LXXX.

(1) A silver cup cost the shopkeeper $4\frac{1}{2}$ guineas; for what must he sell it to obtain a profit of 5 per cent. after allowing the purchaser a discount of $5\frac{1}{2}$ per cent.?

(2) If a watch costs a shopkeeper £3. 14*s.*, at what price must he sell it to make a profit of $12\frac{1}{2}$ per cent. after allowing the purchaser a discount of $7\frac{1}{2}$ per cent.?

(3) A jeweller bought a bracelet for £3. 11*s.* 6*d.*; at what price must he sell it to gain $12\frac{1}{2}$ per cent. and allow a purchaser $2\frac{1}{2}$ per cent. discount for cash?

(4) If a piano costs a dealer £79. 15*s.*, at what price must he sell it to gain 53 per cent. and allow a purchaser a discount of $6\frac{1}{2}$ per cent.?

(5) A piece of furniture cost the manufacturer £3; at what price must he mark it to gain 15 per cent. after allowing a purchaser 10 per cent. for cash?

(6) If wine costs 15*s.* per gallon, at what price must it be sold to gain $17\frac{1}{2}$ per cent. after allowing the purchaser 6 per cent. for cash?

(7) A gold chain cost £9. 10*s.*; at what price must it be marked to give a profit of $18\frac{3}{4}$ per cent. after allowing the purchaser 5 per cent. discount for cash?

(8) Goods costing the manufacturer £4162. 10*s.* were retailed at a profit of 5 per cent. after allowing the purchaser a discount of $7\frac{1}{2}$ per cent.; at what amount were the goods priced?

(9) A tradesman, buying goods for £814. 9*s.*, makes $10\frac{1}{2}$ per cent. after allowing his customers 9 per cent. discount for cash; what is the marked price?

(10) A wholesale grocer buys raisins at £2. 9*s.* per cwt., and retails them at a profit of 8 per cent., allowing the retail grocer 16 per cent. discount for cash; what is the actual price per lb. to the retail grocer?

(11) A tradesman's prices are 30 per cent. above cost price, but he allows his customers 15 per cent. off their bills for cash payments; what is his actual gain per cent.?

(12) A tradesman's goods are marked 22 per cent. above cost price, but he takes off 5 per cent. from his customers bills for cash; how much per cent. profit does he actually make?

(13) A corn merchant's prices are 25 per cent. above cost price; if he allows a customer 12 per cent. off his bill, what profit per cent. does he realise?

(14) How much per cent. must a tradesman add on to the cost price of his goods, so that he may make 10 per cent. profit after allowing customers 12 per cent. off their bills for cash payments?

(15) Find the exact percentage of profit at which goods must be marked for sale so that, after taking 5 per cent. off for cash payments, the actual rate of profit may be 20 per cent.

(16) A publisher sells to a bookseller at 10 per cent. profit, and the bookseller sells to his customers at 25 per cent. profit; how much per cent. more than the prime cost does the purchaser pay?

(17) If a brewer sells beer to a retailer at a profit of 10 per cent., and the retailer sells to a customer at a profit of 50 per cent., what percentage of the price paid by the consumer is profit?

160. CASE III.—Given the selling price and the gain or loss per cent.; **to find the cost price.**

Example i. *Goods are sold for* £11500, *at a profit of* 15 *per cent. on the cost price; what did they cost?*

$$\begin{aligned} £11500 &= \text{cost price} + \tfrac{15}{100} \text{ of cost price} \\ &= \text{cost price} + \tfrac{3}{20} \text{ of cost price} \\ &= \tfrac{23}{20} \text{ of cost price;} \\ \therefore \text{cost price} &= \tfrac{20}{23} \text{ of } £11500 \\ &= \underline{£10000.} \end{aligned}$$

Otherwise, more directly:—

$$\text{cost price} = \tfrac{100}{115} \text{ of } £11500 = \underline{£10000.}$$

Example ii. *A cow is sold for* £19. 4*s.*, *at a loss of* 20 *per cent. on the cost price; what did it cost?*

$$\begin{aligned} £19.\ 4s. &= \text{cost price} - \tfrac{1}{5} \text{ of cost price} \\ &= \tfrac{4}{5} \text{ of cost price;} \\ \therefore \text{cost price} &= \tfrac{5}{4} \text{ of } £19.\ 4s. \\ &= \underline{£24.} \end{aligned}$$

Otherwise, more directly:—

$$\text{cost price} = \tfrac{100}{80} \text{ of } £19.\ 4s. = \underline{£24.}$$

Example iii. *If a manufacturer makes a profit of* 10 *per cent., the wholesale dealer of* 15 *per cent., and the shopkeeper of* 20 *per cent., what was the cost to the manufacturer of an article bought at a shop for* £6. 6*s.* 6*d.*?

Cost price to the shopkeeper $= \frac{100}{120}$ of £6. 6*s.* 6*d.*;
∴ ..... wholesale dealer $= \frac{100}{115}$ of $\frac{100}{120}$ of £6. 6*s.* 6*d.*;
∴ ..... manufacturer $= \frac{100}{110}$ of $\frac{100}{115}$ of $\frac{100}{120}$ of £6. 6*s.* 6*d.*
$= \frac{10}{11} \times \frac{20}{23} \times \frac{5}{6} \times £\frac{2533}{40}$
$= £\frac{2533}{264}$
$= £4.\ 3s.\ 4d.$

EXERCISE LXXXI.

(1) Find the cost price of a diamond ring sold for £27. 19*s.*, at a gain of $7\frac{1}{2}$ per cent.

(2) Goods are sold for £11. 18*s.* $0\frac{1}{2}d.$, at a profit of 16 per cent.; what did they cost?

(3) A house is sold for £2030, at a profit of 12 per cent.; what did it cost?

(4) A horse is sold for £102, at a loss of 15 per cent.; what did it cost?

(5) Goods were sold for £216, at a loss of 10 per cent.; what did they cost?

(6) A watch was sold for £28, at a gain of $16\frac{2}{3}$ per cent.; what did it cost?

(7) By selling 10 acres of land for £4699. 8*s.* 3*d.*, a man gained $5\frac{1}{4}$ per cent.; what was the original price per acre?

(8) What is the cost of lead per cwt., if the sale of 48 cwt. for £45. 11*s.* 3*d.* gives a profit of £12. 10*s.* per cent.?

(9) A grocer by selling sugar at $3\frac{1}{2}d.$ per lb. loses 2 per cent. on his outlay; what did it cost him per cwt.?

(10) A merchant sells cigars at 1*s.* each, and makes a profit of $42\frac{6}{7}$ per cent. on his outlay; what was the cost price of 100 cigars?

(11) If a wholesale dealer makes a profit of 5 per cent., and the retail dealer $16\frac{2}{3}$ per cent., what was the cost to the wholesale dealer of goods sold by the retailer for 140 guineas?

(12) The manufacturer of an article makes a profit of 25 per cent., the wholesale dealer makes a profit of 20 per cent., and the retail dealer makes a profit of 28 per cent.; what is the cost to the manufacturer of an article which is retailed for 16*s.*?

(13) *A* sold some goods to *B*, making a profit of 10 per cent.; *B* sold them for £6. 1*s*., making a profit of 10 per cent.; what did *A* pay for the goods?

(14) *A* sold a horse to *B*, gaining $7\frac{1}{2}$ per cent. on what it cost him; *B* sold it to *C* for £70. 19*s*., gaining 10 per cent. on what it cost him; what did *A* pay for the horse?

(15) *A* makes an article and sells it to *B*, *B* sells it to *C*, and then *C* sells it to *D* for 35*s*.; *A* makes 20 per cent. profit, *B*, 25 per cent., *C*, 40 per cent.; what was the cost of manufacture?

(16) A shopkeeper marks his goods at a price from which he can deduct $7\frac{1}{2}$ per cent. for prompt payment, and still make a profit of 10 per cent. on what the goods cost him; find the cost price of an article marked at £2. 15*s*.

(17) A grocer sells sugar at 4*d*. per lb., and takes off 5 per cent. for cash payment; find what it cost him per cwt. if he makes a profit of 60 per cent.

(18) An ironmonger sells beeswax at 2*s*. 1*d*. per lb., and takes off 5 per cent. for cash payment; find what it cost him per cwt., sold for cash, in order that he may make a profit of 33 per cent.

MIXTURES.

160a. We may mix two or more qualities of wine at different prices so as to produce wine at almost any price between the greatest and least of those that are mixed.

If, for example, we mix one bottle of wine at 2*s*. 6*d*. with one at 3*s*. 6*d*., we can make up two bottles worth 6*s*. together or 3*s*. each; and again if we mix one bottle at 2*s*. 6*d*. with 99 at 3*s*. 6*d*., we can make up 100 bottles at a trifle less than 3*s*. 6*d*.

We will consider how to determine (i) the price of a mixture when the quantity and price of each ingredient are known, and (ii) the quantity of each ingredient to be taken to make a mixture of a particular value.

CASE I. *Example. If 30 tons of coal at 20s. 6d. per ton be mixed with $22\frac{1}{2}$ tons at 24s. per ton, find the price of the mixture.*

Cost of 30 tons at 20*s*. 6*d*. $= £30 \times 1\frac{1}{40}$
$= £30.\ 15s.$;
cost of $22\frac{1}{2}$ tons at 24*s*. $= £22\frac{1}{2} \times 1\frac{1}{20}$

$= £\frac{45}{2} \times \frac{6}{5}$
$= £27;$
$\therefore$ cost of $52\frac{1}{2}$ tons of mixture $= £57\frac{3}{4}$,
$\therefore$ cost of 1 ton of mixture $= £\frac{57\frac{3}{4}}{52\frac{1}{2}} = £\frac{231}{210} = £\frac{11}{10}$
$= £1.\ 2s.;$
$\therefore$ price required $= £1.\ 2s.$ per ton.

EXERCISE LXXXIa.

(1) A man bought 2 qrs. of corn at £1. 19*s.* per qr., and 7 bush. at £2. 8*s.* per qr.; he mixed them and sold the mixture at 6*s.* 3*d.* per bush.; how much did he gain?

(2) A wine merchant mixes 2 casks of wine at 12*s.* 6*d.* a gallon with 1 cask at 18*s.* a gallon; what is the value of the mixture per gallon?

(3) A mixture is made of 1 gallon of ale at 5*d.* per gallon, 3 at 9*d.*, 4 at 1*s.*, and 12 at 4*d.*; how much per gallon is the mixture worth?

(4) If 25 lb. of coffee at 1*s.* $1\frac{1}{2}d.$ per lb. be mixed with 200 lb. at 1*s.* $8\frac{1}{4}d.$, what is the value per lb. of the mixture?

(5) A mixture of two kinds of wine, in the proportion of 4 gallons of one to 11 gallons of the other is worth £1. 8*s.* per gallon. The first kind being worth £1. 15*s.* 4*d.* per gallon, what is the value of the second kind?

(6) A quantity of coffee has chicory mixed with it, so that the chicory is $\frac{1}{9}$th of the whole mixture. If the mixture be worth 2*s.* per lb., and the chicory 4*d.* per lb., what is the value per lb. of the pure coffee?

(7) A mixture is made of 27 tons of coal at 17*s.* 9*d.* per ton, 12 tons at 22*s.* 6*d.* per ton, and 11 tons at 19*s.* 4*d.*; find the value per ton of the mixture.

(8) A grocer mixes 1 cwt. 3 qrs. of sugar at $5\frac{1}{2}d.$ per lb. with 2 cwt. 1 qr. 14 lb. at $2\frac{3}{4}d.$ per lb. At what price per lb. must he sell the mixture so as not to lose by the sale?

(9) A grocer mixes 2 cwt. 15 lb. of sugar at 11*d.* per lb. with 4 cwt. 30 lb. at 5*d.*; at what price per lb. must he sell the mixture so as not to lose by the sale?

(10) An innkeeper buys 10 gallons of spirits at 12*s.*, 15 at 4*s.* 6*d.*, and 18 at 5*s.* 9*d.*; how must he sell the mixture per gallon that he may gain £2. 5*s.* 10*d.* on his outlay?

(11) A grocer mixes 72 lb. of tea at 2*s.* 10½*d.* per lb. with 90 lb. of tea at 2*s.* 6*d.* per lb.; at what price per lb. must he sell the mixture so as to gain 5*d.* per lb.?

(12) A wine merchant buys two sorts of spirit at 16*s.* 6*d.* and 10*s.* 6*d.* per gall. He mixes them in the proportion of 5 parts of the cheaper to 7 of the dearer. At what rate per gallon must he sell the mixture to make 25 per cent. profit?

(13) A man mixes 20 gallons of ale, which cost him 16*d.* a gallon, with 65 gallons of beer, which cost him 9*d.* a gallon; at what price must he sell the mixture to gain 20 per cent.?

(14) If 80 gallons of brandy at £1. 5*s.* per gallon be mixed with 55 gallons at £1. 7*s.* 6*d.* per gallon, at what price per gallon must the mixture be sold to gain 15 per cent. on the outlay?

(15) A wine merchant buys 6 dozen of wine at 72*s.* per dozen, and 20 dozen more at 54*s.* per dozen. What is his gain per cent. if he sells the mixture at 63*s.* per dozen?

(16) A publican mixes 4 gallons of gin worth 15*s.* a gallon, with 4 gallons of water and a gallon of base spirit worth 10*s.*; what will he gain per cent. on his outlay by selling the mixture at 2*s.* 10*d.* per bottle of six to the gallon?

(17) A grocer buys coffee at £8. 10*s.* per cwt. and chicory at £2. 10*s.* per cwt., and mixes them in the proportion of 5 parts chicory to 7 parts coffee; at what rate per lb. must he sell the mixture so as to gain $16\frac{2}{3}$ per cent. on his outlay?

(18) I purchase 2½ cwt. of tea at £13. 1*s.* 8*d.* per cwt. and 1½ cwt. at £18. 2*s.* per cwt. I mix them and sell the mixture at 3*s.* 6*d.* per lb. What is the profit per lb., the profit per cent. on the outlay, and the profit per cent. on the selling price?

160b. CASE II. *Example* i. *Two wines, worth* 32*s. and* 24*s. per gallon respectively, are mixed together. If the mixture be worth* 27*s.* 6*d. per gallon, in what ratio are the wines mixed?*

The price of 1st is 9 sixpences per gall. above that of the mixture, and..... 2nd is 7 ..... below.....
Hence we must take such quantities of each that the loss on the first wine is exactly balanced by the profit on the second.

To do this we mix the wines in the proportion of 7 gallons of the first to 9 of the second.

Example ii. *Mix wine at* 25*s.*, 18*s.*, 16*s.* 6*d.*, *and* 15*s. a gallon respectively so that the mixture may be worth* 17*s.* 6*d. a gallon.*

The prices of the wines are respectively
15 sixpences and 1 sixpence above,
and 2 .....5..... below
that of the mixture; and we have to mix the wines so that the two excesses may make up for the two deficiencies.

One way to do this is to take
1 gallon of the 1st with 3 of the 4th,
and 1 ..... 3rd ..... 2 .....2nd;
for the excess in the value of 1 gallon of the 1st will exactly balance the deficiency in the value of the 3 gallons of the 4th; and the excess in the value of 1 gallon of the 3rd will balance the deficiency in that of 2 gallons of the 2nd. We may therefore mix the wines in the proportion of
1, 2, 1 and 3.

We might also take
2 gallons of the 1st with 15 of the 3rd,
and 5 .....2nd ..... 1 ..... 4th,
in which case the proportion would be
2, 5, 15 and 1.

NOTE. Examples of this kind often have several different solutions.

EXERCISE LXXXIB.

(1) How must a grocer mix black tea at 2*s*. 4*d*. a lb. with green tea at 3*s*. 8*d*. a lb., so as to make a mixture at 2*s*. 8*d*. a lb.?

(2) Wine at 18*s*. a gall. is mixed with wine at 25*s*. a gall., and the mixture is sold for 22*s*. 6*d*. a gall.; how were they mixed?

(3) A grocer buys tea at 2*s*. and at 2*s*. 9*d*. per lb. How must he mix them so as to gain 20 per cent. by selling at 3*s*. per lb.?

(4) A grocer mixes coffee at 2*s*. with chicory at 6*d*. per lb. In what proportion are they mixed, if by selling at 1*s*. 9*d*. per lb. he makes 5 per cent. profit?

(5) Mix ale at 2*s*. 8*d*., 2*s*., and 1*s*. 10*d*. per gallon with water, so that the mixture may be sold at 1*s*. 6*d*. per gallon.

(6) A grocer mixed teas at 4*s*. 5*d*., 4*s*. 9*d*., and 6*s*. 9*d*. per lb. respectively, and sold the mixture at 5*s*. per lb.; how were they mixed?

(7) In what proportion must a grocer mix teas at 4*s*. 7½*d*., 5*s*. 3½*d*., and 3*s*. 4½*d*., so as to sell the mixture at 4*s*. 10*d*. per lb.?

(8) Three kinds of tea at 2*s*., 2*s*. 8*d*., 3*s*. 6*d*. per lb. respectively are mixed together; if the price of the mixture is 3*s*. per lb., how many pounds of each kind are taken?

(9) Currants at 4*d*. per pound are mixed with currants at 6*d*., 9*d*., and 11*d*. per lb. respectively, and the mixture is sold at $7\frac{1}{2}d$. per lb.; in what proportion were they mixed?

(10) In what proportion must wines of 15*s*., 20*s*., 26*s*., and 30*s*. a gallon respectively be mixed, so that the mixture may be sold at 24*s*. a gallon with $14\frac{2}{7}$ per cent. profit?

(11) How much water must be added to a cask containing 70 gallons of spirits worth 13*s*. 4*d*. a gallon, to reduce the price to 11*s*. 8*d*. a gallon?

(12) A dealer buying spirit at 5*s*. a gallon, dilutes it with water so that, by selling it at 4*s*. 6*d*. a gallon, he gains 20 per cent. How much water is there in a gallon of the compound?

(13) A dairyman buys milk at $2\frac{1}{2}d$. per quart, dilutes it with water, and sells the mixture at 3*d*. per quart. His profits are 60 per cent. upon his outlay. How much water is there in each quart that is sold?

(14) A person buys equal quantities of apples at 2 a penny and 3 a penny respectively, and mixes them. How many may he then sell for 5*s*., so as neither to gain nor lose?

(15) If 10 oz. 13 dwt. 16 grs. of gold worth £3. 5*s*. per oz., be mixed with 1 oz. 2 dwt. 8 grs. of silver worth 5*s*. per oz., how much worthless alloy must be added to reduce the value of the compound metal to £2. 16*s*. 8*d*. per oz.?

PARTNERSHIPS.

161. If two or more persons arrange to carry on business together, such an association is called a **Partnership**. Each member of it is called a **Partner**, and will generally supply a portion of the money required to carry on the business. At the end of certain periods, the **gross** profits earned will be calculated; from these will be subtracted the expenses incurred; and the remainder, or **net** profits, will be divided among the partners, each receiving, we will suppose, the same fraction of the net profits as the money furnished by him was of the total amount subscribed by them all. The money used in carrying on a business is called **Capital.**

162. We will consider the distribution of the profits or the losses of a partnership, and we will suppose that they are distributed among the partners in proportion to each one's share of the capital.

Example. Two partners, A and B, begin business together with a capital of £3500, *of which A contributes* £2000 *and B the rest. Their profits at the end of the year are* £2100. *What is the share of each?*

Total capital = £3500;

A's share of capital = £2000 = $\frac{2000}{3500}$ of whole = $\frac{4}{7}$ of whole,

and *B*'s ..... = £1500 = $\frac{1500}{3500}$ ..... = $\frac{3}{7}$ .....;

$\therefore$ *A*'s share of profits = $\frac{4}{7}$ of £2100 = £1200,

and *B*'s ..... = $\frac{3}{7}$ of £2100 = £900.

163. If one or more of the partners join the others *after* they have started, their share of the profits will depend not only upon their share of the capital, but also upon the *time* during which their capital is employed.

Example. A puts £2000 *into a business at the beginning of the year, and is joined* 4 *months later by B with* £1500. *The total profit amounts to* £2100 *for the year. What will be the respective shares of A and B?*

Evidently *A*'s £2000 is employed for 12 months, and *B*'s £1500 for 8 months. Now 12 months' use of £2000 may be considered as equivalent to *one* month's use of £24000, and 8 months' use of £1500 as equivalent to *one* month's use of £12000.

Hence, we may state the case as follows:—"*If A and B trade for* one *month with a capital of* £36000, *of which* £24000 *belongs to A and* £12000 *to B, what will be their respective shares of a profit of* £2100?"

We see then that *A*'s share of profit

= $\frac{24000}{36000}$ of £2100 = $\frac{2}{3}$ of £2100 = £1400;

and *B*'s share of profit

= $\frac{12000}{36000}$ of £2100 = $\frac{1}{3}$ of £2100 = £700.

EXERCISE LXXXII.

(1) Two partners, *A* and *B*, with a capital of £781, gained £156. 4*s*. in a year; *A* had put into the business £352, and *B* the rest. How much of the profits belongs to each?

(2) Three persons, *A*, *B* and *C*, form a partnership; their capital consists of £15000, of which £5000 belongs to *A*, £7500 to *B* and £2500 to *C*. Their profits amount to £3000 at the end of the year. What is the share of each?

(3) Three merchants, *A*, *B* and *C*, invest respectively in a cargo the sums of £5200, £3600 and £2000. When sold there is a clear profit on the whole cargo of £1647. How much ought each to receive?

(4) *A* and *B* are partners in a business in which *A* has £4900 and *B* £1400: *B*, being the working partner, receives 6 per cent. of all the profit; the rest is divided in proportion to the capital. What does each receive of £450 profit?

(5) *A* is a working, *B* a sleeping partner in a business. *A* possesses £1200 of the capital, *B* £2000. *A* receives 10 per cent. of all profits for managing, the rest being proportionally divided. Find their respective shares of £800 profit.

(6) Three persons, *A*, *B* and *C*, invest respectively in a business the sums of £226, £350. 10*s*., and £479. *A*, as manager, receives £50 of the profit, the rest being divided in proportion to the capital invested. What does each receive of £775. 13*s*. 1½*d*. profit?

(7) *A*, *B* and *C* are partners in a business in which each has respectively £5000, £3000 and £2500. *C* receives 12½ per cent. of the profit as manager; the remainder is divided amongst them all in proportion to the amount of capital invested by each. What does each receive of £840 profit?

(8) Three persons, *A*, *B* and *C*, join in business. *A* invests £500 for 3 months, *B* £650 for 8 months, and *C* £300 for 11 months. They gain £420. Find the share of each.

(9) *A* puts into a concern £600, and *B*, three months after, £300; six months after the start they put in £300 each, and they gain £375 in a year and a half. How must they share it?

(10) Four merchants, *A*, *B*, *C* and *D*, trade together. *A*'s capital of £300 was in use for 12 months, *B*'s capital of £330 for 10 months, *C*'s capital of £375 for 8 months, and *D*'s capital of £395 for 6 months. The whole gain was £818. What did each receive?

(11) *A* and *B* enter into partnership. *A* puts in £2100 and *B* £1500. Four months later, *C* joins the firm with £2700. At the end of the year the profit is 10 per cent. on the whole capital; what share of profit belongs to each?

(12) Two partners start in business, one with £2500, and the other with £1500, and they borrow £1200 more at

4 per cent. At the end of the year they have made a profit of 10 per cent. on the whole capital; what should each receive in proportion to his capital?

(13) *A* puts £1500 and *B* £9000 into a business. The gross receipts for the first year are £5600, but 5 per cent. of this is paid for insurance, and $\frac{1}{7}$ for other expenses. *B* receives £325 of the rest for managing, and the remainder is divided between them in proportion to the capital employed. Find the share of each.

(14) *A* and *B* are in partnership in a concern in which *A* has £5000 engaged, and *B* £7500. The gross receipts for a year are £3200; $\frac{1}{8}$ of this is spent in salaries, and £30 in insurance of the premises. By an arrangement between the partners, *A* is to receive 8 per cent. on his capital, and *B* 4 per cent. on his; and the remainder of the profit is to be divided in proportion to the capital. Find the net receipts of *A* and *B* respectively.

BANKRUPTCIES.

164. A man becomes **bankrupt**, or **insolvent**, when the money which he owes is more than that which he possesses.

What he owes is called his **liabilities**, or **debts**; what he possesses is called his **assets**, or his **estate**.

He is the **debtor**; those to whom he owes anything are his **creditors**.

Each creditor is supposed to receive the same fraction of the assets that the money due to him is of the debtor's whole liabilities.

If the assets amount, for example, to $\frac{2}{3}$ of the debts, each creditor receives £$\frac{2}{3}$ for each £1 that is due to him; and the debtor is said to pay a **dividend** of 13*s.* 4*d.* in the £.

165. When a debtor has to make an arrangement with his creditors, his affairs are said to be in **liquidation**.

If the creditors arrange privately with the debtor, without resource to the courts dealing with bankruptcy affairs, the debtor is said to enter into a **composition** with his creditors.

Example i. *If a bankrupt's debts amount to* £1500, *and his assets to* £950, *how much in the pound can he pay?*

Dividend on a debt of £1500 = £950,
∴ ..... £1 = £$\frac{950}{1500}$ = £$\frac{19}{30}$,
∴ ..... £1 = 12*s.* 8*d.*

Example ii. *If a bankrupt pays* 12*s.* 8*d. in the pound, how much will a creditor receive to whom he owes* £190?

Dividend on debt of £1 = £$\frac{19}{30}$,
∴ ..... £190 = £$\frac{19}{30}$ × 190
= £$\frac{361}{3}$
= £120. 6*s.* 8*d.*

NOTE.—Such examples as Example ii. can also be worked by Practice (See Ex. XXIX.).

Example iii. *A bankrupt pays* 12*s.* 8*d. in the pound, and his assets amount to* £950; *what is the amount of his debts?*

To each 12*s.* 8*d.* in the assets corresponds a debt which = £1,
∴ ..... 4*d.* ..... = £$\frac{1}{38}$,
∴ ..... £1 ..... = £$\frac{60}{38}$ = £$\frac{30}{19}$,
∴ ..... £950 ..... = £$\frac{30}{19}$ × 950;
∴ total debts = £1500.

EXERCISE LXXXIII.

(1) If a bankrupt's liabilities amount to £1875 and his assets to £910. 3*s.* 1½*d.*, how much in the £ can he pay?

(2) A bankrupt whose effects are worth £1181. 12*s.* 6*d.* owes £4726. 10*s.*; how much in the £ can he pay?

(3) A bankrupt's debts amount to £7250, and his creditors lose £6011. 9*s.* 2*d.*; what dividend does his estate pay?

(4) A bankrupt owes to one creditor 1000 guineas, to each of two others £500, and to each of three others £150; his assets amount to £600; how much in the £ can he pay, and how much will the largest creditor receive?

(5) A bankrupt fails for £12500, and his estate only realises £3906. 5*s.*; what dividend does he pay, and what will a creditor receive whose claim is for £798. 10*s.*?

(6) A bankrupt whose liabilities are £2480, has assets consisting of goods worth £1077. 5*s.* and book-debts, amounting to £930 and valued at 17*s.* 6*d.* in the £; what dividend in the £ will he be able to pay?

(7) A tradesman owes the following sums:—£56.17*s*.8½*d*., £43. 12*s*. 5*d*., £176. 4*s*. 10*d*. and £16. 9*s*. 3½*d*. He has owing to him in good debts £49. 8*s*. 6*d*., £64. 6*s*. 9*d*. and £13. 14*s*. 8*d*.; and in bad debts £52. 6*s*. 8*d*., £95. 17*s*. 6*d*. and £43. 3*s*. 4*d*., for which he receives respectively 7*s*. 6*d*., 3*s*. 4*d*. and 15*s*. in the £. Is he able to pay his debts in full; and, if not, how much in the £ can he pay?

(8) A bankrupt paid 4*s*. 4½*d*. in the £; what was the loss on a claim of £865. 4*s*.?

(9) A bankrupt pays three dividends of 6*s*. 4½*d*., 3*s*. 9½*d*. and 1*s*. 3*d*. in the £; what do his creditors lose on a debt of £4360?

(10) A bankrupt pays 13*s*. 6*d*. in the £, and his assets amount to £1980; what is the amount of his liabilities?

(11) A creditor received 16*s*. 3*d*. in the £, and thereby lost £135. 10*s*.; how much was originally due to him?

(12) A bankrupt pays 13*s*. 7*d*. in the £, and his liabilities amount to £8372; what is the amount of his assets?

(13) A bankrupt whose liabilities are £987. 15*s*. pays three dividends of 10*s*. 8*d*., 3*s*. 4*d*. and 3*s*. 6*d*. in the £; what is the value of his estate?

(14) A bankrupt's dividend being 11*s*. 11*d*. in the £, how much is lost by a creditor who had sold him 8 cwt. 2 qrs. 8 lb. of merchandise at £1. 2*s*. 6*d*. per cwt.?

(15) A bankrupt owes £5317. 5*s*., and his assets are £4076. 11*s*. 2*d*. The expenses of winding up the estate amount to 10 per cent. of the liabilities; how much in the £ can he pay?

(16) A bankrupt owes his three creditors £525, £630 and £795 respectively, and his assets are £422. 10*s*. Supposing that the trustee gets 5 per cent. of the assets for winding up the bankrupt's affairs, how much in the £ can the estate pay, and what will each creditor receive?

(17) A bankrupt owes £5174. 15*s*., including £1552. 8*s*. 6*d*. for rent, taxes and wages which have to be paid in full. His assets being £4269. 3*s*. 4½*d*., what dividend will his other creditors receive?

RATES AND TAXES.

166. Sums of money paid to the imperial authorities to defray the expenses of the Government of the country are called **taxes**, or **duties**.

167. Duties levied by the Government on certain imports or exports are called the **Customs**.

168. Duties levied by the Government on spirituous liquors and tobacco, and licenses to manufacture the same, are called the **Excise**.

Licenses on guns, dogs, etc., are also excise duties in Great Britain.

169. The tax levied by the Government on yearly incomes is called the **Income Tax**.

Incomes not exceeding £160 per annum are exempt from payment of Income Tax; in the case of incomes not exceeding £400 the tax is not chargeable on the first £160; in the case of incomes over £400 and not exceeding £500 the tax is not chargeable on the first £150; in the case of incomes over £500 and not exceeding £600 the tax is not chargeable on the first £120; and finally, in the case of incomes exceeding £600 and not exceeding £700 the tax is not chargeable on the first £70 (1898).

170. Sums of money paid to the local authorities, *e.g.*, municipal corporations, boards of guardians, etc., to defray expenses connected with matters under their jurisdiction are called **rates**.

171. Property which is fixed, such as houses and lands, is called **real estate**.

Property which is moveable, such as shares in companies, furniture, etc., is called **personal estate**.

172. The value of a person's property (houses, land, and income from trade, profession or investments), as estimated by the authorities, is called the **assessment**, or **rateable value**, of the property.

173. Rates and taxes are reckoned at so much in the pound.

If, for example, the income-tax is 8*d.* in the £, a tax of 8*d.* has to be paid on each £1 of income, *i.e.*, $\frac{1}{30}$ of an income has to be paid to the Government. There are, however, some exceptions, as we have stated above.

Example i. *A district council needs* £416. 13*s.* 4*d.* *for public purposes; the assessment being* £5000, *find how much in the pound the rate will be.*

On an assessment of £5000 the rates = £416. 13*s.* 4*d.*;

∴ .... £1 ..... $= \frac{\text{£416. 13s. 4d.}}{5000}$

$= 1s.\ 8d.$

Example ii. *Find the tax, at* 10*d.* *in the pound, on property assessed at* £760.

Tax on assessment of £1 $= £\frac{1}{24}$;

∴ .....£760 $= £\frac{1}{24} \times 760$

$= £31.\ 13s.\ 4d.$

Example iii. *An income-tax payer pays* 8*d.* *in the pound, and the tax amounts to* £17. 17*s.* 4*d.*; *what is the amount of the income on which he pays tax?*

To each 8*d.* in the tax corresponds an income which = £1,

∴ ..... 1*d.* ..... $= £\frac{1}{8}$,

∴ To the given..... $= £\frac{1}{8} \times 4288$

$= £536.$

EXERCISE LXXXIV.

(1) A school board needs £715. 12*s.* 6*d.* for purposes of education; the assessment being £13740, find how much in the £ the rate will be.

(2) The rateable value of a parish being £5676. 13*s.* 4*d.*, and £745. 1*s.* 3*d.* being required for public purposes, how much in the £ will the rate be?

(3) If a man has to pay £18. 15*s.* as income-tax on an income of £1500, how much in the £ is the tax?

(4) Find the local rates at 13*d.* in the £ on property assessed at £16241. 5*s.*

(5) What is the amount of rates to be paid on an assessment of £74. 10*s.* at 3*s.* 4*d.* in the £?

(6) How much will a poor-rate of 2*s.* 8*d.* in the £ produce in a parish where the rateable value of the whole property is £4736. 5*s.*?

(7) A poor-rate of 2*s.* 7½*d.* in the £ produces £745. 1*s.* 3*d.*; what is the rateable value of the parish?

(8) A householder pays a water-rate of 9*d.* in the £, a district-rate of 1*s.* 3*d.* in the £, and a poor-rate of 1*s.* 5½*d.* in the £, and the whole amounts to £15. 2*s.* 7¼*d.* What is the amount of his assessment?

(9) A man pays a district-rate of 1*s.* 6*d.* in the £ on his rental, a water-rate of 1*s.* in the £, and a poor-rate of 1*s.* 10½*d.* in the £; if the rent and taxes amount to £85. 6*s.* 3*d.*, what is the rent?

(10) A person, after paying income-tax on £515, has £484. 19*s.* 2*d.* left; how much did he pay in the £?

(11) After paying income-tax at the rate of 4*d.* in the £ a man has £491. 13*s.* 4*d.* remaining; what was his income?

(12) The expenses of the poor in a parish amount to £110. 7*s.* 8*d.* A rate of 4*s.* 4¾*d.* in the £ is levied, and proves insufficient by 11½*d.*; what is the assessment of the parish?

STOCKS AND SHARES.

174. When the capital employed in a business and the trade done are small, the whole affair may be in the hands of a very small number of persons. All the partners may then take an active part in conducting the business, and they constitute a **firm.**

But where the capital is very large, and the partners very numerous, as in the case of the construction and maintenance of a railway, it is customary for the general body of partners to elect a few of those most fit among themselves to manage and direct the affairs of the **Company.** These are called the **Directors.**

175. Before starting a company, the **promoters,** who are interested in the particular business which it is proposed to carry on, meet together and choose directors, and decide that the capital shall be fixed at, say, £3000000. The directors will then issue a *prospectus*, setting forth the objects and the powers of the company, and inviting the public to *subscribe* towards it. This capital may be divided into 300000 equal **parts** of £10 each, or into 30000 equal parts of £100 each, or in other ways. In the first case each part is called a £10 **share.** In the second case also each part may be a *share*, but it is more commonly described as £100 **stock,** three such parts as £300 stock, and so on.

Any number of shares, or any amount of stock, may be subscribed for by the same person, and the subscriber's name

will be registered in the books of the company as the proprietor of that portion of the capital.

The proprietors of shares or stock in a company are called **Shareholders,** and the association is called a **Joint-Stock Company.**

176. The distinction between shares and stock is this. When the capital of a company is in the form of *stock*, any multiple of £1 of stock may generally be obtained; but when it is in *shares*, only whole shares can be bought or sold, parts of a share not usually being dealt in.

Moreover, the amount of capital represented by a share is not always fully "*paid up*," there being sometimes an unpaid balance, which the directors may call for as the requirements of the business demand it. Stock, on the other hand, is always paid up in full; and when shares are fully paid up, they may be consolidated, or converted into stock.

The temporary documents which entitle the holder to shares, and which consist of certificates of payment of deposit and calls, are termed **scrip.** They are afterwards exchanged for definitive share certificates.

177. At regular intervals, usually half a year, the accounts are made up, all expenses are allowed for, and the *net* profits are divided among the proprietors, at whatever rate may be agreed upon, at so much for each share or for each £100 stock. What each receives is called his **Dividend.** In the case of shares the dividend is calculated only upon the amount of the share actually paid up. Thus in the case of a £10 share, of which only £5 is paid up, a 4 per cent. dividend is reckoned as 4 per cent. on £5.

178. A company in which no shareholder is subject to any further liability after his shares are fully paid up, is called a **Limited Liability Company.** Such a company must be registered, and for the protection of the public must style itself "Limited"; thus, "Robinson & Co., Limited."

The members of firms, and the shareholders in companies, which are not "limited" are liable, should the undertaking fail, for its debts or liabilities to the full extent of their private property.

179. Shares in Limited Companies are generally divided into *preference* and *ordinary*.

The holders of **preference** shares receive a fixed dividend, usually from 4 to 6 per cent., which must be paid before any dividend is paid on the ordinary shares. Preference shares are often **cumulative**, *i.e.*, if the profits of the company at a particular period, when dividends are due, should be insufficient to meet the fixed interest, the balance must be made up out of subsequent profits.

The holders of **ordinary** shares receive dividends varying in amount with the prosperity of the company.

In some cases there are **deferred** shares, the holders of which receive no dividend until the other shareholders have been paid at a certain rate or rates. The vendors of a business to a joint-stock company sometimes accept deferred shares in part payment of the purchase money.

180. Companies usually have the power to borrow money on the security of their property, *i.e.*, on *mortgage*, by the issue of **bonds**, or **debentures**. The holders of debentures are guaranteed the payment of a fixed rate of interest, and the repayment of the principal under certain conditions. But debentures are sometimes perpetual, or irredeemable, as in the case of railway companies. They are then called "debenture stock."

There is a great difference between the position of a holder of shares and that of a holder of debentures. Shareholders are the **partners** in the company; debenture-holders are the **creditors** of the company. The latter are entitled to the interest on their bonds without regard to the claims of any shareholders, and may even take possession of the buildings and plant belonging to the company if their interest is not paid in full when due.

181. If a proprietor wishes to withdraw from a company, he cannot demand his money back from the directors, but he must sell his stock or shares to some one else, if he can find any one willing to buy them. For this purpose there is a special market called a **Stock Exchange.**

182. The London Stock Exchange belongs to a body of private shareholders. The floor of the Stock Exchange is divided by imaginary lines, which, however, are intimately

known to the members, into various markets, in which the different classes of securities are dealt in. For instance, there is the Consol market, where British and Indian Funds are dealt in, a market for Colonial stocks, and different departments for Home Railways, American stocks, Foreign Railways, Banks, etc., and the Mining market. In each of these markets a certain number of **jobbers**, or dealers, do business. A jobber may only do business in one market at a time, although two members of the same firm may trade in different markets.

The jobbers do not come in contact with the outside public, for no member of the public may enter the Stock Exchange. Any one wishing to invest money, or to sell securities, can only do so through the medium of a **stock-broker**, who, on receipt of an order, will go into the Stock Exchange to the particular market where the security which he has in hand is dealt in, and there he will find a dealer ready to buy or sell as the case may be.

The dealer quotes a price—that is to say, he mentions two prices, at the lower of which he is prepared to buy, while he is prepared to sell at the higher. When he has dealt, it is usual for him to "undo" his bargain in the market as soon as practicable; in theory he can effect this at or about the middle price. The margin between the middle price and the dealing price is his profit, and is known as the jobber's "turn."

The broker's remuneration is in the form of *commission* or *brokerage*, ranging from $\frac{1}{8}$ per cent. on Consols to $\frac{1}{2}$ per cent. on registered railway and other stocks.

The utility of the dealer will be manifest, for if the broker had to go about and find another broker amongst the 3500 members of the Stock Exchange, who had those shares to sell that he wanted to buy, it might be days before they would come across one another, whereas now transactions involving perhaps half a million of money can be completed in a few minutes. Fortnightly settling days are fixed by the committee.

183. If a company does a good business, and its *net* profits increase, it will happen in consequence that the dividend on each share, or on each £100 stock, will become greater. And again, in consequence of this increase in the dividend,

the market value of each share, or of each £100 stock, will also increase.

If, for example, the dividend paid were at the rate of 10 per cent. per annum, and the business done was a steady one and comparatively free from risk of loss, the value of a share of the *nominal* value of £10 might really be £25 or more.

If, on the other hand, the company were to pay only a small dividend, or no dividend at all, for several years, and if there should seem to be but a poor prospect before it, it might be possible to buy a £10 share for £2 or even less.

We must be careful therefore to remember the distinction between the *nominal* value, or *face* value, of a share, which is only the amount of *capital* which it represents, and its *real* value, which is the money one could get for it in the stock-market.

The amount of dividend paid to a shareholder depends on the *nominal* value of a share, and not on the price at which it was bought. Thus, the holder of a £10 share bought for £25 would receive the same amount of dividend as the holder of a similar share in the same company bought for 25 shillings, at a time when the shares were cheaper.

When the market value of a share is *greater* than the nominal value, the share is said to be **at a premium.** On the other hand, when the market value is *less* than the nominal value, the share is said to be **at a discount.** Lastly, when the market value and the nominal value are the same, the share is said to be **at par.**

Similarly, when the market value of £100 stock in a particular company is £125, the stock is said to be at £25 *premium*; and when the market value is £85, the stock is said to be at £15 *discount*.

The market value is called the **price**, or **quotation**, of the share or stock. Stock is always quoted at the price per £100. All marketable stocks, of whatever nature, are called **securities.**

184. There is another kind of *stock*, which might for historical and other reasons have claimed prior consideration, and which we casually named in Art. 182.

In the latter part of the 17th century the cost of the long

and expensive wars which were carried on by England was greater than could be borne by the revenue of the country at the time. And, moreover, as posterity would reap much of the benefit supposed to be derived from them, it was considered fair that posterity should bear part of the cost. It was suggested then that the public should be invited to *lend* money to the state, and that a special tax should be imposed, with the proceeds of which interest should be paid upon the loan at a fixed rate, and the loan itself be gradually paid off. Several loans were raised in this manner at various times, and in each case a special tax was levied and the proceeds set apart. The money thus set apart was called the Funds. Subsequently when the special tax was abolished, and the money was borrowed without any special arrangements being made for repayment, the name was transferred to the loans themselves; and by the term **Funds** we now mean the permanent debt owed by the state, commonly known as the **National Debt.**

185. On March 31st, 1903, the Debt amounted to £798,349,000, comprising £640,086,000 of **Funded Debt** (or the **Funds**), £102,703,000 of **Unfunded Debt**, £55,560,000 representing the capitalized value of **Terminable Annuities**, and £3,831,000 raised by Special Acts. The total amount would give an average share of about £20 per head. In 1816 the total amount, reckoned in the same manner, was **£900,436,000, or £45 per head.**

The Unfunded Debt consists of *Exchequer Bills*, *Exchequer Bonds*, and *Treasury Bills*, all of which are repayable at fixed dates. Exchequer Bills are long-dated in days; Treasury Bills are short-dated in days.

The Terminable Annuities consist partly of Life Annuities, and partly of Annuities granted for stated periods.

No time is fixed for the repayment of the Funded Debt, but a certain portion of the revenue, called the *Sinking Fund*, is set apart for the payment of interest on the debt, and any surplus is generally employed in buying stock in the open market, the stock bought being at once cancelled, and the debt thereby diminished.

186. The money obtained by our Government will never be repaid in the way that a *loan*, properly so called, would

be; and it cannot therefore strictly be called a loan. In fact anyone who now buys £100 of British stock, really buys a right to an annual payment, or **Annuity**, of £2¾, £2½, or £3½, or whatever it may be; and he may hand down this right to his descendants, or he may sell it, or give it away. The annuity will be paid so long as the country is able and willing to pay it, and consequently the market price of a Government stock will depend mainly upon the country's reputation for honesty and solvency. The price may be affected temporarily in many ways, particularly by an outbreak of war. The price of British stock, for example, was as high as 107 in 1737, but it was only 47¼ in 1798. The former was, however, a 4 per cent. stock; the latter, a 3 per cent. stock.

187. The British Funded debt has been subject to several changes from time to time, the last of which took place in 1903. It now consists of

(i) the *Consolidated Annuities*, called briefly **Consols**, and formed by the amalgamation of several loans into one, on which interest now is paid at the rate of 2½ per cent. per annum. Before April 5th, 1903, the interest was at the rate of 2¾ per cent. The whole amount of Consols may be paid off *at par* after April 5th, 1923, but no further change of any sort may be made before that date. The interest is paid quarterly, on Jan. 5th, April 5th, July 5th, and Oct. 5th;

(ii) Some smaller amounts of 2½ per cent. and 2¾ per cent., annuities, and debts to the Banks of England and Ireland.

Consols, Exchequer Bills, and Treasury Bills are sometimes called *Government securities.*

188. Many Foreign States, and some of our Colonies, raise loans to meet exceptional expenditure, promising to pay them off, or *redeem* them, *at par* at a stated time.

The deed by which a State binds itself to pay a stipulated rate of interest, and to pay off the loan at a certain date, is called a *bond*, and is transferable from one person to another like a bank-note. Bonds and shares have a distinguishing number, and can only be identified by this number. At the bottom of each bond small tickets called *coupons* are printed, one of which is cut off and presented for payment of interest each half-year until the bond is redeemed. This

is the usual method by which interest is paid on Foreign Government stocks held by persons in this country.

189. Loans raised by municipal bodies for carrying out the drainage, lighting, water supply, and other public works for the general improvement of towns, are called *Corporation stocks*, the repayment of the loans and the interest being secured on the rates. The period for repayment is limited by Parliament to sixty years. Corporation stocks and most Colonial stocks are known as **inscribed stocks**, *i.e.*, the holders' names and the amount of their holdings are carefully registered in books specially kept at the Bank of England, or by other banking firms, and the interest is paid to the holders by the bank without coupons. Transfers of inscribed stocks can only be made by the personal signatures of seller and buyer.

190. The business of stock-jobbing sprang up just two centuries ago, and the first price list of stocks was published in 1697, three years after the establishment of the Bank of England. It contained six securities, and a hundred years later the number had not increased to more than twenty; while to-day upwards of 3000 are quoted.

In consulting the Official List issued daily by the Stock Exchange Committee, it is important to remember that the supervision of the Committee extends only to the record of "Business Done," and that the "Closing Quotations" are not official.

191. In the stock-market reports, in the leading daily newspapers, it will be found that to each stock are appended two prices, $113\frac{1}{8} - 113\frac{3}{8}$, for example. This means that dealers offer the lower price to brokers who are sellers, and ask the higher price from brokers who are buyers. Again the price of a certain stock may appear in the list $113\frac{1}{8} - 113\frac{3}{8}$ x.d. This means that the vendor sells the stock "*ex dividend*," *i.e.*, without the dividend which is about to be paid. On the other hand, the expression $113\frac{1}{8} - 113\frac{3}{8}$ c.d. means "*cum dividend*," *i.e.*, the purchaser is entitled to receive the dividend.

In reckoning the cost of buying or selling stock or shares

it is necessary to take account also of the expense of doing so, which depends upon stamp duty and broker's commission.

This expense must sometimes be taken account of in questions on stocks and shares, but we will generally suppose it already allowed for in the price quoted. It should however be particularly noticed that these expenses *increase* the outlay of a buyer, and *diminish* the receipts of a seller.

Brokerage is reckoned on the *nominal* value. Thus if the price of Consols is $113\frac{1}{2}$, a buyer would have to pay $113\frac{1}{2} + \frac{1}{8}$, and a seller would receive $113\frac{1}{2} - \frac{1}{8}$.

Transactions in stocks and shares should be calculated to the nearest penny.

NOTE.—In Exercises LXXXV.—XCIV. brokerage is included in the price of the stock.

192. CASE I.—To find the income derived from a given amount of stock.

Example. What annual income will be derived from £2875 *of* 4 *per cent. stock?*

By 4 *per cent. stock* is meant a stock on £100 of which is paid a regular dividend of £4 per annum.

Income from £100 stock = £4,

$\therefore$ .....£2875 ..... = £$\frac{2875}{100} \times 4$

= £115.

EXERCISE LXXXV.

What annual income will be derived from

(1) £3300 of 5 per cent. stock?

(2) £4275 of 4 per cent. stock?

(3) £8000 of $3\frac{1}{2}$ per cent. stock?

(4) £10775 of 3 per cent. stock?

(5) £7650 of $4\frac{1}{2}$ per cent. stock?

(6) £8225 of $2\frac{1}{2}$ per cent. stock?

(7) £3241. 17*s.* 6*d.* of 6 per cent. stock?

(8) £812. 10*s.* of 4 per cent. stock?

(9) £4326. 10*s.* of $2\frac{1}{2}$ per cent. stock?

(10) £6820. 12*s.* 6*d.* of $3\frac{1}{2}$ per cent. stock?

193. CASE II.—To find how much stock at a given price can be bought (or sold) for a given sum of money.

Example. How much $2\frac{1}{2}$ *per cent. stock at* $92\frac{3}{8}$ *can be bought for* £1434. 17*s.* 10*d.*?

Amount of stock obtained for $£92\frac{3}{8}$ $= £100$,

$\therefore$ ..... $£1434\frac{17}{120} = £\frac{1434\frac{17}{120}}{92\frac{3}{8}} \times 100$

$= £\frac{17218\frac{7}{120}}{\frac{739}{8}} \times \frac{8}{739} \times 100$

$= £\frac{2060\frac{2}{3}}{3}$

$= £\frac{6180}{3}$

$= £1553\frac{1}{3}$;

$\therefore$ amount of stock obtained $= £1553.\ 6s.\ 8d.$

EXERCISE LXXXVI.

What amount of

(1) $3\frac{1}{2}$ per cent. stock at 98 can be bought for £4900?
(2) $4\frac{1}{2}$ per cent. stock at 105 can be bought for £2394?
(3) 3 per cent. stock at $85\frac{1}{2}$ can be bought for £3591?
(4) 3 per cent. stock at 90 can be bought for £3249?
(5) 4 per cent. stock at $97\frac{3}{4}$ can be bought for £14076?

What amount of

(6) 3 per cent. stock at $92\frac{1}{2}$ can be sold for £3445. 12*s.* 6*d.*?
(7) 4 per cent. stock at $90\frac{1}{4}$ can be sold for £3249?
(8) $2\frac{1}{2}$ per cent. stock at $87\frac{1}{2}$ can be sold for £787. 10*s.*?
(9) $2\frac{1}{4}$ per cent. stock at $82\frac{1}{8}$ can be sold for £821. 5*s.*?
(10) $2\frac{3}{4}$ per cent. stock at $92\frac{1}{2}$ can be sold for £1202. 10*s.*?

194. CASE III.—To find the cost of a given quantity of stock at a given price.

Example. What must be paid for £1553. 6*s.* 8*d. of* 3 *per cent. stock at* $92\frac{3}{8}$?

Cost of £100 stock $= £92\frac{3}{8}$,

$\therefore$ ..... £1 ..... $= £\frac{739}{800}$;

$\therefore$ cost of $£1553\frac{1}{3}$ stock $= £\frac{6180}{3} \times \frac{739}{800}$

$= £\frac{2060 \times 739}{800}$

$= £\frac{1721187}{120}$

$= £1434.\ 17s.\ 10d.$;

$\therefore$ money required $= £1434.\ 17s.\ 10d.$

Instead of the 3rd and following lines we might multiply £1553. 6*s.* 8*d.* by 739, and divide by 800 in the ordinary way.

EXERCISE LXXXVII.

What must be paid for

(1) £1500 of 3 per cent. stock at 90?
(2) £4550 of 3 per cent. stock at 90?
(3) £4500 of 5 per cent. stock at $112\frac{1}{2}$?
(4) £2250 of 3 per cent. stock at 93?
(5) £2137. 10*s.* of 4 per cent. stock at 80?

What will be received on selling

(6) £3340 of 4 per cent. stock at 98?
(7) £8500 of 3 per cent. stock at $97\frac{3}{4}$?
(8) £1245 of 4 per cent. stock at 84?
(9) £7440 of 3 per cent. stock at $92\frac{7}{8}$?
(10) £7580 of 3 per cent. stock at 91?
(11) £4450 of Turkish stock at 52?
(12) £2050 of 4 per cent. stock at 90?

195. CASE IV.—To find the income derived from the investment of a given amount of money in a given stock at a given price.

Example. What income will be derived from the investment of £4537. 10*s. in* 3 *per cent. stock at* $90\frac{3}{4}$?

Income from £100 stock = £3;
but price of £100 stock = £$90\frac{3}{4}$;
∴ income from £$90\frac{3}{4}$ money = £3,
∴ ..... £1 ..... = £$\frac{3}{90\frac{3}{4}}$ = £$\frac{12}{363}$ = £$\frac{4}{121}$,
∴ ..... £4537$\frac{1}{2}$ ..... = £$\frac{4}{121} \times \frac{9075}{2}$
= £2 × 75
= £150.

Instead of the last three lines we might multiply £4537. 10*s* by 4, and divide by 121 in the ordinary way.

EXERCISE LXXXVIII.

What income will be derived from investing

(1) £21000 in $2\frac{3}{4}$ per cent. consols at 99?
(2) £1274 in $3\frac{1}{2}$ per cent. stock at 91?
(3) £25935 in 3 per cent. stock at 90?
(4) £3519 in $3\frac{1}{2}$ per cent. stock at $97\frac{3}{4}$?
(5) £7560 in 3 per cent. stock at $94\frac{1}{2}$?
(6) £4788 in $3\frac{1}{2}$ per cent. stock at 105?
(7) £3220 in $3\frac{1}{2}$ per cent. stock at $80\frac{1}{2}$?

(8) If *A* invest £1210 in $3\frac{1}{4}$ per cents. at $97\frac{1}{2}$, and a like sum in 5 per cent. stock at $90\frac{3}{4}$, what will be his income?

(9) *A* invests £1695. 16*s*. 8*d*. in $3\frac{1}{2}$ per cents. at $101\frac{3}{4}$, and *B* invests £1767. 3*s*. 9*d*. in 3 per cents. at $94\frac{1}{4}$; what is the difference between the incomes thence obtained?

(10) What income is obtained by investing £154. 14*s*. in the $3\frac{1}{2}$ per cents. at 91, and £183. 12*s*. in the $2\frac{1}{2}$ per cents. at 85?

(11) What annual income will be obtained from £13000 invested in a $3\frac{1}{2}$ per cent. stock at 91; and from the same sum invested in a 4 per cent. stock at 96?

(12) A person invests £25935 in 3 per cent. stock at 90; what is his income? If the first year's dividend be invested in the same stock at 91, and the dividend for the second year at 95, what will be his income for the third year?

(13) What half-yearly dividend is derived from an investment of £3000 in $3\frac{1}{2}$ per cents. at $98\frac{3}{4}$, after deducting income-tax at the rate of 3*d*. in the pound?

(14) What half-yearly dividend is due upon an investment of £2500 in 3 per cents. at $87\frac{3}{8}$, after deducting 7*d*. in the pound for income-tax?

196. CASE V.—To compare the investment of money in various stocks at various prices.

Example. Which is the better investment—$5\frac{1}{2}$ *per cent. stock at* 105, *or* $4\frac{1}{2}$ *per cent. stock at* $87\frac{1}{2}$*?*

In former case, income from £105 money = £$5\frac{1}{2}$,
∴ ..... £1 ..... = £$\frac{11}{210}$;
In latter case, ... £$87\frac{1}{2}$ ..... = £$4\frac{1}{2}$,
∴ ..... £1 ..... = £$\frac{9}{175}$.

It will be found in the usual way that $\frac{11}{210}$ is greater than $\frac{9}{175}$; therefore the former investment is the better.

EXERCISE LXXXIX.

Which is the better investment

(1) Bank stock paying 10 per cent. at $234\frac{1}{2}$, or 3 per cent. railway stock at $92\frac{1}{2}$?

(2) $3\frac{1}{4}$ per cent. stock at $98\frac{5}{8}$, or $3\frac{1}{2}$ per cent. stock at par?

(3) 3 per cent. stock at 85, or 4 per cent. stock at 96?

(4) 3 per cent. stock at $83\frac{1}{2}$, or $3\frac{1}{2}$ per cent. at 97?

(5) $3\frac{1}{2}$ per cent. stock at 88, or 4 per cent. at 91?

(6) 4 per cent. stock at 96, or 5 per cent. at 112?

(7) $1\frac{3}{4}$ per cent. stock at $47\frac{1}{4}$, or $3\frac{2}{3}$ per cent. at 99?

(8) Railway shares paying £6 per share when the £100 share is at 132, or a 3 per cent. stock at 93?

197. CASE VI.—To find the rate per cent. of interest derived from an investment, when the price of a stock and the percentage paid are given.

Example. What rate of interest is obtained by investing in 4 per cents. at $92\frac{1}{2}$?

The rate per cent. required is to be reckoned on the money invested.

Income from £$92\frac{1}{2}$ money = £4,

$\therefore$ ..... £100 ..... = £$\frac{100}{92\frac{1}{2}} \times 4$

= £$\frac{800}{185}$

= £$\frac{160}{37}$

= £$4\frac{12}{37}$;

$\therefore$ rate of interest = $4\frac{12}{37}$ per cent.

EXERCISE XC.

(1) If £100 of 3 per cent. stock is worth £85 cash, what rate of interest is this equivalent to?

(2) What rate of interest will a person receive when 3 per cent. stock is at 84?

(3) At what rate will a person receive interest who invests his capital in 4 per cent. stock when it is at 104?

(4) At what rate will a person receive interest who invests in the $2\frac{3}{4}$ per cents. at 99?

(5) What rate of interest is obtained by investing in $3\frac{1}{2}$ per cents. at 105?

(6) What rate of interest will a person receive who invests in 4 per cents. at $92\frac{1}{2}$?

(7) A company pays a dividend of 9 per cent., and its £10 shares sell for £25; what rate of interest will an investor get for his money?

198. CASE VII.—To find at what price a stock of a given rate per cent. can be bought to give a stated percentage.

Example. If a 4 per cent. stock pay interest at the rate of $4\frac{1}{3}\frac{2}{7}$ per cent. on the money invested, what is the price of the stock?

$£4\frac{1}{3}\frac{2}{7}$ = interest on £100 money,

$\therefore £4 = \ldots\ldots\ldots\ldots £\frac{4}{4\frac{1}{3}\frac{2}{7}} \times 100 \ldots\ldots\ldots,$

$= \ldots\ldots\ldots\ldots £\frac{1\,4\,8}{1\,6\,0} \times 100 \ldots\ldots\ldots,$

$= \ldots\ldots\ldots\ldots £\frac{1\,8\,5}{2} \ldots\ldots\ldots,$

$= \ldots\ldots\ldots\ldots £92\frac{1}{2} \ldots\ldots\ldots;$

but £4 = ... £100 stock;

$\therefore$ price of stock = $92\frac{1}{2}$.

EXERCISE XCI.

(1) A person investing in a 3 per cent. stock buys at such a price that he receives $3\frac{1}{3}$ per cent. interest for his money; what is the price of the stock?

(2) A person investing in a 4 per cent. stock receives $4\frac{3}{8}$ per cent. interest for his money; what is the price of the stock?

(3) If money is worth $3\frac{3}{4}$ per cent., what should be the price of a 3 per cent. stock?

(4) A railway pays $3\frac{1}{4}$ per cent. dividend; at what price will a £100 share give $4\frac{1}{2}$ per cent. interest to a purchaser?

(5) A person invested in a 3 per cent. stock, and found that, after paying an income-tax of 10*d.* in the £, he got $3\frac{1}{3}$ per cent. per annum on his money; at what price did he buy the stock?

(6) What price is paid for a 3 per cent. stock if, after paying an income-tax of 5*d.* in the £, the interest on the money invested is at the rate of exactly 3 per cent.?

199. CASE VIII.—To find the gain or loss in buying stock at one price and selling it at another.

Example. Find the profit made by investing £3445 in $2\frac{1}{2}$ per cent. stock at $86\frac{1}{8}$, and selling out at $91\frac{3}{4}$ (brokerage included in both prices).

Gain on $£86\frac{1}{8} = £(91\frac{3}{4} - 86\frac{1}{8}) = £5\frac{5}{8}$,

$\therefore \ldots\ldots\ldots £3445 = £\frac{3445 \times 5\frac{5}{8}}{86\frac{1}{8}}$

$= £\frac{3445 \times 45}{689}$

$= £225.$

EXERCISE XCII.

(1) A man bought £1950 of $2\frac{1}{2}$ per cent. stock at $97\frac{3}{4}$, and sold out on its rising to $103\frac{1}{4}$; find his gain, brokerage being already included in the prices named.

(2) A person invested £5850 in 3 per cent. stock at $97\frac{1}{2}$, and when the price rose to $99\frac{3}{4}$ he sold out; what did he gain?

(3) A man invests £4095 in 3 per cent. stock at 91; he sells out £3000 stock when the price has risen to $93\frac{1}{2}$, and the remainder when the price falls to 85; how much did he gain or lose by the transaction?

(4) If I buy 10 fully paid up £20 shares at $27\frac{1}{2}$ each, and sell them at $37\frac{1}{2}$ after receiving a dividend of 15 per cent., how much shall I gain in all?

(5) A person invests £7560 in 3 per cent. stock at $94\frac{1}{2}$, and when it falls to 90 he sells out $\frac{1}{4}$ of his stock; afterwards, when it is at $94\frac{3}{4}$, he sells the remainder; find the alteration in his capital.

(6) A person invests £1365 in 3 per cent. stock at 91; he sells out £1000 stock at $93\frac{1}{2}$, and the remainder at 85; how much does he gain or lose by the transaction?

200. CASE IX.—To find how much new stock at a given price can be bought with the proceeds of selling a given quantity of other stock at a given price.

Example. A person has £9000 of 3 per cent. stock; he sells out at 91, and buys 4 per cent. stock at $94\frac{1}{2}$; how much 4 per cent. stock does he obtain?

Proceeds of sale of 3 per cent. stock $= £91 \times 90$
$= £8190$;

amount of 4 per cent. stock obtained for $£94\frac{1}{2} = £100$;

$\therefore$ ..... $£8190 = £\frac{8190}{94\frac{1}{2}} \times 100$
$= £8190 \times \frac{2}{189} \times 100$
$= £\frac{130 \times 200}{3}$
$= £\frac{26000}{3}$
$= £8666.\ 13s.\ 4d.$

EXERCISE XCIII.

(1) How many railway shares at £77 each are equivalent to £50400 stock at $93\frac{1}{2}$?

(2) If I purchase £1000 of 3 per cent. stock at a discount of $3\frac{1}{8}$ per cent., and, selling out again after the stock has fallen to $82\frac{3}{4}$, invest the proceeds in $3\frac{3}{4}$ per cents. at par, what do I lose in capital by the whole transaction?

(3) How much 4 per cent. stock at 131 could be bought with the proceeds of the sale of £13886 of 3 per cent. stock at 93?

(4) A person invests £2184 in 3 per cent. stock at 91, and sells out when the stock has fallen 1 per cent.; he then invests the proceeds of the sale in 4 per cent. stock at 108; find the amount of his new stock?

(5) An investor who had 150 £10 shares allotted to him in a brewery company, sold them out when they were quoted at $£1\frac{1}{5}$ premium, and invested the money in corporation stock at par; how much stock did he buy?

(6) A person possesses £10000 of $3\frac{1}{2}$ per cent. stock and £7500 of $4\frac{1}{2}$ per cent. stock. He sells out when the stocks are at 126 and 168 respectively, and invests the proceeds in $2\frac{3}{4}$ per cent. consols at $100\frac{4}{5}$; what amount of consols does he obtain?

201. CASE X.—To find the change of income caused by transferring capital from one kind of stock to another.

Example. Find the alteration in income caused by selling £10000 *of* 3 *per cent. stock at* $94\frac{1}{2}$, *and with the money obtained buying* 4 *per cent. stock at* 105.

Income derived from 3 per cents. $= £300$;

money obtained by selling 3 per cents. $= £10000 \times \frac{94\frac{1}{2}}{100}$;

$\therefore$ amount of 4 per cent. stock bought $= £94\frac{1}{2} \times 100 \times \frac{100}{105}$;

$\therefore$ income derived from 4 per cents. $= £94\frac{1}{2} \times 100 \times \frac{100}{105} \times \frac{4}{100}$

$= £100 \times \frac{189}{200} \times \frac{4}{105}$

$= £360$;

$\therefore$ change of income $= £60$ increase.

EXERCISE XCIV.

(1) If a person sells £1000 of 3 per cent. stock at 91, and invests the proceeds at 5 per cent., by how much is his income increased?

(2) What change of income is made by transferring £4275 stock from the 4 per cents. at 80 to the $5\frac{1}{2}$ per cents. at 99?

(3) A person sells £2250 of 3 per cent. stock at 93, and invests the proceeds in 6 per cent. stock at 108. Find the change in his income.

(4) If the 3 per cents. be at 92 and the 4 per cents. at 115, how much will one's income be increased by selling £10000 of 3 per cent. stock, and investing the proceeds in the 4 per cents.?

(5) A man sold £19200 of 3 per cent. stock at 85, and invested the proceeds in 4 per cent. stock at 96. What was his income before and after the transaction?

(6) A person transfers £1000 stock from the 4 per cents. at 90 to the 3 per cents. at 72. How much of the latter stock will he hold, and what will be the difference in his income?

(7) If I sell £43400 stock out of the 3 per cents. at 96, and buy 4 per cent. stock at 105, find the difference in my income.

(8) If I hold £6400 stock in the 3 per cents., and sell out at $86\frac{1}{2}$, investing the money obtained in 4 per cent. railway debentures at 115, what change would I make in my income?

(9) If the 3 per cents. be at 91, and the 5 per cents. at 117, how much will the income of a man be affected by his selling £2250 of 3 per cent. stock, and investing the proceeds in the 5 per cents.?

(10) What income will be obtained by investing £1232. 10*s.* in the 3 per cents. at $90\frac{5}{8}$? If the stock is sold out at $88\frac{3}{4}$, and the proceeds invested in railway stock at $106\frac{1}{2}$, paying a dividend of 6 per cent., what will be the increase in income?

(11) A man has £6680 stock in the 4 per cents. at 98. When this stock has fallen 2 per cent., he transfers his

capital to the $3\frac{1}{2}$ per cents. at $83\frac{1}{2}$. Find the alteration in his income.

(12) The price of £100 of bank stock, which pays $6\frac{1}{2}$ per cent., is $185\frac{3}{4}$; £6250 of this stock is transferred to 3 per cent. corporation stock when the price of the latter is $92\frac{7}{8}$; find the loss of annual income.

(13) A person whose annual income is £450 transfers his capital from the 3 per cents. at 84 to the $3\frac{1}{4}$ per cents. at $87\frac{1}{2}$. What change does he make in his income?

(14) A man invests £26180 in the 3 per cents. at $93\frac{1}{8}$, but after a time he sells out half at $92\frac{1}{4}$, and invests the proceeds in the 5 per cents. at 123. Find the difference in his income.

202. CASE XI.—Questions in which brokerage must be taken account of.

Example. A man has £7220 of 3 per cent. stock; he sells out at $102\frac{3}{8}$, and invests the proceeds in $2\frac{1}{2}$ per cent. stock at $90\frac{1}{8}$. Find the change in his income, allowing brokerage at $\frac{1}{8}$ per cent.

Income derived from 3 per cents. $= £7220 \times \frac{3}{100}$
$= £216.\ 12s.;$

money derived from sale of £100 stock $= £(102\frac{3}{8} - \frac{1}{8}) = £102\frac{1}{2}$,

$\therefore$ money derived from sale of £7220 stock $= £\frac{7220 \times 102\frac{1}{2}}{100}$
$= £7400.\ 10s.;$

money paid for £100 of $2\frac{1}{2}$ per cent. stock $= £(90\frac{1}{8} + \frac{1}{8}) = £90\frac{1}{4}$,

$\therefore$ income derived from $2\frac{1}{2}$ per cents. $= £\frac{7400\frac{1}{2} \times 2\frac{1}{2}}{90\frac{1}{4}}$
$= £205;$

$\therefore$ diminution of income $= £11.\ 12s.$

EXERCISE XCV.

(1) How much stock can be obtained by investing £3772. 6*s*. 3*d*. in $2\frac{1}{2}$ per cents. at $88\frac{3}{8}$, brokerage being $\frac{1}{8}$ per cent.?

(2) What sum will be obtained by selling out £8975 stock in the $2\frac{1}{2}$ per cents. at $89\frac{1}{2}$, brokerage being $\frac{1}{8}$ per cent.?

(3) What sum of money must be paid to purchase £2600 of $3\frac{1}{2}$ per cent. stock at $93\frac{3}{4}$, allowing $\frac{1}{8}$ per cent. brokerage?

(4) What income will be obtained by investing £10098 in the 3 per cents. at $93\frac{3}{8}$, brokerage being $\frac{1}{8}$ per cent.?

(5) How much money must be invested in 3 per cent. stock at $95\frac{3}{8}$, brokerage being $\frac{1}{8}$ per cent., to obtain an income of £1000?

(6) What rate of interest is obtained by investing in 5 per cent. stock at $119\frac{7}{8}$, brokerage being $\frac{1}{8}$ per cent.?

(7) What is gained by investing £1950 at $97\frac{3}{8}$ and selling out at 104, brokerage being $\frac{1}{8}$ per cent. on each transaction?

(8) Find the alteration in income occasioned by transferring £3200 stock from the 3 per cents. at $86\frac{3}{8}$ to a 4 per cent. stock at $114\frac{7}{8}$, the brokerage being $\frac{1}{8}$ per cent. on each transaction.

(9) If £500 be spent in buying 3 per cent. stock at $99\frac{3}{8}$, and in paying the broker's commission of $\frac{1}{8}$ per cent., what amount of stock will be obtained, and what income will be derived from the investment?

(10) A man invests £19040 in 5 per cent. stock at $74\frac{1}{4}$; the stock rising 2 per cent. he sells out; what does he gain, brokerage being $\frac{1}{8}$ per cent. on each transaction?

(11) A person invested £1911 in 3 per cent. stock at $79\frac{1}{2}$; he sold out, and realised a gain of £150 after paying $\frac{1}{8}$ per cent. brokerage on each transaction; at what price did he sell?

(12) If 3 per cent. stock be at $90\frac{5}{8}$, what sum must I invest in order to secure a yearly income of £470 after paying an income-tax of 5*d.* in the £, brokerage being $\frac{1}{8}$ per cent.?

(13) £500 of $4\frac{1}{2}$ per cent. preference stock is bought at $111\frac{1}{2}$, the stamp and fee cost £2. 10*s.*, and commission is charged at the rate of 10*s.* per £100 stock; what is the actual interest produced by the money invested?

(14) If £10425 of $3\frac{1}{2}$ per cent. stock be sold out at $102\frac{3}{8}$, and the proceeds be invested in $2\frac{1}{2}$ per cents. at $69\frac{3}{8}$, what will be the change of income, brokerage of $\frac{1}{8}$ per cent. being charged on each transaction?

(15) I invested £4257 in $2\frac{3}{4}$ per cent. consols at $96\frac{5}{8}$, and subsequently sold out half my holding at the same price, investing the proceeds in 5 per cent. Indian Railway stock

at $120\frac{1}{2}$; find the difference in my income, brokerage being reckoned at $\frac{1}{8}$ per cent.

(16) Find the yearly income obtained from investing £3960 in railway stock standing at 103, and paying $5\frac{1}{4}$ per cent., brokerage being $\frac{1}{8}$ per cent.

(17) What rate and what amount of interest will an investor receive who invests £19500 in the 3 per cents., standing at $89\frac{1}{8}$, brokerage being $\frac{1}{8}$ per cent.?

(18) If I sell out $2\frac{1}{2}$ per cent. stock at $93\frac{1}{8}$ and buy 3 per cents. at $101\frac{1}{8}$, and by this transaction increase my income by £11. 10*s.* a year how much stock had I in the $2\frac{1}{2}$ per cents. at first, allowing $\frac{1}{8}$ per cent. brokerage?

THE BRITISH COINAGE. CHEQUES.

203. When a person buys goods in a shop, the shopkeeper generally requires payment in *ready money*. This may be given either in coins, or in bank-notes, or by a cheque; bank-notes and cheques being equivalent to ready money, since they bear either a promise, as in the case of a bank-note, or an order, as in the case of a cheque, that the bearer shall receive the amount stated, on presenting the note or cheque at the proper place, *whenever he pleases*.

204. The equivalent for which individuals readily give their goods or service is called **money**. Money is therefore not only a medium of exchange, but also a measure of value. In the United Kingdom we have coins of gold, silver and copper, but **gold** is the *standard* of value.

Standard gold contains $\frac{11}{12}$ths of pure gold and $\frac{1}{12}$th of alloy, and is said to be $\frac{11}{12}$ths fine (or 916·66 millièmes, which means 916·66 parts of pure gold in a thousand).

By the Coinage Act passed in 1870, out of 40 Troy-pounds weight of standard gold, 1869 sovereigns are to be coined: a sovereign should therefore weigh 123·27448 grains (or 7·98805 grammes, metric weight), but the Mint is permitted to issue sovereigns whose weight does not differ from this by more than 0·2 grains, and a sovereign is legal tender, provided it weighs not less than 122·5 grains. The former variation is known as *the remedy*, and the latter as *allowance for abrasion*.

Standard silver contains $\frac{37}{40}$ths of pure silver and $\frac{3}{40}$ths of alloy, and by the Coinage Act 66 shillings are to be coined out of 1 Troy-pound weight of standard silver: a shilling should therefore weigh 87·27272 grains, and other silver coins in proportion.

Bronze is an alloy consisting of 95 parts of copper, 4 parts of tin, and 1 part of zinc, and 48 pennies must weigh 1 lb. Avoirdupois.

In the United Kingdom English gold coins are legal tender up to any sum of money; silver coins are legal tender up to £2; pence and halfpence up to one shilling.

English sovereigns are taken in payment at their full value throughout the world.

205. The **Bank of England** was established in 1694, when the Government obtained a loan of £1,200,000, at 8 per cent., from 40 subscribers, who received in return a charter conferring certain privileges, including the right to trade in bills of exchange, gold and silver. The charter was last renewed in 1892, and the loan to the Government now amounts to £11,015,100, upon which the Government pays interest at the rate of $2\frac{3}{4}$ per cent. The Bank is allowed the free issue of notes to the full amount of this Government debt; but notes beyond this amount must be represented by certain securities, and by gold coin or bullion stored in the bank cellars.

The Bank of England keeps the banking account of the Government; transfers consols, etc. (Art. 187); pays the interest on the National Debt; buys gold at £3. 17*s*. 9*d*. per ounce, and sells sovereigns at £3. 17*s*. $10\frac{1}{2}$*d*. per ounce; keeps the reserve balances of all the larger banks; discounts bills (Art. 247) and issues notes. These notes are legal tender in England and Wales, but at the Bank and its branches gold may be demanded.

206. A **cheque** is a written order to a banker to pay on demand a stated sum of money to the person named. A cheque is usually written on a printed form supplied by the banker in books, the forms being consecutively numbered and each being impressed with a penny stamp.

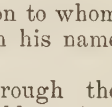
The person who draws, or writes out and signs, the cheque is called the **drawer**; the banker on whom it is

drawn is called the **drawee**; and the person to whom it is made payable is called the **payee**.

207. Cheques are of two forms:—

(1) *Payable to Bearer*, in which case payment is to be made to any person presenting the cheque.

(2) *Payable to Order*, in which case the person to whom the cheque is payable must **endorse** it, *i.e.*, sign his name on the back.

For the sake of security, cheques remitted through the post should be **crossed**, *i.e.*, two parallel lines should be drawn across the body of the cheque, and the words "& Co.," written thus 

The words "*not negotiable*," written across a cheque, are an additional safeguard. They do not prohibit the passing of such a cheque from one person to another, but they give a holder from whom it may be stolen the right to claim its value from any one into whose hands it may subsequently pass.

A banker will not pay cash for a crossed cheque, but the amount will be placed to the credit of a person who has a banking account. If the payee has not a banking account, he must pay the cheque to a person who has.

To prevent difficulties arising from differences in endorsements, care should be taken that the names of the payees of the cheques are correctly spelt, and that a firm is described by its exact title.

208. The following is the usual form of a cheque:—

No. 12345 SOUTHAMPTON, *April 4th*, 1898.

IMPRESSED STAMP. 1*d*.

HAMPSHIRE & DORSET BANKING & Co. COMPANY LIMITED.

Pay Mr. John Jones *or Order*
Three hundred and twenty-eight pounds and ten shillings.

£328. 10. 0 JAMES ALLEN.

BILLS OF EXCHANGE.

Inland Bills.

209. It is a common custom for a merchant to pay for goods, not in gold and silver, nor in bank-notes and cheques payable on demand, but in what are called **Drafts** or **Bills of Exchange**. In principle these are similar to cheques, but, usually, instead of being exchangeable for money *at once*, each bears a written promise that the amount shall be paid after a certain interval, which may be a week, or 3 months, or any other period whatever. This is a distinguishing feature of the most common form of a Bill of Exchange.

210. By the *Bills of Exchange Act* passed in 1882 a **Bill of Exchange** is "an unconditional order in writing, addressed by one person to another, signed by the person giving it, requiring the person to whom it is addressed to pay on demand, or at a fixed or determinable future time, a certain sum in money to, or to the order of, a specified person, or to bearer."

211. The following is the usual form of a bill of exchange :—

IMPRESSED STAMP. 4*s*.

£328. 10*s*. 0*d*.

PORTSMOUTH, *January* 1*st*, 1898.

ACCEPTED. Payable at Hampshire & Dorset Bank, Southampton. James Allen.

Three months after date pay *to me or my Order*
Three hundred and twenty-eight pounds and ten shillings
for value received.

JOHN JONES.

To MR. JAMES ALLEN,
19, TITCHFIELD STREET,
SOUTHAMPTON.

212. Bills may be payable either *by* a person or *to* him.

A record of the former is kept in a book called the **Bills Payable** Book, specially ruled to show all particulars of each bill; whilst particulars of the latter are entered in a book called the **Bills Receivable** Book, whether he keeps them to maturity or pays them away.

In the example, the bill is a *Bill Payable* to James Allen ; to John Jones it is a *Bill Receivable.*

213. The creditor, *i.e.*, the person who draws, or writes out and signs, the bill is called the **drawer**; the debtor or person to whom it is addressed is called the **drawee**; and the person to whom the money is payable is called the **payee.**

The drawee *accepts* the bill, *i.e.*, promises to pay it, by writing across the face of it the word *accepted*, with the name of the bank (if any) at which it is made payable, and his signature.

When the bill is accepted, the person on whom it is drawn is then called the **acceptor**, and the bill itself is called an **acceptance.**

The period of time between the date at which the bill is drawn and the date at which it arrives at maturity, *i.e.*, at which the money is to be paid, is called the **currency** of the bill.

214. Bills may be made payable

(1) *On demand,*
(2) *At sight,*
(3) *At presentation* ;

} All of which are equivalent terms, meaning that the bill is payable on demand like a bank-note.

(4) So many *days* (*or months*) *after date*, *i.e*, from the date of the bill.

(5) So many *days* (*or months*) *after sight*, *i.e.*, from the date the drawee stated his acceptance of the bill, in which case he must put this date on the bill.

By the term *months* is meant calendar months.

215. Bills both drawn and payable in the British Isles are called *Inland Bills.* Bills drawn abroad, or payable abroad, are called *Foreign Bills* (Art. 244).

216. All bills drawn in the British Isles must be drawn on stamped forms bearing impressed stamps according to the following table of duties :—

Bills payable on demand	1*d.*
Bills payable otherwise than on demand :	
Not exceeding £5	1*d.*
Exceeding £5 and not exceeding £10	2*d.*
£10 £25	3*d.*
£25 £50	6*d.*
£50 £75	9*d.*
£75 £100	1*s.*
For every additional £100, and also for any fractional part of £100	1*s.*

217. In the British Isles a bill drawn "after date" or "after sight" is not legally due until **three** days after the day on which it is nominally due. These three days are called **days of grace.**

Thus a bill drawn at "three months after date," and dated January 1st, becomes legally due on April 4th.

A bill drawn at 60 days from January 1st becomes legally due on March 5th (leap year, March 4th).

If the last day of grace be a Sunday, Christmas Day, or Good Friday, the bill is due and payable on the *preceding* business day ; if it be a Bank Holiday the bill is due and payable on the *succeeding* business day. Also, if the last day of grace be a Sunday, and the preceding day be a Bank Holiday, the bill is due and payable on the *succeeding* business day.

Presentation for payment must be made at maturity, *i.e.*, on the exact date at which the bill is legally due.

218. Bills in the United Kingdom may be drawn either "to bearer," or "to order." Most bills are drawn *to order*, as in the example (Art. 211). This means that the payee may himself receive the money when due, or may, by an order written on the back of the bill, instruct the drawee to pay some other person.

A bill may be thus *negotiated*, *i.e.*, it may be transferred, in exchange for goods or money, from one person to another, who may again transfer it, and so on, before it is presented for payment to the person upon whom it is drawn. Each person in turn writes his name on the back of it, *i.e.*, he *endorses* the bill.

When the bill is paid at the time agreed upon, the bill is said to be *honoured* or *retired*.

Should the acceptor fail to pay the bill at its maturity, the bill is said to be *dishonoured*, and every endorser is liable for the payment of the amount of the bill on which the endorsement is written.

219. The buying and selling of bills form a large part of the business of a banker. There are also many companies, private firms, and individuals in all large business centres, who make what is called **bill-broking** their special business.

The negotiation of bills is dealt with in the pages treating of Banker's Discount.

220. There is one other kind of bill not so common as Bills of Exchange which should be mentioned.

A **Promissory Note** is a promise in writing made by one person to another to pay, at a certain date named, a sum of money to, or to the order of, a specified person, or to bearer. A promissory note does not require to be "accepted" (Art. 213); in other respects it is subject to the laws which govern bills of exchange.

221. The following is the usual form of a promissory note:—

STAMP. 1*s*.

£100.

SOUTHAMPTON, *January* 1*st*, 1898.

Three months after date I promise to pay to Mr. John Jones, or his order, the sum of One hundred pounds.

JAMES ALLEN.

222. A **Bill of Lading** is a receipt given by the captain of a ship to the shipper for goods put on board by the latter, and in it he undertakes to deliver the goods to the person

whom the shipper names for that purpose, on being paid the cost of carriage, commonly called the "*Freight.*"

There are sundry conditions inserted in every Bill of Lading, but these are too numerous to mention.

For the sake of greater security *three* stamped bills of lading are usually signed by the captain; the original intention of which was that the shipper should retain one, that one should be posted to the consignee of the goods, and that the third should be held by the captain. This arrangement did not prove satisfactory, and is now obsolete.

For many years past it has been customary for the shipper to obtain money for the goods by drawing bills of exchange upon the consignee of the goods; and to complete his security he requires the complete set of three. For these bills of exchange the bills of lading are considered good collateral security, and the shipper is thus enabled to pay the cost of the goods shipped. The captain has now only an unstamped copy of the bills of lading, but this is of no consequence, inasmuch as the ship's *manifest* (*i.e.*, the description of, and details concerning, her cargo) contains the same particulars.

The shipper may make the goods deliverable to John Jones, London, in which case no one but John Jones can get them on arrival, and the bill of lading is then useless as a security to any one but John Jones himself. On the other hand, and this is the usual course, the goods may be made deliverable *to order*, *i.e.*, to the shipper's order, and then, by endorsement the bills of lading become a negotiable instrument.

A copy of a Bill of Lading is given below:—

Bill of Lading.

SHIPPED in good order and well-conditioned by Peter Jackson, in and upon the good steamship called the *Euphrates*, whereof is master for the present voyage William Parry, and now riding at anchor in Madras, and bound for London, *Five hundred bags Rice*, being marked and numbered as in the margin, and are to be delivered in the like good order and well-conditioned at the aforesaid Port of London, unto Order or to Assigns. Freight for the said goods, *Twenty-five shillings per ton of twenty hundredweight.* IN WITNESS whereof the master of the said ship hath affirmed to three

Bills of Lading, all of this Tenor and Date, the one of which Bills being accomplished, the other two to stand void.

Dated in Madras, the seventh day of June, 1898.

(Signed) WILLIAM PARRY.

EXERCISE XCVI.

Write out in proper form the following bills, and say in each case *who* will have to pay the money, and *when* :—

(1) March 4th, 1898, Alfred Allen draws on Charles Field at 10 months for £1275.

(2) March 2nd, 1898, John Walker draws on Alfred Allen at 8 months for £666. 13*s*. 4*d*.

(3) December 22nd, 1895, Charles Field draws on John Walker at 8 months for £170. 12*s*. 6*d*.

(4) May 3rd, 1898, Robert Smith accepts Charles Field's draft at 6 months for £463. 10*s*.

(5) June 12th, 1898, Charles Field gives Robert Smith his acceptance at 7 months for £696. 10*s*. 10*d*.

(6) May 10th, 1898, John Walker gives Charles Field his acceptance at 6 months for £1931. 9*s*. 2*d*.

TRUE DISCOUNT.

223. If a creditor *A* holds a bill of exchange, and wants ready money before the time when *B* will pay him, it will be sufficient if he can find a third person *C* willing to give ready money for the right to receive from *B*, when that time comes, the amount stated in the bill. They will only have to determine *how much* money *C* shall give *A* in exchange for this right.

The *theoretical* ready-money value of a bill is called its **True Present Worth**, and will clearly be less than the amount of the bill itself.

The difference between the present worth and the amount of the bill is called the **True Discount**.

224. In determining the present worth of a bill we must consider that *C* loses the use of his money until he receives the amount of the bill from *B*. The money which he receives from *B* at the stated time should therefore repay to him the money which he gave *A*, and also the money which this might have earned.

Hence we see that the **true present worth of a bill is that sum which, with interest on itself for the time until the bill is paid, would produce the amount stated in the bill.** This would place *A* and *C* on exactly equal terms. It follows that

True Discount = Interest on Present Worth.

When a bill is exchanged or *sold* for ready money it is said to be **discounted.**

225. In questions involving discount there are *four* quantities: (1) The Amount of the Bill; (2) the Rate of Interest; (3) the Time which the bill has to *run*; (4) the Present Worth.

These quantities correspond exactly to the four quantities involved in questions on simple interest. The *Time* and the *Rate of Interest* appear in both; the *Present Worth* corresponds to the *Principal*; and the *Amount of the Bill* (or its *face value*) corresponds to the *sum to which the Principal will amount.*

Example. Find the present worth of £1356. 13*s.* 4*d.*, *due* 3 *months hence, reckoning simple interest at* 7 *per cent.*

This corresponds to CASE II. of Simple Interest.

Interest on £100 for 3 mos. at 7 per cent. $= £\frac{1}{4} \times 7 = £1\frac{3}{4}$,

$\therefore$ Amount of £100 in ..... $= £101\frac{3}{4}$,

$\therefore$ P. W. of £101¾ due in 3 mos..... $= £100$,

$\therefore$ ..... £1356⅔ ..... $= £\frac{100}{101\frac{3}{4}} \times 1356\frac{2}{3}$

$= £\frac{400}{407} \times \frac{4070}{3}$

$= £\frac{4000}{3}$

$= £1333.\ 6s.\ 8d.$

$\therefore$ the discount on £1356. 13*s.* 4*d.* $= £23.\ 6s.\ 8d.$

The discount could be found directly, since it is $\frac{1\frac{3}{4}}{101\frac{3}{4}}$ of the bill.

NOTE.—In working examples in true discount it will be useful to remember that

Amount of Bill − P. W. of Bill = Discount;

$\therefore$ Interest on Bill − Int. on P. W. = Int. on Discount;

i.e., **Interest on Bill − Discount = Interest on Discount.**

226. Strictly speaking, discounts for periods longer than a year should be true discount at compound interest, and this is the principle adopted by Life Assurance Companies in calculating the actual values of deferred payments. (Art. 234).

Time is saved by the use of books which give the amount of £1 at different rates per cent., and for various periods.

EXERCISE XCVII.

Find, to the nearest penny, the present worth and the true discount of the following bills :—

(1) £177. 5*s*. 6*d*., due 1 year hence, int. at 5 per cent

(2) £5747, due 9 months hence, interest at $3\frac{1}{2}$ per cent.

(3) £225. 11*s*., due 6 months hence, int. at 8 per cent.

(4) £4120. 8*s*. 7*d*., due 9 months hence, int. at 4 per cent.

(5) £1842. 15*s*., due 3 months hence, int. at 5 per cent.

(6) £377. 12*s*. 6*d*., due in 18 months, int. at 4 per cent.

(7) £550. 13*s*. 4*d*., due in 15 months, int. at 4 per cent.

(8) £249. 1*s*., due 15 months hence, int. at $2\frac{1}{2}$ per cent

(9) £201. 4*s*. $7\frac{1}{2}$*d*., due 25 days hence, int. at $3\frac{1}{2}$ per cent.

(10) £75. 0*s*. $1\frac{1}{3}$*d*., due 75 days hence, int. at $7\frac{1}{2}$ per cent.

BANKERS' DISCOUNT.

227. We have shown how to determine the discount on bills of exchange so that buyer and seller may stand on equal terms, but that principle is theoretical only, and is not acted on by business men. A merchant holding a *bill receivable* may take it to a banker, or to a Discount Company, or to a private firm of bill-discounters, who will pay him the amount of the bill less a charge at a certain rate per cent. per annum, which is their fee for discounting the bill before it is legally due. The amount retained by the bill-broker is the **interest on the face value of the bill** for the time the bill

has to run, instead of the interest on the *true present worth* of the bill.

The banker, or bill-broker, has therefore the advantage.

The difference between True Discount and Banker's Discount is equal to the interest on the True Discount.

When a bill is discounted all the parties to it are still liable.

Example. Find the cash value of a bill for £328. 10*s.*, *which is dated January* 1*st*, 1898, *is payable in* 3 *months, and is discounted on February* 12*th; the rate of discount being* $2\frac{1}{2}$ *per cent.*

It will be noticed that 1898 is not a leap year, and that there are therefore only 28 days in February. Also the bill becomes payable, *nominally* on April 1st, *legally* on April 4th.

The number of days between February 12th and April 4th $=16+31+4=51$.

Interest on £1 for 365 days $= £2\frac{1}{2} \times \frac{1}{100}$,

$\therefore$ ..... £328½ for 51 days $= £328\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{100} \times \frac{51}{365}$

$= £\frac{657}{2} \times \frac{5}{2} \times \frac{1}{100} \times \frac{51}{365}$

$= £\frac{657}{4} \times \frac{1}{100} \times \frac{51}{73}$

$= £\frac{9}{400} \times 51$

$= £\frac{459}{400}$

$= £1{\cdot}1475$;

$\therefore$ banker's discount $= £1.\ 2s.\ 11d.$;

$\therefore$ cash value $= £328.\ 10s. - £1.\ 2s.\ 11d.$

$= £327.\ 7s.\ 1d.$

Bankers and bill-brokers use books containing interest tables which give the interest on various amounts at various rates, thus facilitating calculations.

NOTE.—The pupil should compare Exercise XCVIII., Questions 1–6, with XCVI.

EXERCISE XCVIII.

(1) What will a banker retain on discounting a bill of £1275, drawn on the 4th of March at 10 months, and discounted on the 14th of August at 5 per cent.?

(2) What will a banker retain on discounting a bill of £666. 13*s.* 4*d.*, drawn on the 2nd of March at 8 months, and discounted on the 8th of June at $3\frac{1}{2}$ per cent.?

(3) A bill of £170. 12*s*. 6*d*., drawn on the 22nd Dec. 1895, at 8 months, is discounted on the 29th of April at $3\frac{3}{4}$ per cent.; how much will the banker retain?

(4) A bill of £463. 10*s*., drawn on May 3rd at 6 months, is discounted on May 18th; find the discount at $2\frac{3}{4}$ per cent.

(5) Find the banker's discount on a bill of £696. 10*s*. 10*d*., drawn on June 12th at 7 months, and discounted on Sept. 26th at $6\frac{1}{4}$ per cent.

(6) What will a banker give on the 1st of July for a bill of £1931. 9*s*. 2*d*., drawn on the 10th of May at 6 months, interest being reckoned at $5\frac{1}{3}$ per cent.?

(7) A bill of £347. 19*s*. $4\frac{1}{2}$*d*., nominally due on the 1st of January, is discounted on the 29th of May; what will the banker retain, reckoning interest at $3\frac{1}{3}$ per cent.?

(8) A bill of £307. 19*s*. $4\frac{1}{2}$*d*., drawn on April 1st at 6 months, is discounted on May 31st; find the banker's discount at $6\frac{1}{2}$ per cent.

(9) On the 27th of December, 1897, a bill of £423. 7*s*. 6*d*. was discounted at $4\frac{3}{4}$ per cent.; the bill was drawn on the 31st of July at 12 months; what was the discount?

(10) A bill of £1231. 17*s*. 6*d*., drawn on April 6th at 9 months, is discounted on June 4th; find the banker's discount at $3\frac{1}{4}$ per cent.

(11) A bill of £342. 17*s*. 10*d*., drawn on May 1st at 4 months, is discounted on June 15th; find the discount at $1\frac{1}{2}$ per cent.

(12) Find the banker's discount on a bill of £362. 13*s*. 4*d*., drawn and discounted on 13th of March, 1895, and nominally due on the 26th of March, 1896, interest being reckoned at 5 per cent.

EQUATION OF PAYMENTS.

228. When a number of bills for various amounts fall due at various times, it is convenient to be able to determine a date on which the aggregate amount of the bills may be settled in one payment, with equal regard to the interests of payer and payee.

The date on which this may be done is called the **equated time**, and the period between the initial date and the equated time is called the **equated period.** The problem itself is called the **Equation of Payments.**

Suppose, for example, that on one and the same day a merchant draws a bill for £350 payable in 3 months, another for £600 payable in 4 months, and a third for £230 payable in 6 months. At what time may he pay the aggregate amount of £1180 in one sum?

It will be seen that the periods of the bills allow him the use of

(i) £350 for 3 mos., wh. is equivt. to that of £1050 for 1 mo.;
(ii) £600 for 4 mos., wh. is equivt. to that of £2400 for 1 mo.;
(iii) £230 for 6 mos., wh. is equivt. to that of £1380 for 1 mo.

Altogether ∴ he is entitled to the use of £4830 for 1 mo.,
or to the use of £1180 for $\frac{4830}{1180}$ mos.,
i.e., „ „ £1180 for $4\frac{110}{118}$ mos.

Hence, the *equated period* is $4\frac{110}{118}$ months, and the *equated time* is $4\frac{110}{118}$ months after the date on which the bills were drawn.

Example. Find the equated period of four bills drawn on the same day, viz., £250 *due in* 2 *months,* £123 *due in* 3 *months,* £432 *due in* 4 *months, and* £56 *due in* 6 *months.*

$$\text{Number of months in equated period} = \frac{250\times2+123\times3+432\times4+56\times6}{250+123+432+56} = \frac{2933}{861} = 3\frac{350}{123}.$$

The *equated time* can be ascertained by reducing $\frac{350}{123}$ of a month to days, and reckoning onwards from the date on which the bills were drawn.

229. The principle is the same if the bills are drawn on different dates, and we will now consider a practical case in which a merchant supplies goods to a customer from time to time, and from time to time also receives cash to be placed to his customer's credit in part payment of the account. The matter to be determined is the date on which the balance should be paid on closing the account; the process is sometimes called **Averaging Accounts.**

Example. A merchant supplies goods, and receives cash, as stated below. Determine the date on which the balance should be paid on closing the account.

1897.	Dr.	£	s.	d.	1897.	Cr.	£	s.	d.
15 Mar.	To goods, at 2 mos.	253	4	2	22 Mar.	By cash	110	10	0
18 Mar.	„ 3 „	326	15	0	16 May	„	450	0	0
19 June	„ 4 „	572	18	5	5 Sept.	„	394	17	6

We might reckon dates either from 15th March, the date of the first bill, or from the first day of that month. We will adopt the latter course.

The balance of the account is £197. 10*s.* 1*d.* To determine when this should be paid, we multiply each amount by the number of days between 1st March and the day on which the bill is due or the cash is paid, and we find the sum of these products on each side of the account. We then subtract the smaller sum from the larger, and divide the remainder by the balance, £197. 10*s.* 1*d.* The quotient will be the number of days between 1st March and the date on which this balance should be paid. We will express each amount as a decimal of a £ to 3 places (Art. 59).

The number of days from 1 March, 1897, to 15 May, 18 June, 19 October, 22 March, 16 May, 5 September are respectively 75, 109, 232, 21, 76, 188.

Dr.			Cr.		
	£253·208×75	=£ 18990·600		£110·500×21	=£ 2320·500
	£326·750×109	=£ 35615·750		£450·000×76	=£34200·000
	£572·921×232	=£132917·672		£394·875×188	=£74236·500
	£1152·879	£187524·022		£955·375	£110757·000
	£ 955·375	£110757·000			
	197·504)	76767·022 (388·6			
		1751582			
		1715502			
		135470			

The equated period is therefore 388·6 days, and the balance, £197. 10*s.* 1*d.*, would be due at this interval after 1st March, 1897. It would therefore be due on 25th March, 1898. Discount would be allowed if the amount were paid before this date, and interest would be added if it were not paid till after.

NOTE.—The balance of the above account is in favour of the merchant. If it had been in favour of the customer, *i.e.*, if the customer's payments had exceeded the amounts due, the equated period would be reckoned *backwards* from 1st March; and the

merchant would have to repay him the amount of the balance, together with interest on it for the interval between the equated time and the date on which the repayment is made.

EXERCISE XCIX.

(1) £300 is due in 2 months, £250 in 3 months, and £200 in 4 months; find the equated time of payment.

(2) Find the equated time of payment of £700 due in 4 months, £650 due in 5 months, and £150 due in 8 months.

(3) Find the equated time of payment of £5000, of which one half is due in 3 months, one quarter in 4 months, and the remainder in 5 months.

(4) One man owes £259. 10*s*. 6*d*., whereof £100 is to be paid in one month and the balance in 4 months; find the equated time.

(5) A man owes £780 to be paid in 8 months; but he pays £300 at the end of 4 months, and £260 at the end of 6 months. When will the remainder be due?

(6) Find the equated time of payment of £647. 12*s*. 6*d*. due in 3 months, £325. 9*s*. 4½*d*. due in 4 months, and £500 due in 6 months.

Determine the dates on which the cash balances of the following accounts should be paid:—

(7)

1894.	Dr.	£	*s*.	*d*.	1894.	Cr.	£	*s*.	*d*.
5 June	To goods, 3 mos.	237	0	0	20 June	By cash	150	0	0
18 July	„ 2 „	1365	0	0	25 July	„	1000	0	0
10 Aug.	„ 5 „	128	0	0	5 Nov.	„	200	0	0

(8)

1893.	Dr.	£	*s*.	*d*.	1893.	Cr.	£	*s*.	*d*.
15 Feb.	To goods, 4 mos.	1105	10	4	21 Mar.	By cash	100	0	0
23 May	„ 6 „	2318	15	5	16 June	„	1007	15	6
14 Aug.	„ 3 „	527	18	2	7 July	„	323	13	4

(9)

1897.	Dr.	£	*s*.	*d*.	1897.	Cr.	£	*s*.	*d*.
18 Jan.	To goods, 3 mos.	375	2	5	25 Jan.	By cash	350	0	0
10 Feb.	„ 4 „	503	16	9	19 Mar.	„	259	10	0
7 Apr.	„ 6 „	264	0	0	4 May	„	320	0	0
14 May	„ 3 „	437	12	9	21 Aug.	„	180	0	0

COMPOUND INTEREST.

230. Money is said to be lent at **Simple Interest**, when it is agreed between the lender and the borrower that the interest shall be paid over periodically as soon as it becomes due (Art. 86–97).

It may, however, sometimes be agreed that the interest shall not be paid over, but shall be retained by the borrower, and shall be considered as an additional loan upon which he must subsequently pay additional interest.

The loan thus becomes increased at regular periods, and with it is increased the sum upon which interest must be paid.

For example, suppose £100 be lent in this way, the rate of interest being 5 per cent., and the interest being payable yearly. At the end of one year the interest is £5. This is retained by the borrower, and the loan becomes £105. For the second year the interest is £5. 5*s*. This again is retained, and the loan becomes £110. 5*s*. The interest for the third year is £5. 10*s*. 3*d*., and the loan becomes £115. 15*s*. 3*d*.

Money lent upon these conditions is said to be lent at **Compound Interest.**

231. Case I.—Given the Principal, Time and Rate per cent.; **to find the Amount, and hence the Increase.**

When the rate is 5 per cent. per annum, the Interest on any Principal for one year is $\frac{5}{100}$, *i.e.*, $\frac{1}{20}$ of the Principal. The quotient can in this case be obtained at sight, and the whole calculation may be arranged as in the following example.

Example. Find the amount, and the increase, of £634. 9*s*. 4*d* *in* 3 *years at* 5 *per cent. per annum compound interest.*

	Principal	= £634. 9*s*. 4*d*.
	Interest for 1st year ($\frac{1}{20}$)	= £31. 14*s*. 5·60*d*.,
∴	amount in 1 year	= £666. 3*s*. 9·60*d*.
	Interest for 2nd year ($\frac{1}{20}$)	= £33. 6*s*. 2·28*d*.,
∴	amount in two years	= £699. 9*s*. 11·88*d*.
	Interest for 3rd year ($\frac{1}{20}$)	= £34. 19*s*. 5·99*d*.,
∴	amount in 3 years	= £734. 9*s*. 5·87*d*.,
	and increase in 3 years	= £100. 0*s*. 1·87*d*.

Note.—We may proceed in this way whenever the rate per cent. leads to an easy divisor. If, for example, the rate per

cent. be 10, $12\frac{1}{2}$, $8\frac{1}{3}$, etc., the corresponding vulgar fractions would be $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{12}$, etc., and therefore the divisors would be 10, 8, 12, etc. In such cases as these the quotients can be written down as in the above example.

EXERCISE C.

Find the amount, and the increase, of the following sums at compound interest :—

(1) £1234 at 5 per cent. in 3 years.
(2) £625. 10*s.* at 5 per cent. in 2 years.
(3) £923. 17*s.* 6*d.* at 5 per cent. in 3 years.
(4) £795. 3*s.* 10*d.* at 5 per cent. in 4 years.
(5) £258 at 10 per cent. in 3 years.
(6) £1724. 15*s.* at 10 per cent. in 4 years.
(7) £2794 at $12\frac{1}{2}$ per cent. in 3 years.
(8) £2618. 5*s.* at $12\frac{1}{2}$ per cent. in 3 years.
(9) £842. 12*s.* 6*d.* at $8\frac{1}{3}$ per cent. in 3 years.
(10) £374. 16*s.* 7*d.* at $8\frac{1}{3}$ per cent. in 3 years.

232. When the rate of interest is not one of those which lead to an easy divisor, we adopt the following method of performing the necessary calculations.

Example. Find the amount, and the increase, of £345. 13*s.* 8*d.* *in* $2\frac{3}{4}$ *years at* 3 *per cent. compound interest.*

We first express the Principal as a decimal of a pound. This can always be done at sight as explained in Art. 59. In this case the Principal is equivalent approximately to £345·683.

It will be noticed that division by 100 is effected simply *by moving the decimal point two places to the left.*

To find the amount at the end of the *first* year we multiply £345·683 by 3, divide the product by 100 (by moving the decimal point), and add the result to the original sum.

To find the amount at the end of the *second* year we multiply the amount at the end of the *first* year by 3, divide the product by 100 as before, and add the result to the amount at the end of the first year.

The final period being $\frac{3}{4}$ of a year, the interest on £100 for this period is $\frac{3}{4}$ of £3, *i.e.*, £$2\frac{1}{4}$. To find the amount at the end of this final period we therefore multiply the amount at the end of the second year by $2\frac{1}{4}$, divide the product by 100 as before, and add the result to the amount at the end of the second year.

The final amount is £374·987, which, *neglecting the fraction of a penny*, is £374. 19*s.* 8*d.*; whence the increase is £29. 6*s.* 0*d.*

The work, stated in full, appears as below:

£345·683 = Principal
3
10·37049 = int. for 1st year
345·683
356·0535 = amt. after 1 year
3
10·681605 = int. for 2nd year
356·0535
366·7351 = amt. after 2 years
$2\frac{3}{4}$
7334702
916838
8·251540 = int. for $\frac{3}{4}$ of a year
366·7351

Amount in $2\frac{3}{4}$ yrs. = £374·9866
= £374. 19*s.* 8*d.*
Increase in $2\frac{3}{4}$ years = £29. 6*s.* 0*d.*

EXERCISE CI.

Find the amount, and the increase, of the following sums at compound interest, neglecting fractions of a penny.

(1) £1256. 10*s.* in 2 years at 3 per cent.
(2) £7205. 5*s.* in 2 years at $4\frac{1}{2}$ per cent.
(3) £3264. 11*s.* 7*d.* in 3 years at 3 per cent.
(4) £3257. 13*s.* 9*d.* in 3 years at 4 per cent.
(5) £2573. 18*s.* 4*d.* in 3 years at $3\frac{1}{2}$ per cent.
(6) £5264. 7*s.* 10*d.* in 3 years at $3\frac{1}{4}$ per cent.
(7) £2969. 16*s.* 3*d.* in 3 years at $2\frac{1}{4}$ per cent.
(8) £1586. 13*s.* 6*d.* in 3 years at $4\frac{1}{2}$ per cent.
(9) £2685. 19*s.* 5*d.* in $2\frac{1}{2}$ years at 3 per cent.
(10) £1896. 15*s.* 2*d.* in $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.
(11) £1678. 9*s.* 11*d.* in $2\frac{1}{4}$ years at $4\frac{1}{2}$ per cent.
(12) £3689. 8*s.* 3*d.* in $2\frac{1}{3}$ years at $4\frac{1}{2}$ per cent.
(13) £2314. 17*s.* 2*d.* in $2\frac{2}{3}$ years at $3\frac{3}{4}$ per cent.
(14) £7201. 1*s.* 9*d.* in $2\frac{3}{4}$ years at $3\frac{1}{5}$ per cent.
(15) £3628. 7*s.* 5*d.* in $2\frac{5}{12}$ years at 4 per cent.

233. The intervals at which interest is to be reckoned and added may be any periods agreed upon, and, moreover, the rate of interest may vary from period to period.

Example. Find how much £243. 3*s.* 7*d. will amount to if lent at compound interest for* $\frac{3}{4}$ *of a year, interest being reckoned and added every three months: at the rate of* 5 *per cent. per annum for the first three months,* $3\frac{1}{2}$ *per cent. for the second, and* 3 *per cent. for the third.*

The interest on £100 for the successive periods of three months is £$1\frac{1}{4}$, £$\frac{7}{8}$, and £$\frac{3}{4}$ respectively.

Original sum = £243·179

$1\frac{1}{4}$

243179

60795

3·03974 = int. for 1st period

243·179

246·219

$1-\frac{1}{8}=\frac{7}{8}$

246219

30778

2·15441 = int. for 2nd period

246·219

248·373

$1-\frac{1}{4}$ $=\frac{3}{4}$

248373

62093

1·86280 = int. for 3rd period

248·373

Amount in 9 mos. = £250·236

= £250. 4*s.* 8*d.* (neglecting fraction of 1*d.*)

Increase = £7. 1*s.* 1*d.*

EXERCISE CII.

Find the amount of the following sums lent at compound interest, interest being added at the periods stated.

(1) £325. 3*s.* 6*d.* for one year at 4 per cent. per annum, added quarterly.

(2) £513. 12*s.* 8*d.* for four months at 4 per cent. per annum, added monthly.

(3) £672. 8*s.* 10*d.* for $1\frac{1}{2}$ years at 5 per cent. for the first year and $3\frac{1}{2}$ per cent. per annum for the final period, interest being added half-yearly.

(4) £1236. 17*s.* 9*d.* for 2 years at 3 per cent. per annum for the first year and 2 per cent. for the second, interest being added half-yearly.

(5) £2693. 5*s.* 7*d.* for 2 years at $2\frac{1}{2}$ per cent. per annum for the first six months and $1\frac{1}{2}$ per cent. for the remainder of the time, interest being added half-yearly.

234. Case II.—Given the Amount, Time, and Rate per cent., **to find the Principal or Present Worth.**

Example i. *Find the sum of money which, lent at* $2\frac{1}{4}$ *per cent. per annum compound interest, would in* $2\frac{1}{3}$ *years amount to* £262. 13*s.* 5*d.*

$$\text{The required sum} \times \frac{102\frac{1}{4}}{100} \times \frac{102\frac{1}{4}}{100} \times \frac{100\frac{3}{4}}{100} = £262.\ 13s.\ 5d.,$$

$$\therefore \text{ the required sum} = £\frac{262{\cdot}671}{1{\cdot}0225 \times 1{\cdot}0225 \times 1{\cdot}0075}$$

$$= £\frac{262{\cdot}671}{1{\cdot}0533475}$$

$$= £249{\cdot}368$$

$$= £249.\ 7s.\ 5d.$$

In the last line we have reckoned the fraction of a penny as a whole penny.

Example ii. *Find the present worth of a debt of* £325. 13*s.* 2*d.* *due in* $2\frac{3}{4}$ *years, reckoning compound interest at* 3 *per cent.*

$$\text{Amount of £1 in } 2\tfrac{3}{4} \text{ years} = £\frac{103}{100} \times \frac{103}{100} \times \frac{102\frac{1}{4}}{100},$$

$$\therefore \text{ present worth of £1} = £\frac{100}{103} \times \frac{100}{103} \times \frac{100}{102\frac{1}{4}},$$

$$\therefore \text{ present worth of £325. 13}s.\ 2d. = £325\tfrac{58}{240} \times \tfrac{100}{103} \times \tfrac{100}{103} \times \tfrac{400}{409}$$

$$= £300.\ 4s.\ 2{\cdot}28d.$$

The result will be correct to the *nearest farthing* if we determine the number of pounds to 3 decimal places.

235. When the Interest is Simple, the Principal, Rate per cent., Time and Amount are connected by the relation

$$\text{Amount} = \text{Principal} \times \left(1 + \frac{\text{rate} \times \text{no. of years}}{100}\right).$$

When the Interest is Compound, the corresponding relation is

$$\text{Amount} = \text{Principal} \times \left(1 + \frac{\text{rate}}{100}\right)^{\text{no. of years}}.$$

EXERCISE CIII.

Find the sums of money which, lent at compound interest, will amount to the following in the given times and at the given rates per cent. respectively, interest being added yearly.

(1) £100 in 2 years at 5 per cent

(2) £375 in 2 years at 5 per cent.

(3) £436. 10*s.* in 3 years at 3 per cent.

(4) £518. 5*s.* in 3 years at 4 per cent.

(5) £723. 14*s.* 8*d.* in $2\frac{1}{2}$ years at 4 per cent.

(6) £391. 8*s.* 5*d.* in $2\frac{3}{4}$ years at 4 per cent.

(7) £1276. 17*s.* 2*d.* in $2\frac{1}{2}$ years at $2\frac{1}{2}$ per cent. for the first year and 3 per cent. afterwards.

(8) £2085. 3*s.* 9*d.* in $2\frac{2}{3}$ years at 2 per cent. for the first 2 years and $2\frac{1}{4}$ per cent. afterwards.

(9) £7621. 12*s.* 4*d.* in $2\frac{2}{3}$ years at $4\frac{1}{2}$ per cent. for the first year and 3 per cent. afterwards.

(10) £5013. 11*s.* 3*d.* in $2\frac{1}{2}$ years at $2\frac{3}{4}$ per cent. for the first year and $2\frac{1}{2}$ per cent. afterwards.

EXERCISE CIV.

Find the true present worth and the discount in the following cases, reckoning compound interest and neglecting fractions of a penny.

(1) £100, due in 2 years, at 5 per cent.

(2) £600, due in $2\frac{1}{2}$ years, at 5 per cent.

(3) £500, due in 3 years, at 5 per cent.

(4) £800, due in $2\frac{1}{4}$ years, at 4 per cent.

(5) £1000, due in $1\frac{3}{4}$ years, at 4 per cent.

(6) £1350, due in $2\frac{3}{4}$ years, at 3 per cent.

(7) £562. 10*s.*, due in $2\frac{1}{2}$ years, at $3\frac{1}{2}$ per cent.

(8) £371. 15*s.*, due in $2\frac{1}{4}$ years, at $3\frac{1}{2}$ per cent.

(9) £546. 17*s.* 6*d.*, due in $1\frac{3}{4}$ years, at $3\frac{1}{2}$ per cent.

(10) £861. 12*s.* 6*d.*, due in 3 years, at $4\frac{1}{2}$ per cent.

236. Large sums of money are every year advanced to Local Government Boards and other public bodies, for improvements, building of workhouses, schools, harbours, bridges, etc., to be repaid in a given number of years, including interest. The following scheme of tables for the repayments of Loans was compiled under the Local Government Act, 1858, and examined by the actuary of the National Debt Office. It gives the sums needed as the annual amount of principal combined with interest required for the liquidation of a debt of £100, at the stated percentage, in 10, 20, 30, 40, and 50 years:—

Years.	3 per Cent.			3½ per Cent.			4 per Cent.			4½ per Cent.			5 per Cent.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
10	11	14	5½	12	0	5¾	12	6	7	12	12	9	12	19	0
20	6	14	5¼	7	0	8¾	7	7	2	7	13	9	8	0	5¾
30	5	2	0½	5	8	9	5	15	8	6	2	9½	6	10	1¼
40	4	6	6¼	4	13	7¾	5	1	0½	5	9	8¼	5	16	6¾
50	3	17	8¾	4	5	3¼	4	13	1¼	5	1	2½	5	9	6¾

NOTE.—Of course, if the loan be for £1000, each annual instalment as shown above must be multiplied by 10; if for £2000, then by 20, and so on.—(*Whitaker's Almanack.*)

MONETARY SYSTEMS AND EXCHANGE.

237. In consequence of the intercourse and traffic between different countries, it becomes continually necessary to determine how much in the monetary system of one country is equivalent to a sum of money expressed in the monetary system of another.

For example, a person about to leave London for Paris may wish to change a certain sum in English money for its equivalent in francs and centimes; or a merchant in London with money in the form of pounds, shillings and pence may wish to pay for goods bought in Berlin, the value of which is stated in marks, etc.

In former times such a calculation was much more troublesome than it is now, owing to the large number of small

states, with independent systems of coinage, into which Europe was then divided. At the present time, however, one uniform system is in use throughout what is now the German Empire, while another uniform system has been adopted in common by the states which together form the Latin Monetary Union.

238. Until 1872 some of the Continental States practised **Bi-metallism**, that is, they coined at their Mints *on demand* both gold and silver, the worth of any quantity of gold being fixed at $15\frac{1}{2}$ times that of an equal weight of silver. The relative values of the two metals were consequently fixed within narrow limits, for no one would sell silver for less than could be obtained by sending it to the French Mint. Since 1872, however, no country has coined both gold and silver as *standard money*.

As we have already said (Art. 204), gold is the standard in England, and silver pieces are merely *tokens* of certain fractions of the standard gold coin, being legal tender only up to 40*s*. Gold is the standard in France also, but the legal tender of silver pieces is not limited as in England. In India the standard is silver.

239. For purposes of exchange we can ascertain the comparative values of a gold coin and a silver coin only by estimating the value of the pure metal in each at the market price of silver for the day. There is no fixed exchange value for silver, or for a silver coin, any more than for an article of ordinary merchandise.

A florin is now worth in England half as much again as a rupee, although it actually contains less silver; for the value of a florin is its token value, $\frac{1}{10}$ of £1, while a rupee has only its exchange value, which is now about 1*s*. 4*d*. In 1861 the exchange value of the rupee was as high as 2*s*. 2*d*.

A troy ounce of standard gold contains 440 grains of fine gold and 40 grains alloy. It sells for £3. 17*s*. $10\frac{1}{2}$*d*.

A troy ounce of standard silver contains 444 grains of fine silver and 36 grains alloy. It used to sell in England for 5*s*., but now sells for about 2*s*. 3*d*. only.

240. The principal monetary systems are the following; most of them are now decimal.

Gold-Standard Countries.

Name.	*Standard coin.*	*Grains of fine gold.*	*Exchange value.*	*Sub-divisions.*
United Kingdom	Sovereign	113·0016	£1	20 shillings
Latin Union	Franc	4·4803	9·51*d.*	100 centimes
Germany	Mark	5·5313	11·745*d.*	100 pfennige
Austria-Hungary	Florin	9·4099	19·985*d.*	100 kreutzers
Holland	Florin	9·3459	19·84*d.*	100 cents
Norway, Sweden & Denmark	Krone	6·2227	13·2*d.*	100 öre
United States	Dollar (gold)	23·2199	49·29*d.*	100 cents
Turkey	Pound	102·0804	18*s.* 0·8*d.*	100 piastres

Silver-Standard Countries.

Name.	*Standard coin.*	*Grs. fine silver.*	*Exchange value.*	*Subdivisions.*
India*	Rupee	165·0	Except in the case of India, exchange value in gold depends upon market price of silver.	16 annas
Russia	Rouble	277·7221		100 copeks
Mexico	Dollar (silver)	377·168		100 cents
China	Shanghai tael (a weight)	516·405		1000 cash

The states which form the "Latin Monetary Union" are France, Italy, Switzerland, Belgium and Greece, and their coins are alike in quality and weight. The same system has been adopted in part by Spain, Roumania, Servia and Bulgaria, but they have not joined the Union. The *francs* and *centimes* of France are however called *lire* and *centesimi* in Italy, *dinars* and *paras* in Servia, *pesetas* and *centimos* in Spain, *drachmai* and *lepta* in Greece, *leïs* and *banis* in Roumania, *leva* and *stotinkis* in Bulgaria.

Nearly all the South American countries have issued a standard coin termed a *peso*, which is supposed to be equal in quality (or fineness) and weight to the French 5-franc piece, but the money is mostly represented by a paper currency. In British Colonies, English money is current. The currency of Ceylon, Mauritius, and the East Africa Protectorate is based, however, on the rupee of British India; that of the Straits Settlements, Hong Kong and Labuan, on the Mexican dollar; and that of Canada and British Honduras on the United States dollar. A British dollar, identical in weight and fineness with the Japanese silver *yen* was authorized by order in Council in 1894, and has been made legal tender in the Straits Settlements, Hong Kong, and Labuan. At Shanghai and other Chinese ports the

* Indian Exchange is at present in a very unsettled state, and is not subject to ordinary rules.

silver *tael* is the medium of commerce. In Russia a depreciated paper currency is used for all transactions of 1 rouble and upwards. The exchange value of the silver rouble is 1*s.* 2¼*d.*, when silver is at 26*d.* per ounce.

241. In Art. 183 we noticed how a share in a Joint Stock Company has two values:—(i) its *nominal* value, or the amount of capital it represents, and (ii) its *marketable* value, or the amount of money it will sell for, which may vary from day to day.

Similarly a sum of money expressed in £. *s.* *d.* has two values with reference to francs and centimes; (i) its *nominal* or *intrinsic* value, which depends only upon the quantity of gold which it represents, and (ii) its *marketable* value, which may differ slightly from the intrinsic value, and may vary from day to day. The latter depends upon the state of trade.

In determining the equivalent in the monetary system of one country of a given amount expressed according to that of another, we determine what is called the **rate of exchange.**

242. When English money is exchanged for French money, the *nominal* rate is 25 francs (or, more accurately, 25·22½ francs) for £1; and in the case of American money, one dollar for 4*s.* 2*d.* These rates are called the **par of exchange.**

The use of the term *par*, to express the equality between the nominal rate and the actual rate, is similar to the use of the same word to express that the nominal value of a share or of stock is the same as its marketable value.

Example. When standard silver, $\frac{37}{40}$ths fine, is worth 3*s.* 6*d.* *per oz. Troy, find the par of exchange between a sovereign and a rupee, given that a rupee contains $\frac{3}{8}$ oz. of silver, $\frac{11}{12}$ths fine.*

1 oz. Troy, $\frac{37}{40}$ths fine contains $\frac{37}{40}$ oz. Troy of pure silver;
also $\frac{3}{8}$ oz. Troy, $\frac{11}{12}$ths fine contains $\frac{11}{32}$ oz. Troy .....
But value of $\frac{37}{40}$ oz. pure silver = $3\frac{1}{2}$ shillings
∴ ..... $\frac{11}{32}$ ..... $= \frac{11}{32} \times \frac{40}{37} \times \frac{7}{2} s.$
$= \frac{385}{296} s.$;
∴ value of silver in 1 rupee = $\frac{385}{296} s.$;
∴ 1 sovereign = $\frac{20}{\frac{385}{296}}$ rupees
$= \frac{20 \times 296}{385}$ rupees
$= 15\frac{84}{77}$ rupees
$= 15{\cdot}38$ rupees.

243. The following are examples for practice in the exchange of English money into an equivalent sum of foreign money, and *vice versa*.

EXERCISE CV.

Exchange

ITALY.

(1) £335 at 25l. 20c. per £1.
(2) £460 at 25l. 22c. per £1.
(3) 640l. 50c. at 25l. 20c. per £1.
(4) 3769l. 70c. at 25l. 30c. per £1.

GREECE.

(5) £370 at 25dr. 10l. per £1.
(6) £149 at 25dr. 30l. per £1.
(7) 9602dr. 20l. at 23dr. 42l. per £1.
(8) 9424dr. 25l. at 25dr. 30l. per £1.

SPAIN.

(9) £85. 10*s.* at 9½*d.* per peseta.
(10) £92. 11*s.* at 9*d.* per peseta.
(11) 4494 pesos at 2*s.* 3*d.* per peso.
(12) 5763 pesos at 2*s.* 11*d.* per peso.

PORTUGAL.

(13) £450 at 4mil. 110 reis per £1.
(14) £565 at 4mil. 130 reis per £1.
(15) 1920mil. 250 reis at 4*s.* 5½*d.* per milreis.
(16) 2765mil. 300 reis at 4*s.* 5¼*d.* per milreis.

GERMANY.

(17) £632. 10*s.* at 20mk. 36pf. per £1.
(18) £742. 15*s.* at 20mk. 64pf. per £1.
(19) 2675mk. at 20mk. 43pf. per £1.
(20) 3284mk. at 20mk. 58pf. per £1.

AUSTRIA.

(21) £320 at 11fl. 54kr. per £1.
(22) £860 at 10fl. 3kr. per £1.
(23) 6251fl. 0kr. at 9fl. 50kr. per £1.
(24) 6733fl. 20kr. at 9fl. 30kr. per £1.

HOLLAND.

(25) £850 at 12fl. per £1.
(26) £222 at 11fl. 93c. per £1.
(27) 5672fl. 0c. at 12fl. per £1.
(28) 4217fl. 50c. at 12fl. 5c. per £1.

NORWAY.

(29) £1250 at 18 krone 17 öre per £1.
(30) £3574 at 18 krone 22 öre per £1.
(31) 1125 krone 50 öre at 18 krone 7 öre per £1.
(32) 4250 krone 78 öre at 18 krone 28 öre per £1

UNITED STATES.

(33) £150. 10*s.* at 47½*d.* per dollar.
(34) £480. 5*s.* at 49*d.* per dollar.
(35) 2916dols. at 4dols. 79c. per £1.
(36) 6500dols. at 4dols. 90c. per £1.

RUSSIA.

(37) £370 at 6r. 25co. per £1.
(38) £645 at 6r. 45co. per £1.
(39) 1461r. 2co. at 6r. 38co. per £1.
(40) 4160r. 25co. at 6r. 45co. per £1.

TURKEY.

(41) £650 at 110 piastres 28 paras per £1.
(42) 4000 pias. at 109·50 piastres per £1.

EGYPT.

(43) £725 at 97 piastres 38 paras per £1.
(44) 5410 pias. at 98 piastres per £1.

CHINA.

(45) £460. 1*s.* 4*d.* at 2*s.* 9½*d.* per tael.
(46) 4350 taels at 2*s.* 5⅞*d.* per tael.

JAPAN.

(47) £730. 1*s.* 8*d.* at 1*s.* 10*d.* per yen.
(48) 3167 yens at 1*s.* 10*d.* per yen.

FOREIGN BILLS OF EXCHANGE.

244. Business transactions between one country and another are commonly carried on with what are called **Foreign Bills of Exchange**, or briefly **Foreign Bills**, by means of which large accounts may be settled without sending from one country to another any large quantities of gold or silver.

For example, suppose *A* in London sends goods to *B* in Paris, and suppose *C* in Paris sends goods of exactly the

same value to *D* in London; *C* then *draws a bill* on *D*, which bill *D accepts* (Art. 254).

This is a case in which *D* in London owes *C* in Paris exactly as much as *B* in Paris owes *A* in London, and the whole might be settled by *D* paying *A*, and *B* paying *C*. For this purpose *B* in Paris may buy the *bill* which *C* has drawn on *D* who is in London, and may send it by post to *A* who is also in London. *A* then gets the money from *D* and the matter is at an end, without any gold or silver having passed between the two places.

245. The case we have imagined is a simple one, but it illustrates the way in which such transactions are arranged. A merchant as far as possible avoids remitting gold and silver, for that is a costly process. Instead of doing so, he buys *bills of exchange*, and sends them to his creditor by post. There may however be a balance which can be paid only by remitting gold.

The medium between a merchant who wants to buy a bill and a merchant who wants to sell one is generally a *bill-broker*, who sells to the one and buys from the other (Art. 219).

When Paris owes London more than London owes Paris, *bills on London* are sought after in Paris, and a merchant there is willing to pay more for them than their *nominal* value; in Paris bills on London are then at a *premium*, and in London at the same time *bills on Paris* are at a *discount*.

The premium is of course never more than the cost of transmitting gold, either as **specie**, in the form of coins, or as **bullion**, in the uncoined form. These limits of exchange are called **specie points**. The cost of sending bullion from London to Paris is about 10 centimes per 25 francs. Thus taking the mint par of exchange at 25·22½ francs for £1, a merchant in London, if he sent bullion, would have to pay in Paris at the rate of only 25·12½ francs for £1; and a merchant in Paris, on the other hand, if he sent bullion, would have to pay in London at the rate of 25·32½ francs for £1. The specie points for London and Paris are therefore about 25·12½ francs and 25·32½ francs per £1.

The rate of exchange which varies from day to day, but is limited more or less by the "specie points," is called the **course of exchange**.

When bills are neither at a premium, nor at a discount, they are said to be *at par*. Pounds and francs then exchange at their intrinsic values in gold, namely £1 for 25·22½ francs. In Paris, when bills on London are at a premium, more than 25·22½ fr. are given for £1, and the course of exchange is said to be favourable to London; when at a discount, less than 25·22½ are given for £1, and the course of exchange is said to be unfavourable to London.

246. The Money Market comprises bankers, discount brokers, bill brokers and bullion merchants. When the banks hold more money than there is at the time a demand for, they reduce their rates of *discount*, *i.e.*, the rate of interest they charge for lending money on securities. Money is then said to be *cheap*, and the Money Market, *easy*. When the demand for loans is brisk, the bank rate rises, money is said to be *dear* or *scarce*, and the Money Market is *firm*. Thus, in a London newspaper the Money Market report may read :—

"The Money Market remained in a very firm state. Brokers quoted 2 15-16 for three months' fine paper, while four months' bills were done at 2¾, and six months' at 2⅞. Gold was in increased demand in the open market for New York, and the price for bars rose to 77*s.* 10½*d.* per ounce; while, at the Bank of England, United States coin to the value of £102000 was sold. Sovereigns to the number of 114000 were received at the Bank from Australia, and French gold coin to the value of £10000 was bought."

The rates of discount given above are so much per annum, and "fine paper" means bills drawn on firms of first-class standing or on banks.

247. The lowest rate at which the Bank of England discounts "fine" bankers' bills, or lends money on high-class securities, is called the **bank-rate**. Brokers generally charge a lower rate of discount than the Bank.

248. When the Government of India finds it necessary to raise money, it generally issues **promissory notes** for, say, Rs. 1000 and offers them for sale. A fixed rate of interest is paid to the holders of these notes until they are paid off. Money spent in buying such notes is said to be invested in Government of India Securities. Thus we often

read in the Money Market column of the leading London newspapers paragraphs similar to the following :—

"A special sale of 50,00,000[1] rupees was made by the India Council to-day; 30,00,000 rupees on Calcutta, and 10,00,000 rupees on Bombay, all bills, being at 1*s.* 3 15–16*d.* per rupee; and the remainder in telegraphic transfers on Calcutta, at 1*s.* 4 1–16*d.* per rupee."

The revenues of the Government of India are received in silver, *i.e.*, rupees, and the money is kept in the treasuries at Calcutta, Bombay and Madras. The Government of India has to pay to the British Government the home charges of troops serving in India, pensions, interest on loans, etc., in gold. To do this the India Council draws bills on the Government of India; the latter sells its bills in London, where it receives gold equivalent to the amount of rupees payable in India. "Council bills" are bills on demand, drawn by the Secretary of State for India on the Government Treasury (in the extract, Calcutta and Bombay). "Telegraphic transfers" mean that the Secretary of State telegraphs to the Government Treasury (in the extract, Calcutta) to pay certain banks or firms (the nominees of the purchasers of the transfers) so many rupees.

249. Foreign bills are bought and sold in the Royal Exchange on Tuesdays and Thursdays, and prices are published in the "Money Market" column of the leading London newspapers in the following form:—

On 'Change this afternoon rates on Continental cities were as under, the quotations being for bills of three months' date, except where otherwise stated:—Holland, 12·3¼ to 12·3⅜; ditto short, 12·1¼ to 12·1⅜; Belgium, 25·40 to 25·47½; Paris, 25·37½ to 25·45; ditto short, 25·25 to 25·30; Berlin, 20·60 to 20·64; St. Petersburg, 25 to 25⅛; Copenhagen, 18·35 to 18·39; Stockholm, 18·36 to 18·40; Vienna, 12·15 to 12·17½; Naples, 26·82½ to 26·92½; Genoa, 26·82½ to 26·92½; Madrid, 35⅜ to 35⅝; Cadiz, 35⅜ to 35⅝; Lisbon, 35⅛ to 35⅜; Oporto, 35⅛ to 35⅜.

From this a merchant knows that he could buy from the brokers bills on Paris, payable in three months' time, at the rate of £1 for each 25·37½ francs payable in Paris; and that he could sell similar bills payable in Paris to a broker at the rate of £1 for each 25·45 francs on the bills.

[1] The number 100000 is called a *lac* in India, and is written 1,00,000; therefore 50,00,000 represents *fifty lacs*, and 30,00,000 represents *thirty lacs*.

"Short," or "cheque," means bills of exchange payable at sight, *i.e.*, on demand or within a few days.

The difference between the quotations for "short" rate and "long" rate depends upon the rate of discount in the country on which the bill is drawn (Art. 247), and upon the state of credit.

Holland, 12·3¼ to 12·3¾, means so many florins and stivers per £1. [1 florin = 20 stivers.]

Berlin, 20·60 to 20·64, means so many marks and pfennigs per £1.

Copenhagen, 18·35 to 18·39, means so many krone and öre per £1.

Vienna, 12·15 to 12·17½, means so many florins and kreutzers per £1.

St. Petersburg, 25 to 25⅛, states the number of pence for 1 silver rouble.

Madrid and Cadiz, 35⅜ to 35⅝, states the number of pence for 1 peso (=5 pesetas).

Lisbon and Oporto, 35⅛ to 35⅜, states the number of pence for 1 milreis.

250. From the quotations in Art. 249 we see that there are two terms in London rates of exchange, one being fixed, the other fluctuating. Thus in the exchange between London and Paris the fixed term is the pound sterling, and the fluctuating term is the number of francs and centimes received for the pound sterling.

By this method a merchant knows at a glance from the quotation list the price at which he can *buy* bills on Paris and *sell* bills on Paris.

In the exchange between London and Lisbon the milreis is the fixed term, and the value given in pence is the fluctuating term.

These statements may be briefly expressed by saying that London *receives* from Paris a fluctuating number of francs and centimes per £1 sterling, and London *gives* to Lisbon a fluctuating number of pence per milreis. The greater the number of foreign standard coins obtainable for £1 sterling in the former case, or the fewer pence required in the latter, the more favourable is the exchange to this country (Art. 245).

In the quotation list the fluctuating term only is given, the fixed term being understood.

251. Quotations such as the following appear as financial news in the daily papers

The following rates of exchange for London have been received by telegraph from the centres named:—Paris, 25·28½; Berlin, 20·45½; Hamburg, 20·30½; Frankfort, 20·46; Amsterdam, 12·08⅛; Vienna, 120·25; Madrid 33·92½; Lisbon, 35⅝; Rome (sight) 26·58; St. Petersburg (three months), 93·95; Constantinople (three months), 109·20; Antwerp, 25·32 to 25·36.

This means that bills on London could be bought on that particular day in Paris at the rate of £1 for every 25·285 francs.

Amsterdam, 12·08⅛ on London, means £1=12 florins 8⅛ cents.

Vienna, 120·25 on London, gives the number of florins for £10.

St. Petersburg (three months), 93·95, gives the number of roubles per £10, for which bill-brokers will sell a bill on London payable in three months' time.

Constantinople, 109·20, gives the number of piastres per £1.

Lisbon, 35⅝, gives the number of pence for one milreis.

Madrid, 33·92½, gives the number of pesetas per £.

NOTE.—The London rate of exchange is quoted differently in Amsterdam.

252. Here is another example of a daily quotation in the leading London newspapers:—

The Silver market to-day was weak, and the quotation for bars was ⅜*d.* lower, at 25*d.* per ounce. The Indian transfer rates were cabled 1*s.* 4 1-32*d.* per rupee; while the China exchanges came easier, at 1*s.* 10½*d.* per dollar from Hong Kong, and at 2*s.* 5⅞*d.* per tael from Shanghai.

The first sentence states the lowness of the price of silver, since silver in bars was sold at 25*d.* per ounce. The second sentence means that bills on India, the necessary instructions for which were given by telegraph, cost 1*s.* 4$\frac{1}{32}$*d.* per rupee; bills on Hong Kong cost 1*s.* 10⅛*d.* per dollar, and bills on Shanghai 2*s.* 5⅞*d.* per tael.

253. Foreign Bills of Exchange which have to be sent a long distance are usually drawn in sets of three, to prevent inconvenience should one of them be lost in transmission. The first is generally despatched by the first mail steamer, and the others by following mails. Each copy contains a condition that it shall be payable only while the others remain unpaid.

254. The following is a specimen of a Foreign Bill:—

No. 999. £501. 17*s.* 4*d.* *Due*.....

CALCUTTA, *July 4th*, 1898.

At six months after sight of this First of Exchange (Second and Third of same tenor and date being unpaid) pay to the order of Messrs. Allen & Reid Five hundred and one pounds seventeen shillings and four pence sterling as advised.

JONES & SMITH.

TO THE ANGLO-INDIAN BANK,
LONDON, E.C.

NOTE.—The second bill would read "At six months after sight of this Second of Exchange (First and Third of same tenor and date being unpaid)," etc.

255. The amount of a foreign bill is always given in the money of the country where it is to be paid.

The drawer must always send advice to the drawee that he has drawn upon him (or them); hence the insertion of the words "as advised."

256. The following are examples of direct rates of exchange between two countries.

In actual business transactions, brokerage, stamp duty, interest or discount, and commission, if any, would have to be taken into consideration.

The usual charges of bill-brokers for buying bills are $\frac{1}{8}$ to $\frac{1}{16}$ per cent.

Example i. *A merchant in London buys a bill on Paris for* 8534·4 *francs; how much will have to be paid, when the course of exchange on Paris is* 25·4 *francs?*

The cost of a bill on Paris for 25·4 fr. = £1,

$\therefore$ ..... 8534·4 fr. = $£\frac{8534{\cdot}4}{25{\cdot}4}$

= £336.

Example ii. *A merchant in Paris buys a bill on London when the course of exchange on London is* 25·2 *francs; for how much must the bill be drawn to be worth* 8534·4 *francs in Paris?*

The face value of a bill on London worth 25·2 fr. = £1.

$\therefore$ ..... 8534·4 fr = $£\frac{8534\cdot4}{25\cdot2}$

= £338. 13*s.* 4*d.*

Example iii. *A debt of* 8534·4 *francs is owed by a London merchant to one in Paris. The London rate of exchange is* 25·4, *the Paris rate is* 25·2. *Find which is cheaper:* (1) *to buy direct bills on Paris, or* (2) *to instruct his creditor to draw upon him in London.*

From the solutions to Ex. i. and ii. it will be seen that by the first method there is a saving of £2. 13*s.* 4*d.*

Example iv. *Short exchange on Paris is quoted at* 25·25, *and* 3 *months' bills at* 25·37½, *the market rate of discount in Paris being* 3 *per cent.; which is the cheaper way of remitting to Paris?*

£1 buys 25·25 fr. payable in Paris *now.*

£1 buys 25·37½ fr. ..... 3 *months hence.*

Discount on 25 fr. 37½ c. for 3 months = $\frac{3}{400}$ of 25 fr. 37½ c.

= 19 c.;

$\therefore$ sight rate = 25 fr. 37½ c. − 19 c. = 25 fr. 18½ c.;

$\therefore$ short rate is cheaper.

Example v. *Exchange* 600 *American dollars for English money, when English money is at a premium of* 20 *per cent.*

English money being at 20 per cent. prem. means that it is worth 20 per cent. more American than it would be if it were at par.

At par 4*s.* 2*d.* = 1 dollar;

$\therefore$ at 20% prem. 4*s.* 2*d.* = $(1+\frac{1}{5})$ dollar

= $\frac{6}{5}$ dollar;

$\therefore$ 1 dollar = $\frac{5}{6} \times 50d.$;

$\therefore$ 600 dollars = $600 \times \frac{5}{6} \times 50d.$

= 25000*d.*

= 2083*s.* 4*d.*

= £104. 3*s.* 4*d.*

EXERCISE CVI.

Write out in proper form, and find the cost of a Bill of Exchange in London for

(1) 65000 francs on Paris at 25·30 (francs per £1).

(2) 86000 marks on Berlin at 20·60 (marks per £1).

(3) 52500 florins on Vienna at 12·15 (florins per £1).

(4) 68000 florins on Amsterdam at 12·1¼ (12 fl. 1¼ stivers per £1; 1 fl. = 20 stivers).

(5) 75000 kronas on Stockholm at 18·40 (kronas per £1).

(6) 62500 lire on Genoa at 26·82½ (lire per £1).

(7) 40800 dollars on New York at 4·85 ($ per £1).

(8) 7528 rupees on Bombay at 1*s.* 4*d.* (for 1 rupee).

(9) 9250 roubles on St. Petersburg at 25½ (pence for 1 rouble).

(10) 8360 milreis on Lisbon at 35½ (pence for 1 milreis).

(11) 4070 taels on Shanghai at 2*s.* 5*d.* (for 1 tael).

(12) 5080 dollars on Hong Kong at 1*s.* 10*d.* (for 1 dollar).

(13) If £1 in France exchanges for 25·2 francs, and if 20 francs in England exchange for 15*s.* 9*d.*, how much do you lose in £100 by the two exchanges?

(14) A London merchant wishes to pay a debt of 4500 francs to one in Paris, when exchange is quoted at 25·4 in London and 25·45 in Paris. What gain is there to the former if, instead of remitting, he allows himself to be drawn upon?

(15) A London merchant wishes to pay a Berlin merchant for goods received, and finds the rates of exchange are London on Berlin, at 3 months, 20·53 marks for £1; Berlin on London, at 3 months, 20·27 marks for £1. Which will be most advantageous—for him to remit bills to Berlin, or allow himself to be drawn upon in London? And how much per cent. better, interest being allowed at the rate of 4 per cent. per annum for the time the bills have to run?

(16) When the course of exchange is London on Paris 25 fr. 25 c. at sight, what should be paid for 3 months' bills on Paris, the rate of discount in Paris being at 4 per cent. per annum?

(17) If the price in Lisbon of 3 months' bills on London be 35$\frac{1}{16}$ pence per milreis, and the rate of discount in London be 1½ per cent. per annum, what is the corresponding short rate and what is the cost to a merchant in Lisbon on that day of paying £1000 in London?

(18) Short exchange on London (cheques) is quoted at 25·30, and 3 months' bills at 25·42½, the market rate of discount in London being 3 per cent; which is the most advantageous way for a merchant in Paris to remit to London?

(19) If the cheque exchange in London on Paris is 25·30, and the rate of discount for 3 months' bills in London is at 5 per cent. per annum, what debt in Paris can be discharged by a person in London who holds a 3 months' bill on London for £1000?

(20) Exchange 5220 dollars for English money when it is at a premium of 7½ per cent.

(21) Exchange 12000 francs for English money when it is at a discount of 5 per cent.

(22) A merchant in New York wishes to remit to London 5110 dollars, a dollar being equal to 4*s*. 6*d*. English; for what sum in English money must he draw his bill when bills on London are at a premium of 9½ per cent.?

(23) A merchant in Calcutta wishes to remit Rs. 4100 to London, a rupee being equivalent to 1*s*. 3*d*. For what sum in English money must he draw his bill when bills on London are at a premium of 2½ per cent.?

(24) If the par of exchange be Re. 1 = 2*s*., but an Indian bill of exchange for Rs. 540. 12 as. be negotiated in London for £51. 10*s*., how much per cent. is the rate of exchange below par?

(25) If 25·50 francs are obtained in Paris for £1 sterling, is English money at a premium or at a discount in Paris, and how much per cent.?

(26) When the rate of exchange between London and Paris is £1 = 25·25, how much per cent. is gained by taking English sovereigns at 25 fr. each, and how much per cent. is lost by parting with them at that price?

257. The *course* of exchange between any two countries depends upon the state of their *direct* trade. It might therefore sometimes be advantageous to the merchant in London, instead of buying bills on Paris, to buy bills on Antwerp, and to instruct his agent at Antwerp to buy bills there on Paris. Or again he might remit to Paris by way of New York, or by way of Antwerp, Vienna and New York, or even by a still more circuitous route.

The determination of the course of exchange, when one

or more places intervene, is called the **arbitration of exchange.**

Example i. *If the exchange between London and Amsterdam be* 36*s. Flemish per* £1, *and between Paris and Amsterdam* 2 *fr. per* 3*s. Flemish, how much must be remitted to Paris, by way of Amsterdam, to pay a debt of* 20000 *fr.? Find also the rate of exchange between London and Paris.*

We have 1 franc $= \frac{3}{2}s.$ Flemish,

and 1*s.* Fl. $= \frac{1}{36}$£. sterling,

$\therefore$ 20000 fr. $= 20000 \times \frac{3}{2}s.$ Fl.

$= 20000 \times \frac{3}{2} \times \frac{1}{36}$£

$= £\frac{25000}{3}$

$=$ £8333. 6*s.* 8*d.*;

$\therefore$ sum to be sent to Amsterdam $=$ £8333. 6*s.* 8*d.*

Again, £1 sterling $= 36s.$ Fl.

$= 36 \times \frac{2}{3}$ francs

$= 24$ francs;

$\therefore$ rate of exchange between London and Paris is given by

£1 sterling $= 24$ francs.

Example ii. *A person in London wishes to remit a debt of* 1016 *pistoles to Madrid when the exchanges are*

1 *Sp. pistole* $= 15s.$, £1 $= 25{\cdot}4$ *francs*, 19 *francs* $= 1$ *Spanish pistole.*

Is it more advantageous to remit directly to Madrid or circuitously through Paris?

Direct:— 1016 pistoles $=$ £$1016 \times \frac{3}{4}$

$=$ £254×3

$=$ £762.

Viâ Paris:—1016 pistoles $= 1016 \times 19$ francs

$=$ £$1016 \times 19 \times \frac{1}{25{\cdot}4}$

$=$ £760;

$\therefore$ by remitting through Paris there is a saving of £2.

Example iii. *If* £3 $= 20$ *thalers*, 25 *thalers* $= 93$ *fr.*, 27 *fr.* $=$ 5 *scudi*, *and* 62 *scudi* $= 135$ *gulden, how many gulden can be got for* £11*?*

£11 $= 11 \times \frac{20}{3}$ thalers

$= 11 \times \frac{20}{3} \times \frac{93}{25}$ francs

$= 11 \times \frac{20}{3} \times \frac{93}{25} \times \frac{5}{27}$ scudi

$= 11 \times \frac{20}{3} \times \frac{93}{25} \times \frac{5}{27} \times \frac{135}{62}$ gulden[1]

$= 11 \times \frac{2 \times 2 \times 5}{3} \times \frac{3 \times 31}{25} \times \frac{5}{9 \times 3} \times \frac{9 \times 5 \times 3}{2 \times 31}$ gulden

$= 11 \times 2 \times 5$ gulden

$= 110$ gulden.

[1] This line illustrates the method known as the "Chain Rule."

EXERCISE CVII.

(1) How many francs must be transmitted from Paris to Berlin to discharge a debt of 420 thalers, a thaler being equivalent to 3*s.*, and 24 francs to £1 sterling?

(2) A merchant at Hong Kong wishes to pay for goods in Bombay worth Rs. 15000; how many dollars will he have to pay, exchange at Hong Kong on Bombay being Rs. 250 for 100 dollars?

(3) How many rupees must be transmitted from Bombay to Shanghai to discharge a debt of 420 taels, a tael being equivalent to 2*s.* $5\frac{7}{12}$*d.*, and Rs. 16 to £1?

(4) A bill bought in Calcutta at 1·92 francs per rupee is sold in London at $9\frac{3}{8}$*d.* per franc; what is the exchange between London and Calcutta?

(5) The exchange between London and Hamburg is 13 marks 10 schillings per £ sterling, and between Hamburg and Paris it is 150 marks for 275 francs; what is the exchange between London and Paris? (1 mark = 16 schillings.)

(6) A bill bought in London at 25·6 francs per £ sterling is sold in Lisbon at 172 reis per franc; what is the exchange between London and Lisbon? (1000 reis = 1 milreis.)

(7) A bill on Amsterdam is bought at 12 florins per £ sterling. The proceeds purchase at Amsterdam bills on Hamburg at $34\frac{1}{4}$ fl. for 40 marks. These are forwarded to Paris, and sold at 185 fr. per 100 marks; what is the exchange between London and Paris?

(8) Is it profitable for a Bombay merchant to buy a Paris Hoondi (bill) in order to pay a debt of 10000 francs, when 1 franc = 10 as. 6 p.; or to remit the amount through London, the course of exchange being 1*s.* 3*d.* for Re. 1, and 25 francs for £1?

(9) The exchange in London on Paris and Amsterdam being respectively 25·15 and 12·08, which would be the cheaper way of transmitting money from Paris to Amsterdam—by selling French bills in London and buying Dutch ones with the money, or purchasing Dutch bills in Paris, the rate in Paris being 2·08 francs for a florin?

(10) A London merchant owes a sum of money in Paris; which method of payment will be most advantageous to him—a direct exchange, or a circuitous remittance from London

to Venice, from Venice to Hamburg, and from Hamburg to Paris, the exchanges being as follows: £1 = 24·6 francs, 19 francs = 10 Hamburg marks, 1 mark = 4 lire of Venice, and 55¼ lire = £1?

(11) A merchant in London is indebted to one at St. Petersburg to the amount of 15000 roubles: the exchange between St. Petersburg and London is 50*d.* per rouble, between St. Petersburg and Amsterdam 91*d.* Fl. per rouble, and between Amsterdam and London 36*s.* 3*d.* Fl. per £ sterling. What difference will it make if the London merchant is drawn upon through Amsterdam or direct?

(12) A merchant in London owes a debt of 1000 pistoles to one in Cadiz; find what he gains by sending it to him through France, the exchange being £1 = 25·4 francs, 19 francs = 1 pistole, and 4 pistoles = £3.

(13) A person in London owes another at St. Petersburg 920 roubles, which must be sent him through Paris. He pays the needful sum to his broker at a time when the exchange between London and Paris is 25 fr. 15 c. for £1, and between Paris and St. Petersburg 1·2 fr. for one rouble. The broker delays remitting until the rates are 25 fr. 35 c. for £1, and 1·15 fr. for one rouble. What does the broker gain or lose by the delay?

(14) A person in London owes another in St. Petersburg 460 roubles, which must be remitted through Paris. He pays the requisite sum to his broker when the exchange between London and Paris is 23 francs for £1, and between Paris and St. Petersburg 2 francs for one rouble. The remittance is delayed until the rates of exchange are 24 francs for £1, and 3 francs for 2 roubles. What does the broker gain or lose by the transaction?

(15) If in London I get £1 for 25 fr. 20 c., what shall I gain or lose per cent. by taking French money into Bavaria, where the exchange is 11 guld. 40 kr. for £1, and 8 guld. 20 kr. for 20 francs? (1 gulden = 60 kreutzer.)

(16) Express 32 Spanish piastres in French money, at the rate of 25⅓ fr. for £1, and £3. 7*s.* 6*d.* for 20 piastres.

(17) What would £1 be worth at Paris if a bill at Amsterdam of 12 fl. 15 c. sells for £1, and 9 fl. 50 c. are worth 20 fr. at Paris?

(18) If Rs. 33 be worth 43*s.*, and 25 fr. be worth 20*s.*, change 1000 fr. into rupees.

(19) If 5 silver roubles be worth 15*s.* 7½*d.*, and £4 be worth 5 Napoleons, and 1 Napoleon be worth 4 dollars 25 cents, how many dollars are worth 45·056 silver roubles?

(20) If £1 be worth 11·75 Dutch guilders, 101 thalers worth 175 guilders, and 2 thalers worth 7·35 francs, how many francs should be received for £40?

(21) The exchange at Paris upon London is at the rate of 25 fr. 70 c. for £1 sterling, and the exchange at Milan upon Paris is at the rate of 42 Austrian lire for 20 francs. How many Austrian lire should be paid at Milan for £20?

(22) A person on leaving England exchanged his English money for French at the rate of 25 francs for a sovereign, and on arriving at Munich received 135 Bavarian gulden for 300 francs; what was his loss (i) in English money, (ii) in French money, supposing a gulden to be worth 1*s.* 8½*d.*?

(23) What rate of exchange is established between London and Paris by bills upon Amsterdam bought in London at 12·2¼, the proceeds being forwarded from Amsterdam to Paris in direct bills at 48 florins per 100 francs?

(24) If a London merchant has to remit to Paris at 25·63, and to draw upon Lisbon at 56, at what rate may he draw upon Lisbon if the Paris rate falls to 25·52?

(25) A merchant in Paris owes another in Vienna 6670 Austrian florins; what will he gain in English money if, instead of paying direct, he pays through a London broker; the exchange between London and Paris being 25 francs for £1, between London and Vienna 11·60 florins for £1, and between Paris and Vienna 100 florins for 216 francs?

WEIGHTS AND MEASURES OF CERTAIN FOREIGN COUNTRIES.

258. As has already been stated (Art. 134) the metric system of weights and measures has been adopted not only in France and Switzerland, but in many other countries.

In Italy the names are Italianised, but are easy to distinguish, and Spanish terms are used in Spain.

Germany avoids French nomenclature as far as possible; thus the millimetre is called *strich*; the centimetre, *neuzoll*;

the metre, *stab*; the decametre, *kette*; the litre, *kanne*; the half-litre, *schoppen*; the hectolitre, *fass*; the decagramme, *neuloth*; the half-kilogramme, *pfund*. The *centner*, equal to 50 kilogrammes (110 lb.), and the *tonne*, equal to 1000 kilogrammes, are often used.

Austria uses the metric system with the German names.

In Belgium too there is some difference in the names; thus the metre is called *aune* or *el*, the litre, *litron* or *kan*, the kilogramme, *livre* or *pond*.

Holland retains the old Dutch names; thus the gramme is called *wigtje*, the metre *el*, the litre *kan*, the kilogramme *pond*.

Greece uses different names in some cases; thus the gramme is called *drachme*, the decigramme *obolos*, the centigramme *kokkos*, and $1\frac{1}{2}$ kilogrammes are called the *mna*; the metre is named *pecheus*, and the litre, *litra*.

259. In Russia the unit of measure is practically the British foot; the unit of weight, called a *pood*, is divided into 40 "pounds," and is equivalent to 36 lb., British.

Denmark has a system of weights and measures very similar to that of Great Britain.

260. In the United States of America and in the West Indies the weights and measures in common use are those of the United Kingdom, with the old wine gallon and the Winchester bushel. A hundredweight, however, is also called a *quintal*, and is equal to 100 lb. The old wine gallon = ·8331 imp. gallon, and the Winchester bushel = ·9694 imp. bushel.

261. In China the chief measure of weight is the *picul* ($133\frac{1}{3}$ lb. avoir.), which is also used at Singapore, Penang and Hong Kong. Silk goods are sold by weight.

Piece goods are measured by the *chang*; 12 chang = 47 English yards.

In Japan the weights and measures are the same as in China, but with different names.

The weights and measures used in India and Ceylon have been mentioned in Arts. 152–155.

EXAMINATION PAPERS.

(1) If 30 tons of coal at 20*s.* 6*d.* per ton be mixed with $22\frac{1}{2}$ tons at 24*s.* per ton, find the cost of 18 tons 6 cwt. of the mixture.

(2) Multiply 3273·5620976 by 5·3698527 to 5 places *only*, by the short method.

(3) In 1891 the population of a town was 5 per cent. less than in 1881. In 1891 the population was 11628; what was it in 1881?

(4) What would it cost to carpet a room, 17 ft. $4\frac{1}{2}'$ by 18 ft., with carpet 2 ft. 3′ wide at 3*s.* $11\frac{1}{2}d.$ per yard?

(5) Make out an invoice for the following:—

10 gall. 3 qts. oil at 1*s.* 2*d.* per gall.; 10000 boxes matches at 3*d.* per doz.; 64 cakes blacking at $11\frac{1}{4}d.$ per doz.; $5\frac{1}{2}$ dozen lb. candles at $8\frac{1}{2}d.$ per lb.; $\frac{3}{4}$ cwt. soap at 3*s.* 3*d.* per stone (14 lb.); 500 nails at 3*s.* 4*d.* per thousand; $2\frac{1}{2}$ cwt. lead pipe at $4\frac{1}{2}d.$ per lb.; 130 yards wire fencing at 8*d.* per ft.; $7\frac{1}{2}$ gross screws at $3\frac{1}{2}d.$ per doz.; 51 gimlets at 5*s.* 9*d.* per doz.

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(6) Find the cost of 2 tons 11 cwt. 60 lb. of sugar at £1. 10*s.* 11*d.* per cwt., and find what profit per lb. will be obtained by selling it at 8 lb. for 2*s.* $6\frac{1}{2}d.$

(7) Find the simple interest on £168. 15*s.* for 6 years at $1\frac{2}{3}$ per cent. per annum.

(8) A 9-inch wall is to be 27 ft. 9′ 4″ long and 6 ft. 2′ 3″ high. The dimensions of a brick being 9′ by $4\frac{1}{2}'$ by $2\frac{1}{2}'$, how many bricks will be required to build the wall?

(9) Divide ·00729 by ·2735 correctly to 5 places; and obtain as shortly as you can the product of 125 and 3458697.

(10) Make out an invoice for the following:—

90 boxes matches at $4\frac{1}{2}d.$ per score; 7 gall. 2 qts. oil at 1*s.* 9*d.* per gall.; $2\frac{1}{2}$ cwt. putty at $4\frac{1}{2}d.$ per lb.; $5\frac{1}{2}$ dozen lb. candles at $8\frac{1}{2}d.$ per lb.; 9 gross wick at $2\frac{1}{4}d.$ per dozen; 6 dozen knives at 11*s.* 6*d.* per dozen; 4 sets of carvers at 29*s.* 6*d.* per set; 3 dozen plated forks at 1*s.* 11*d.* each; 2 dozen spoons at $6\frac{1}{2}d.$ each; 5 sets of fire-irons at 10*s.* 9*d.* per set.

(11) The cost of 5 cwt. 2 qrs. 14 lb. of coffee is £56. 13*s.* 9*d.* If the whole is sold at 1*s.* 11½*d.* per lb., what is the total gain or loss?

(12) How many yards of velvet trimming, $\frac{1}{16}$ yard wide, can be cut from 1½ yards of velvet which is $\frac{3}{4}$ yard wide? And find the value of the whole trimming at 2½*d.* per yard.

(13) Take 7 per cent. discount for cash off a bill for £33. 0*s.* 5*d.*

(14) A "rod of brickwork" has an area of a square rod and a thickness of a brick and a half. Find the number of "rods of brickwork" in a wall 297 ft. long, 6 ft. 10½′ high, and a brick thick.

(15) Make out an invoice for the following:—

½ doz. pairs socks at 2*s.* 6*d.* per pair; 52 collars at 6*s.* 3*d.* per doz.; 3 doz. pairs gloves at 2*s.* 3*d.* per pair; 3¼ doz. handkerchiefs at 4*d.* each; 102 scarfs at 7*s.* 9*d.* per doz.; 11½ yds. flannel at 1*s.* 5*d.* per yd.; 29 yds. edging at 2¼*d.* per yd.; 37 yds. ribbon at 1*s.* 0½*d.* per yd.; 4½ doz. yds. dress material at 4*s.* 11*d.* per yd.; 5 lb. wool at 3*d.* per oz.

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(16) If 152 sacks hold 65 qrs. 2⅓ bush. of corn, how many will be required for 18 qrs. 7¼ bush.?

(17) If a cubic yard of clay make 460 bricks, each 9 in. by 4½ in. by 2½ in., how much does clay contract in the baking?

(18) The populations of three towns in 1881 were 20325, 41304, and 6117; in 1891 they had increased respectively 9, 10, and 12 per cent. Find the average population of the three towns in 1891.

(19) Express 2 cwt. 1 qr. 7 lb. as the decimal of 1 ton, and reduce £293. 12*s.* 8½*d.* to the decimal of £1, correct to the $\frac{1}{1000}$th part of a farthing.

(20) Make out the following invoice, with names and dates:—

8 lb. moist sugar at £2. 2*s.* per cwt.; ¾ cwt. soap at 3*s.* 3*d* per stone (14 lb.); 5½ dozen lb. candles at 8½*d.* per lb.; 1¢ cwt. rice at 2½*d.* per lb.; 23¾ lb. tea at 2*s.* 6*d.* per lb.; 31 lb. rice at 3¼*d.* per lb.; 3½ lb. tobacco at 4*d.* per oz.; 5 stones salt at 4*s.* 8*d.* per cwt.; 3½ gall. vinegar at 5½*d.* per pint; 3 cwt. soap at 2¼*d.* per lb.

(21) If 1 ton 13 cwt. 1 qr. 9 lb. 5 oz. of merchandise cost £186. 13*s.* 3¾*d.*, what will 17 cwt. 7 oz. cost?

(22) The external length, width, and depth of an open box are 6 ft. 2 in., 3 ft. 8 in., and 2 ft. respectively, and it is made of wood 1 inch thick. Find the quantity used.

(23) A contractor contracts to finish a piece of work in 30 days, and immediately employs 15 men upon it; at the end of 24 days the work is only half done. How many additional men must he employ to fulfil the contract?

(24) Take 23 per cent. discount for cash off a bill for £15. 6*s.* 3*d.*, and 2½ per cent. off an account for £39. 15*s.*

(25) Make out a bill for the following:—
1 chest tea, 3½ cwt., at 1*s.* 11*d.* per lb.; 2 barrels sugar, each 3 cwt. 84 lb., at £3. 5*s.* per cwt.; 4 cheeses, each 56 lb., at 6½*d.* per lb.; 7 bars soap, each 3½ lb., at 2*d.* per lb.; 4 boxes starch, each 12 lb., at 5¾*d.* per lb.; 1 cwt. 10 lb soap at 2¾*d.* per lb.; 3 qrs. bacon at £3. 15*s.* per cwt.; 1 cwt. 2 qrs. rice at 3*d.* per lb.; 29½ lb. cheese at 11*d.* per lb.; 12 hams, 15 lb. each, at 10½*d.* per lb.

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(26) If 2 ro. 15 per. of land are worth £59. 7*s.* 6*d.*, what is the value of 8 ac. 17 per. of similar land?

(27) The rental of an estate is £1850, and the rates are taken on 80 per cent. of the rental; what is the amount of a rate at 9*d.* in the £?

(28) What will be the cost of decorating the walls of a room at 3*s.* 4*d.* per square yard, the length being 19 ft. 10¼′, the width 16 ft. 1¾′, and the height 20 ft. 6′?

(29) Assuming that if 66420666 be divided by 7358 the quotient is 9027, write down the quotient of
(i.) 6642·0666 ÷ 7358000;
(ii.) ·066420666 ÷ ·007358.

(30) Make out a bill for 2 doz. lb. sperm candles at 2*s.* 1*d.* per lb.; 5½ lb. paraffin candles at 1*s.* 2*d.*; 10 lb. tapers at 1*s.* 3½*d.*; 8½ lb. carriage candles at 10½*d.*; 7 doz. lb. composite

candles at 4*s.* 3*d.* per 6 lb.; 5½ doz. lb. candles at 8½*d.* per lb.; 8¼ lb. wax candles at 2*s.* 3*d.*; 7 galls. 2½ pints oil at 7*d.* per pint; 2 lamps at 13*s.* 9*d.* each.

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(31) The French metre = 39·37 inches, express 3600 metres in yards; and, assuming that £1 = 25 francs, express 24875 francs in English money.

(32) A steam-engine of 4¼ horse-power, working 51 days of 6 hours, consumes 25 tons of coal. How much coal will be consumed by a 17 horse-power engine working at the same work for 3 days of 8½ hours?

(33) The population of a certain town increased 8 per cent. from 1881 to 1891, and its population in the latter year was 29160; what was its population in 1881?

(34) Find, by Practice, the net cost of an iron gate weighing 8 cwt. 2 qrs. 14 lb. at £3. 14*s.* 6*d.* per cwt., after allowing 18*s.* 6*d.* per cwt. for the old gate, which weighs 6 cwt. 1 qr. 15 lb.

(35) Make out a bill for 27 yds. flannel at 1*s.* 4*d.*; 32 yds. calico at 5½*d.*; 3½ doz. pairs stockings at 28*s.* 4*d.* per doz.; 6 pairs gloves at 3*s.* 6*d.* per pair; 8 collars at 8*s.* 9*d.* per doz.; 300 buttons at 5½*d.* per score; 3¼ doz. collars at 8½*d.* each; 27 yds. lace at 3*s.* per doz. yds.; 14¼ yds. cloth at 4*s.* 6*d.* per yd.; 60 yds. calico at 7*s.* 6*d.* per doz. yds.

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(36) If $\frac{7}{8}$ of a ton of copper cost £109. 7*s.* 6*d.*, how much would $\frac{2}{3}$ of a ton cost?

(37) A path is 120 yds. long and 18 ft. wide. Which will be cheaper, and by how much; to cover it with asphalt 2 in. deep at 18*s.* per cubic yard, or with gravel 6 in. deep at 7*s.* per cubic yard?

(38) A book is published in France at 4·50 francs; how many copies can be bought for 562·50 francs, if discount for cash be allowed at the rate of 16$\frac{2}{3}$ per cent.?

(39) Find, to the nearest penny, the simple interest on £912. 6*s.* 8*d.* from 21st October, 1897, to 10th January, 1898, at 7½ per cent.

(40) Make out a bill for 4 stoves at 5 guineas each; 3 brass locks at 30*s.* per doz.; $2\frac{1}{2}$ doz. latches at $1\frac{3}{4}d.$ each; 2 cwt. putty at $2\frac{1}{2}d.$ per lb.; 3 gall. varnish at $5\frac{1}{2}d.$ per pint.; 12 bars soap, each 3 lb., at $3\frac{1}{2}d.$ per lb.; 164 lb. tea at 30*s.* per doz. lb.; 12 packets candles, each $6\frac{1}{2}$ lb., at $11\frac{1}{2}d.$ per lb.

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(41) Find the difference between the value of 63 cwt. 2 qrs. 24 lb. of sugar at a guinea per cwt., and that of 1 qr. 27 lb. 8 oz. of tobacco at £28 per cwt.

(42) The population of a town was 128000 in 1897; what will it be at the end of three years, if it increases at the rate of 5 per cent. each year?

(43) The cost of constructing a railway, 87 miles $7\frac{1}{2}$ furlongs long, was £2674709. 18*s.* $7\frac{1}{2}d.$ What was the average cost per mile?

(44) All round the floor of a room, which is 28 ft. long and 22 ft. wide, there is a border 2 ft. wide which is left uncarpeted. Find the cost of staining the border at 1*s.* $1\frac{1}{2}d.$ per sq. yard. Find also the number of yards of carpet, 27 inches wide, required for covering the rest of the floor, and the cost of this carpet at 3*s.* 9*d.* per yard.

(45) Make out a bill for 1500 envelopes at $8\frac{3}{4}d.$ per 100; $10\frac{1}{2}$ doz. pencils at $\frac{3}{4}d.$ each; $2\frac{1}{2}$ gross pen nibs at $2\frac{1}{2}d.$ per doz.; 12 reams foolscap at $5\frac{1}{2}d.$ per quire; 6 doz. slates at $4\frac{3}{4}d.$ each; 72 copybooks at 1*s.* $11\frac{1}{2}d.$ per doz.; 4 doz. slates at $4\frac{1}{2}d.$ each; 650 slate pencils at $6\frac{1}{2}d.$ per hundred; $2\frac{1}{2}$ gross exercise books at 1*s.* 6*d.* per doz.

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(46) If a merchant buys 500 bags of jute, each containing 400 lb., at £12. 17*s.* 6*d.* per ton, and sells again at £14. 5*s.* per ton, paying on this latter transaction 1 per cent. for brokerage, what is his net gain, to the nearest penny?

(47) What is the amount of wages due to five servants:—*A* for two months at £25 per annum, *B* for four months at £22, *C* for five months at £20, *D* for 1 month at £18, *E* for 1 month at £13?

(48) For what sum should a ship worth £2000 be insured, so that in the event of her total loss within a year, her value, together with the premium of 15 per cent., may be recovered?

(49) The inhabitants of four towns numbering 6000, 4250, 3500 and 1250, five years ago, have since increased 5, 10, 15, and 20 per cent. respectively. Find (i) the present average population of the four towns, and (ii) the average increase per cent.

(50) Make out the following bill:—

750 nails at 2*s.* 7*d.* per 1000; 1½ cwt. lead pipe at 4½*d.* per lb.; 2¾ gross screws at 4¾*d.* per doz.; 32 yds. wire fencing at 5½*d.* per ft.; 35 pairs hinges at 2½*d.* per pair; 69500 bricks at 18*s.* 9*d.* per 1000; 486 tiles at 1*s.* 5*d.* per doz.; 47 loads of sand at 2*s.* 6*d.* per load; 39 ft. wood at 4½*d.* per ft.; 4 men, 8 hours a day for 5½ days, at 7½*d.* per hour.

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(51) Write down, by inspection, the equivalents of £16·25, £1·025, £7·16, £2034·875 in the common form; and express £2. 12*s.* 6*d.*, £134. 10*s.*, 7*s.* 6*d.*, £1. 1*s.* as decimals of a pound.

(52) Five tenders were sent in for building an institution, viz., £14372. 14*s.* 4*d.*, £13589. 12*s.* 8*d.*, £11876. 13*s.* 10*d.*, £15346. 15*s.* 4*d.*, £12610. 0*s.* 6*d.*; by how much per cent. did the first exceed the average of the five tenders, and the last fall short of it?

(53) A man's premium of Life Assurance, at £2. 2*s.* 8*d.* per cent., amounts to £53. 6*s.* 8*d.*; for what sum is he insured? Why is the charge per cent. for Life Assurance different for men of different ages, and why different also frequently for men engaged in different occupations?

(54) Gas in London costs 2*s.* 4*d.* per 1000 cubic feet; if it costs half as much again in Paris, find the price (in French money) per cubic metre, assuming that 1 metre = 39⅓ inches, and £1 = 25 francs.

(55) Make out and receipt the following invoice:—

9½ yds. flannel at 1*s.* 5½*d.*; 26 yds. calico at 2*s.* 1¾*d.*; 15 yds. velvet at 9*s.* 11*d.*; 23 yds. muslin at 3*s.* 7*d.*; 18 yds. linen at 2*s.* 9¼*d.*; 15 yds. ribbon at 11½*d.*; half-a-dozen pairs gloves at 2*s.* 3½*d.*; 48 pairs blankets at 12*s.* 6¾*d.*; 3000 yds. calico at 4*s.* 5*d.* per doz. yds.; 936 reels cotton at 15*s.* per gross. [What abbreviating processes can be made use of in the above calculations?]

(56) A tradesman who is selling off makes a reduction of 10 per cent. from the marked prices. At what price should he mark an article for which he is to receive £1. 11*s.* 6*d.*?

(57) How many francs and centimes should be obtained for £1. 10*s.*, when one franc is worth 9·386*d.*? Express 787·4 miles in kilometres and metres. [1 kilometre = 39370 inches.]

(58) The price of diamonds per carat varies as the number of carats. If a diamond of 3 carats be worth £75, what is the value of a diamond of 2 carats?

(59) A railway incline rises 1 ft. in 80 in the first $\frac{1}{4}$ mile, then 1 in 120 for the next 1500 yds., and 1 in 110 for the next $\frac{1}{2}$ mile. Find the average rise.

(60) Make out and receipt the following invoice :—

300 oranges at 10$\frac{1}{2}$*d.* per dozen; 1250 apples at 5 for 3*d.*; 2500 eggs at 9*d.* for 10 eggs; 16 boxes of figs, each 5 lb. 4 oz., at 7*d.* per lb.; 3$\frac{1}{2}$ sacks of potatoes, each 3 bushels, at 4$\frac{1}{2}$*d.* per gall.; 5 gall. strawberries at 5$\frac{1}{2}$*d.* per qt.; 3 score lb. potatoes at 5 lb. for 3$\frac{1}{2}$*d.*; 2$\frac{1}{4}$ lb. tomatoes at 11*d.* per lb.; 2$\frac{3}{4}$ pk. peas at 1*s.* 5*d.* per pk.; 75 eggs at 9*d.* per dozen.

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(61) A block of gold-ore has a volume of 1 cubic metre; the ore is 2·75 times as heavy, bulk for bulk, as water, and contains 2 oz. of gold to the ton. Find the weight of the block, also the value of the gold it contains. [1 cubic centimetre of water weighs ·0022 lb., and 1 oz. of gold is worth £3. 13*s.* 6*d.*]

(62) Explain the following extract from a daily paper:—"The following were the principal rates of exchange on London: Paris, 25·16$\frac{1}{2}$; Amsterdam, 12·11$\frac{1}{2}$; Antwerp, 25·18 to 25·22; Berlin (short), 20·39$\frac{1}{2}$."

(63) A hexagonal room, 15 foot side and 13 feet high, with one door 8 ft. by 3 ft., one fireplace 5 ft. by 4 ft., and two French windows, each 7 ft. by 3 ft., has to be papered with paper at 4*s.* 6*d.* per piece of 12 yards by 21 inches, also with a frieze 18 in. wide at 4$\frac{1}{2}$*d.* per yard. What should be a tradesman's tender for the work, to the nearest penny, allowing himself a profit of 10 per cent., the workman's rate of wages being 3*d.* per piece of paper and $\frac{1}{2}$*d.* per yard of frieze?

(64) What is meant by the following terms:—consols, preference stock, debenture stock?

A certain amount of 3 per cent. stock was bought at $96\frac{1}{8}$, and afterwards sold at the same price, the proceeds being re-invested in $3\frac{1}{2}$ per cents. at $102\frac{3}{8}$. The broker who transacts the business makes £30. 2*s*. Find the original amount the stockholder possessed, and the change in his income. (Brokerage, $\frac{1}{8}$ per cent. on each of the three transactions).

(65) A merchant has two kinds of tea, the first costing him 2*s*. 8*d*. per lb., the second 3*s*. 6*d*. per lb. In what proportion should he mix them so as to gain 20 per cent. by selling the blend at 3*s*. 9*d*. per lb.?

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(66) A person invested £10000 in railway 5 per cent. stock at 104; and the remainder of his capital in consols, which yielded $2\frac{5}{8}$ per cent. on the actual money invested. Finding his income to be only £800 a year, he transferred all to the railway stock. What did his income then become, to the nearest penny?

(67) What do you understand by the expression "par of exchange"?

Explain the following paragraph:—"India Council bills for 96538 rupees were sold by the Bank of England to-day, all being on Bombay at 1*s*. $1\frac{3}{32}d$. per rupee."

(68) What sum of money will amount to £2400 in 4 years at 5 per cent. per annum, compound interest?

(69) A tradesman began business on the 4th of March with a capital of £900; on the 12th of September he was joined by a partner who put £500 into the business; on the 31st of December the profits amounted to £204. 5*s*. Find the share of each partner.

(70) If the pure gold in an American double eagle be worth 81*s*. 6*d*., while that in a Greek 40-drachma piece be worth 28*s*. $1\frac{1}{2}d$., how many drachmai might be coined in Greece from 10000 double eagles, assuming the loss of gold in working to be ·5 per cent., and the cost of the process equal to three times the value of the gold lost?

(71) Three persons, *A*, *B*, *C*, possess shares in a business concern. *A* receives 10 per cent. of the profits; *B*, £100 and 3 per cent. of the profits; *C*, 3 per cent. on the money he invested. It is found that *A* receives twice as much as *B*, and that *B* receives three times as much as *C*. What did the profits amount to, and what sum of money did *C* invest?

(72) A block of copper 4 feet long, 3 feet broad, and 4 inches thick, is pierced by four rectangular holes, each 6 inches by 3 inches in section. What is the total surface? What weight of gold leaf, $\frac{1}{1000}$ of an inch thick, would completely cover it? [1 cubic foot of gold weighs 1150 lb.]

(73) Find, to the nearest penny, the value of 563·25 oz. of gold, 896 fine (*i.e.* 896 of 1000 parts in weight are pure gold), at 77*s.* 10½*d.* per oz. standard. [12 oz. of standard gold contain 11 oz. of pure gold.]

(74) A youth puts £30 in the Post Office Savings Bank on his 17th birthday, and the amount accumulates at 2½ per cent. per annum, compound interest. Find, to the nearest penny, the amount on his 21st birthday.

(75) A man's rates and taxes are based on his house rent, and amount to £15. 5*s.* per annum. The poor-rate is 10*d.* in the £, water-rate 6 per cent. of the rent, and the remaining taxes amount to two-thirds of the whole. What is his gross income if he allows himself one-eighth of his net income for rent, after paying income-tax at 8*d.* in the £?

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(76) What is meant by the true present worth of a bill? The present worth of a bill due 18 months hence is £1120, the rate of interest being 3 per cent. per annum. What is the face value of the bill?

(77) If silver is worth 2*s.* 6*d.* per ounce, what weight in grams, etc., would be worth 11·325 francs, assuming £1 to be equivalent to 25·11 francs, and 1 kilog. to be equivalent to 2·2 lb.?

(78) A man possessing £7200 Canadian 4 per cent. stock sells out at 108, and invests in Argentine 5 per cent. stock at 64. Find the change in his income, and the broker's charges for effecting the change. (Brokerage ⅛ per cent.)

(79) If two cubic yards of gravel be required to gravel a path 30 yards long and 2 yards wide, how much will be required to cover (to double the depth) a path, 3 feet wide, round a circular bed of 20 feet diameter?

What will be the cost, assuming the price of gravel to be 9*s.* per load of 1 cubic yard, and the cost of laying, rolling, etc., to be 1*d.* per square yard of path?

[Area of a circle $= \frac{22}{7}$ times the square on its radius.]

(80) A bankrupt, whose assets are £4566. 4*s.* and liabilities £13872, owes sums of £3200, £2070, and £1168 to his three principal creditors. What can the bankrupt pay in the £, and how much will the creditors severally receive?

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(81) What is the difference between true discount and banker's discount? Does the latter favour the holder of the bill, or the banker who negotiates it? Explain the expression, "Due at . . . days' sight."

What is the difference between the true and banker's discount on a bill of £1350 due 6 months hence, the rate of interest being 4 per cent. per annum?

(82) A company is formed to acquire a theatre. The share capital is £35000, divided into 10000 six per cent. preference shares of £1 each, and 25000 ordinary shares of £1 each. The preference shares, in addition, participate equally with the ordinary shares in all surplus profits remaining after a dividend of 10 per cent. has been paid on the ordinary shares. The average weekly takings are £50 per performance (6 performances weekly and 50 weeks to the year); income from letting offices, etc., £200; the profits on refreshment bars, etc., £800; the profits on advertisements, £250. The working expenses for the year were £10900; the sinking fund contribution, £942. What dividend might be declared on the ordinary shares?

(83) Explain the following quotation from a London paper:—"The Paris cheque exchange was 25·11. Vienna, 11·95. Silver remained unchanged at $27\frac{1}{2}d.$ The Calcutta exchange was $1s.\ 2\frac{19}{32}d.$"

(84) At the beginning of a certain year the population of a town was 16400; the deaths during the year were 3 per cent. of the population, and the births 3·75 per cent. What was the population at the end of the year, neglecting changes caused by removals?

(85) Having invested a sum of money in $2\frac{3}{4}$ per cent. consols at 113, I held the stock until I had received one year's dividend. I then sold out at $111\frac{1}{8}$, investing the proceeds (including the dividend) in London and South-Western 4 per cents. at 135. My new income was £455. What money did I invest originally? (Brokerage $\frac{1}{8}$ per cent.)

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(86) If 6 per cent. be gained by selling 680 lb. of tea for £119, at what rate per lb. should it be sold so as to gain 18 per cent.? Give your result true to the nearest farthing.

(87) Three persons, *A*, *B*, *C*, enter into partnership with sums of £1800, £1500, and £1400 respectively. *A* withdraws £600 at the end of 3 months, the deficit in the capital being made up by *B* subscribing £300 and *C* £300. The profits at the end of a year's business amount to £840. How should they be divided?

(88) In what proportion should a merchant mix two kinds of tea, costing him respectively 2*s.* 6*d.* and 3*s.* 6*d.* per lb., so as to realize a profit of 20 per cent. by selling the mixture at 3*s.* 9*d.* per lb.?

(89) On January 1st the population of a town was 18500: the death-rate was 20 per 1000, and the birth-rate 26 per 1000; 120 people moved into the town and 46 left it during the year. Find the percentage of increase at the year's end.

(90) A merchant in London buys goods in Paris, the price of which is a million francs. How will he probably pay for them, and what will be their final cost, allowing 10 per cent. for duty and cost of transit? [Exchange in London on Paris 25·25.]

(91) The capital of a railway company consists of £500000 preference stock, paying 4 per cent., and £700000 ordinary stock. The working expenses amount to £25000 a month, and the receipts average £1000 daily. What dividend can be declared on the ordinary stock after £24000 is added to a reserve fund? What would be the net income of the holder of £8500 ordinary stock (income-tax being 8*d.* in the £)?

(92) The cost of workmanship of a metal amounts to 25 per cent. of its actual value in the unworked state, the cost of distribution amounts to 10 per cent. of the cost of the manufactured goods, while the retailer realizes a further profit of 15 per cent. What is the selling price of goods made from £80 worth of metal?

(93) Find the cost of flooring and decorating a room 100 feet long, 80 feet broad, and 20 feet high, the floor being parquetted with pieces, 10 inches by 3 inches, costing 2*s.* 6*d.* per hundred; and the walls and ceiling being painted at a cost of 1*s.* 6*d.* per square yard. There are six windows, 10 feet by 5 feet, and two doors, 8 feet high and 4 feet broad.

(94) A person having £170000 invests it in 3 per cent. stock at $101\frac{7}{8}$; the value of the stock then falls to $99\frac{1}{8}$, whereupon he sells out and invests the proceeds in 6 per cent. stock at $147\frac{7}{8}$. Find the change in his income. (Brokerage $\frac{1}{8}$ per cent.)

(95) A man's liabilities amount to £23450; his assets, including two bills for £5000 and £3000 (due respectively 4 months and 8 months hence), amount to £12270. Supposing the bills can be discounted by a banker at 5 per cent. per annum, how much can the bankrupt pay in the £, and how much, to the nearest farthing, will a creditor recover on a debt of £6500?

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(96) If 3 cwt. 3 qrs. 12 lb. of zinc can be bought for £7. 4*s.*, at what price must 2 tons 8 cwt. 2 qrs. be sold to yield 20 per cent. profit?

(97) If the value of silver fall from 52*d.* to 34*d.* per oz., find the additional profit in the coinage of 20000 shillings, the weight of a shilling being 87·27 grains, $\frac{37}{40}$ fine.

(98) Find the equivalent value in Russian roubles of

1000 shillings, if 100 roubles weigh $5\frac{1}{16}$ Russian pounds, and each pound contains 409·5 grammes. [The silver rouble is ·868 fine, and the shilling weighs 5·65 grammes.]

(99) If I pay £135 an acre for a field, consisting of 30 acres 3 roods 25 perches, which yields, after deducting all charges, £216 a year, what rate of interest do I get for my capital outlay?

(100) The half-yearly profit of a railway company is apportioned as follows:—£102000 to pay interest on preference stock at the rate of 8 per cent. per annum, £45000 to pay interest on debenture stock at the rate of 6 per cent., and £15000 to pay a dividend at the rate of $1\frac{1}{4}$ per cent. on the ordinary stock. What is the whole amount of the capital?

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(101) What amount of $2\frac{3}{4}$ per cent. stock should be given in exchange for £100 of 3 per cent. stock, so that the exchange may be effected without loss of income?

(102) The capital of a company is £5672500, and the profit for the half-year is £197565. The dividend is to be reckoned to the nearest eighth per cent., which will leave a balance. What dividend for the half-year can be paid, and what will the balance be?

(103) Exchange £1256. 14*s*. 10*d*. into Russian silver roubles at 96·74.

(104) A cottage cost £243. 15*s*. 9*d*. What must be the weekly rent so that, after allowing 10 per cent. of the gross receipts for repairs, the investment may yield 5 per cent. per annum?

(105) Which is the cheaper, and by how much per annum: an article costing 12*s*. which has to be replaced every 7 months, or one costing 15*s*. and lasting 9 months?

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(106) The gross income of an estate is £5520. What is the net income after paying income-tax at 6*d*. in the £, and rates to the amount of $18\frac{3}{4}$ per cent. on the gross income?

(107) A London merchant wishes to pay a debt of 12800 francs to a Paris merchant when the course of exchange is quoted at 25·40 in London and 25·45 in Paris; how much does the former merchant gain if, instead of remitting, he be drawn upon by the latter?

(108) A blended tea is made up of three kinds worth 2*s.* 2*d.*, 1*s.* 10*d.*, and 1*s.* 6*d.* per lb. respectively, in the proportions 7, 11, 7. It is sold at 2*s.* $3\frac{1}{2}$*d.* per lb. What is the gain per cent.?

(109) Having a bill for £10000 payable 19 months hence, a man wishes to discount the bill and invest the proceeds in $2\frac{3}{4}$ per cent. consols at $95\frac{1}{8}$ (brokerage $\frac{1}{8}$), so that the income from the consols may not be less than £275 per annum. What is the greatest rate of trade discount that he may accept?

(110) The receipts of a railway company were apportioned as follows:—41 per cent. for working expenses, 56 per cent. to pay the shareholders a dividend of $3\frac{1}{2}$ per cent., and the remainder, £15000, to be set aside as a reserve fund. What is the capital of the company?

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(111) A publisher sells books to a retail dealer at 5*s.* a copy, but allows 25 copies to count as 24. If the retailer sells each of the 25 copies at 6*s.* 9*d.*, what profit per cent. does he make?

(112) A bankrupt's assets are £1472. 7*s.* 6*d.*, and his debts amount to £7634. 5*s.* If the expenses of the bankruptcy amount to £200, what dividend will the creditors receive?

(113) A bill for £1000 at 90 days' sight has been presented for acceptance on February 10th, 1888, and on March 10th is discounted at 5 per cent. per annum; how much will the discounter pay for the bill?

(114) A merchant in London having purchased goods in New York, the price of which is 3475 dollars, wishes to pay for them; explain some usual method of doing so. Find what the goods will cost the merchant, exchange in London on New York being $4\cdot83\frac{3}{4}$.

(115) A man purchases £1400 stock in 3 per cent. West Australia inscribed stock at $94\frac{1}{2}$, and also invests £3150 in the purchase of Russian inscribed 5 per cent. loan at $94\frac{1}{4}$; how much stock has he standing in his name?

If he sells the West Australian at $95\frac{1}{8}$, and the Russian

at $96\frac{1}{2}$, what does he gain or lose by the transaction? (Brokerage on West Australian $\frac{1}{8}$, on Russian $\frac{1}{4}$.)

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(116) By the conversion of a stock from 3 per cent. to $2\frac{3}{4}$ per cent., a person's income is diminished by £27 per annum. What cash would he obtain by selling out at $97\frac{1}{8}$, brokerage being neglected?

(117) A man expects to let a house for £80 per annum, and calculates that, after deducting an annual sum for repairs, etc., he will get $7\frac{1}{2}$ per cent. on the purchase money. He lets the house for £76 per annum, spends yearly £5 more than he intended on repairs, and gets 6 per cent. on the purchase money. What did he give for the house?

(118) Sugar passes through the hands of three dealers, each of whom makes 10 per cent. profit on his outlay; and is sold by the last of the three at $2\frac{3}{4}d.$ per lb. Find, to the nearest penny, the original price of the sugar per cwt.

(119) A publican buys beer at 10*d.* per gallon. To every 2 gallons of beer he adds a pint of water, and he sells the mixture at 2*d.* per pint. What is his gain per cent.?

(120) After deducting life insurance premium of £100, and paying income-tax at 8*d.* in the £ on the remainder, a man's net income is £870. What is his gross income?

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(121) In 1871 the census showed that the population of a certain town had increased 5 per cent. on that of 1861. In 1881 the increase was 10 per cent. on the population of 1871, and in 1891 it was 15 per cent. on that of 1881. What was the increase per cent. in the 30 years from 1861 to 1891?

(122) A seller reduced the price of oranges per score by 2*d.*, and in consequence received only 85 per cent. of what he had been receiving before the reduction; what does he now get for four cases containing 150 oranges each?

(123) The books of a company with a capital of £3300000 show the following facts bearing on a year's operations:— Working expenses were 47 per cent. of the gross receipts, and a sum of £4979. 3*s*. 4*d*. was paid for interest on loans. What was left after these payments was subject to income-tax at the rate of 6*d*. in the £, and the remainder was distributed among the shareholders, except £4900, which was carried to the reserve fund? If the gross receipts were £306250, at what rate was the dividend paid?

(124) An estate has been let for £480 a year, less landlord's taxes amounting to 15*d*. in the pound; if the rent is reduced to £450 a year, and taxes rise to 16*d*. in the £, by how much per cent. will the landlord's net receipts be reduced?

(125) A person buys $2000 United States 6 per cent. bonds at 86¾; find what they cost him, brokerage being ¼ per cent., and the nominal par of exchange 4*s*. 6*d*. per dollar. Find also his income, the course of exchange being 4*s*. 0½*d*. per dollar.

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(126) A tradesman, who allows 1½*d*. in the shilling discount on all goods sold, makes 10 per cent. on his capital; at how much per cent. above the cost price does he mark the articles for sale?

(127) A man possesses £22400 of 2¾ per cent. consols, and sells them at 99⅜. He then invests one-half of the proceeds in 4 per cent. railway debenture stock at 132½, lends £7266. 13*s*. 4*d*. on mortgage at 4½ per cent., and loses the rest. What alteration is produced in his annual income?

(128) James Wilson and Thomas Jackson enter into partnership on January 1st, 1898, to trade as Wilson, Jackson & Co. with a capital of £8000; Wilson contributing £6000, and his partner the remainder. Interest on capital is to be reckoned at 5 per cent. per annum, and the former is to take two-thirds of profits or losses, the latter one-third. At the close of the year the balance-sheet shows a profit of £1125. 15*s*. Render the account of each partner with the firm, not reckoning interest on the amounts drawn out for personal expenses, which were £480 by Mr. Wilson and £330 by Mr. Jackson.

(129) A wine merchant buys 10000 litres of wine at 2 francs a litre, and immediately sells it for cash at 2*s.* 6*d.* per quart. Find the amount of his gross profit, assuming that 8 litres are equal to 7 quarts, and that a franc is equal to $9\frac{1}{2}d.$

(130) What is the price per lb., to the nearest farthing in English money, of Brittany butter, which costs 3 francs per kilogramme? [£1 = 25·17 francs, and 1 kilo. = 2·204 lb.]

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(131) A company makes in the year a profit of £75118. 17*s.* 6*d.* on a capital of £872325; find, to the nearest quarter per cent., the highest dividend that can be paid, and how much will be carried forward.

(132) A bookseller is allowed by a publisher 20 per cent. discount on the nominal price, and 5 per cent. on the reduced price for cash; he is charged also for 13 copies as 12. The bookseller allows his cash customer 3*d.* in the shilling off the nominal price. What is the bookseller's profit per cent., to the nearest tenth, on cash sales?

(133) Brown sells his goods 20 per cent. cheaper than Black Brothers, and 20 per cent. dearer than White & Grey. How much would a customer of Black Brothers benefit by taking £500 worth of goods (i) from Brown, and (ii) from White & Grey?

(134) A person, holding £14350 Natal $4\frac{1}{2}$ per cent. stock, sells out at 105, and with the proceeds purchases Exchequer bills, bearing interest at the rate of 8*s.* 9*d.* per cent. monthly, when the bills are at a premium of £2. 10*s.* per cent. Find the alteration in his income.

(135) Wheat is quoted in London at £2. 15*s.* per quarter, and in Antwerp at 26 francs per hectolitre; freight to London is 5*s.* per quarter, and to Antwerp 2 francs per hectolitre; and exchange with London is $1\frac{1}{2}$ per cent., and with Antwerp $2\frac{1}{2}$ per cent. Determine to which of these ports a corn merchant should consign his wheat, and what he will thereby gain per quarter [£1 = 25·26 francs, 1 quarter = 2·908 hectolitres.]

(136) A shopkeeper marks his goods with a price from which he can deduct 15 per cent. for ready money, and still have $10\frac{1}{2}$ per cent. profit. What is the marked price of an article which costs him £3. 19*s*. 2*d*. ?

(137) A dealer was making a profit of 25 per cent. by selling whisky at 15*s*. per gallon. Owing to a rise in duty on spirits he paid 6*d*. a gallon more than before for similar whisky, and sold it at 15*s*. 6*d*. per gallon. What was the profit per cent. after the change ?

(138) A merchant buys 700 quarters of wheat. He sells 280 quarters at $7\frac{1}{2}$ per cent. profit, 320 at 10 per cent. profit, and the rest at $12\frac{1}{2}$ per cent. profit. His whole gain amounts to £100. 19*s*. 7*d*. At what price per quarter did he buy ?

(139) The capital of a company consists of £1100000 in $4\frac{1}{2}$ per cent. debenture stock, the same amount in 6 per cent. preference shares, and the same amount also in ordinary shares. If the net annual profits are £209000, calculate the amount per cent. available for dividend on the ordinary shares after payment of the debenture and preference claims.

(140) A person intending to purchase enough $2\frac{1}{2}$ per cent. stock to yield an income of £579. 15*s*. per annum when the stock is at 95, delays till the price has risen to $95\frac{3}{4}$. How much more will he then have to pay ?

ANSWERS.

REVISED EDITION.

I. Page 2.

A. 397596.	B. 152145.	C. 138707.	D. 261968.	E. 189002.
F. 587607.	G. 341123.	H. 306801.	I. 312046.	J. 332543.
1. 144014.	2. 140848.	3. 190528.	4. 307907.	5. 195427.
6. 204622.	7. 108641.	8. 256823.	9. 109520.	10. 110073.
11. 159895.	12. 193116.	13. 120532.	14. 160976.	15. 146506.
16. 224943.	17. 70723.	18. 174414.	Gross Total 3019538.	

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1. 428934.	2. 1643799.	3. 7594.	4. 5487445.
5. 909299.	6. 1969129.	7. 685755.	8. 1414337.
9. 3493718.	10. 1227223.	11. 5858777.	12. 83389.
13. 9089.	14. 28131.	15. 22787.	16. 32016.
17. 48889.	18. 7749.	19. 38588.	20. 38889.

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1. 35449589000.	2. 184106130000.	3. 296740525000.
4. 19535382828.	5. 22249874400.	6. 24488552242.
7. 57119863500.	8. 31881845600.	9. 63357201600.
10. 356099254000.	11. 375674574240.	12. 486404256000.

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1. 6915555.	2. 27212724.	3. 822724452.	4. 6573632571.
5. 51289138.	6. 484725094.	7. 3487415420.	8. 4746767440.
9. 3253155393.	10. 3664270.	11. 14609925.	12. 106836625.
13. 230938750.	14. 2404453125.	15. 131966000.	16. 40996494.
17. 55654140.	18. 77202073.	19. 96490058.	20. 86838210.
21. 109492752.	22. 97636032.	23. 124822404.	24. 169336664

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1. 24127632.	2. 17819543.	3. 3597705.	4. 18866175.
5. 15468441.	6. 22618432.	7. 21286356.	8. 40155786.
9. 61817620S.	10. 531916875.	11. 193625747.	12. 65720808.
13. 331824675.	14. 307736000.	15. 4806273384.	

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1. 17394083928.	2. 29720058996.	3. 12861344175.
4. 38346523875.	5. 57783767936.	6. 24685316040.
7. 6070595256.	8. 22307329341.	9. 23985264576.
10. 12018873792.	11. 57218121072.	12. 18680854125.

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1. 441. **2.** 1681. **3.** 1089. **4.** 2809. **5.** 2209.
6. 3136. **7.** 5476. **8.** 7921. **9.** 4624. **10.** 7225.

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1. $2^2 . 3 . 5$. **2.** $2 . 3 . 13$. **3.** $2 . 3 . 17$. **4.** $2 . 3 . 5 . 7$. **5.** $2^2 . 3^2 . 7$.
6. $3^2 . 5 . 7$. **7.** $3 . 5^2 . 7$. **8.** $2^2 . 3^2 . 17$. **9.** $5 . 11 . 13$. **10.** $2 . 7 . 61$.
11. 11^3. **12.** $2^4 . 7 . 13$. **13.** $3 . 5^2 . 7^2$. **14.** $2^2 . 3 . 5 . 7 . 11$.
15. $2 . 3 . 5^3 . 7$. **16.** $3^2 . 5^2 . 7^2$.

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1. 4288596; rem. 6. **2.** 2073283; rem. 13. **3.** 1014960; rem. 47.
4. 231889; rem. 279. **5.** 65439; rem. 216. **6.** 174991; rem. 66.
7. 158746; rem. 101. **8.** 104840; rem. 177. **9.** 61484; rem. 359.
10. 12347794; rem. 2. **11.** 549877; rem. 12. **12.** 75786; rem. 178.
13. 2929873; rem. 12. **14.** 307406; rem. 238. **15.** 27572; rem. 1634.
16. 493911; rem. 103. **17.** 24693; rem. 572. **18.** 5879; rem. 2246.

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4. 647975. **5.** 433517. **6.** 711885. **7.** 620382.
8. 297289; rem. 498. **9.** 371162; rem. 705. **10.** 515929; rem. 538.
11. 336703; rem. 680. **12.** 740732; rem. 640. **13.** 184525; rem. 256.
14. 455795; rem. 585. **15.** 636903; rem. 354. **16.** 401897; rem. 1187.
17. 120725; rem. 981. **18.** 222943; rem. 1736. **19.** 103249; rem. 875.
20. 24158; rem. 1918.

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4. 8; 80; 800. **5.** 41·9; 419; 4190. **6.** 157·9; 1579; 15790.
7. 8·7; 87; 870. **8.** 1·5; 15; 150. **9.** 10·5; 105; 1050.
10. ·4; 4; 40. **11.** ·9; 9; 90. **12.** 53·45; 534·5; 5345.
13. 6·28; 62·8; 628. **14.** ·15; 1·5; 15. **15.** ·05; ·5; 5.
16. 5·283; 52·83; 528·3. **17.** 20·06; 200·6; 2006.
18. 1·579; 15·79; 157·9. **19.** 140·03; 1400·3; 14003.
20. ·528; 5·28; 52·8. **21.** ·005; ·05; ·5. **22.** ·05781; ·5781; 5·781.
23. ·00057; ·0057; ·057. **24.** ·00001; ·0001; ·001.

XII. Page 21.

1. 10·46; 1·046; ·1046. **2.** 16·77; 1·677; ·1677.
3. 15·79; 1·579; ·1579. **4.** 36·11; 3·611; ·3611.
5. 1·579; ·1579; ·01579. **6.** 1·40003; ·140003; ·0140003.
7. 3·70007; ·370007; ·0370007. **8.** 2·803205; ·2803205; ·02803205.
9. 1·705706; ·1705706; ·01705706. **10.** 4·376201; ·4376201; ·04376201.
11. 3·00012; ·300012; ·0300012. **12.** 8·300156; ·8300156; ·08300156.
13. ·419; ·0419; ·00419. **14.** ·105; ·0105; ·00105.
15. ·5345; ·05345; ·005345. **16.** ·2006; ·02006; ·002006.
17. ·11003; ·011003; ·0011003. **18.** ·5372; ·05372; ·005372.
19. ·05; ·005; ·0005. **20.** ·087; ·0087; ·00087.
21. ·0628; ·00628; ·000628. **22.** ·001; ·0001; ·00004.
23. ·0015; ·00015; ·000015. **24.** ·0005; ·00005; ·000005.

XIII. Page 23.

1. 182·67. **2.** 458·96. **3.** 2·19. **4.** 26·69. **5.** 12·97.
6. 19·02. **7.** 160·92. **8.** 110·92. **9.** 43·23. **10.** 43·43.
11. 5·44. **12.** 94·21. **13.** 47·77. **14.** 305·18.

XIV. Page 24.

1. 11·9974. **2.** 41·9375. **3.** 5·943654. **4.** 83·338365745.
5. 4938·2602. **6.** 1840·01069. **7.** 12·3497. **8.** 12·3488.
9. 8954·09885. **10.** 133746·6292.

XV. Page 24.

1. ·54. **2.** 2·49. **3.** 2·12. **4.** ·90. **5.** 4·28. **6.** 10·85.
7. 6·44. **8.** 1·55. **9.** 11·88. **10.** 14·63. **11.** 31·08. **12.** 6·97.
13. 1·55. **14.** 12·97.

XVI. Page 28.

1. 18·077. **2.** 3·652. **3.** ·0022. **4.** ·0185. **5.** ·0566.
6. ·52806. **7.** 190042·18627. **8.** 26416·40239. **9.** 10·75188.
10. 27·02898. **11.** 985·428339. **12.** 24·327689. **13.** 4125·785.
14. 5·87723. **15.** 3·99142748. **16.** ·00188922. **17.** ·084.
18. ·37592. **19.** ·04. **20.** ·2821.

XVII. Page 31.

1. 2·78. **2.** 1·563. **3.** 1·190. **4.** 254·428. **5.** 2115·1922.
6. ·0733. **7.** ·0229. **8.** 129·73217. **9.** 726·361296.
10. 285119·0879907. **11.** ·013302. **12.** 1·527. **13.** 30·30.
14. ·4783. **15.** ·3358. **16.** ·0108.

XVIII. Page 32.

1. £7·204. **2.** £5·305. **3.** £2·407. **4.** £8·709. **5.** £9·810.
6. £1·911. **7.** £9·314. **8.** £4·519. **9.** £3·624. **10.** £2·181.
11. £6·284. **12.** £5·394. **13.** £4·891. **14.** £7·798. **15.** £3·682.
16. 3*s.* 1½*d.* **17.** 7*s.* 2¼*d.* **18.** 5*s.* 2*d.* **19.** 13*s.* 1½*d.* **20.** 15*s.* 0¾*d.*
21. 11*s.* 1¾*d.* **22.** £3. 7*s.* 4¼*d.* **23.** £2. 8*s.* 4*d.* **24.** £4. 13*s.* 5½*d.*
25. £8. 14*s.* 8½*d.* **26.** £6. 10*s.* 10½*d.* **27.** £2. 16*s.* 6¾*d.*
28. £2. 19*s.* 8*d.* **29.** £3. 13*s.* 3*d.* **30.** £5. 10*s.* 2½*d.*

XIX. Page 33.

Lines.	A. £	*s.*	*d.*	B. £	*s.*	*d.*	C. £	*s.*	*d.*
(15)	25601	9	8	30012	2	7	11105	14	3
(13)	27389	17	8	35819	2	1	19001	19	2
(10)	38207	10	2	42650	2	5	34398	13	4
(7)	38616	6	3	43799	17	9	50803	10	6

Lines.	D. £	*s.*	*d.*	E. £	*s.*	*d.*	F. £	*s.*	*d.*
(15)	8914	13	4	34557	19	0	19391	2	8
(13)	12507	10	9	41627	4	5	29056	4	2
(10)	29901	18	2	57283	6	7	39790	8	3
(7)	36366	16	10	57889	17	7	52193	18	6

XX. Page 34.

	£	s.	d.		£	s.	d.		£	s.	d.
A.	67	9	9	B.	73	4	4	C.	72	10	4
D.	391	12	5	E.	726	11	5	F.	4110	12	10
G.	6121	3	9	H.	56473	5	7	1.	7992	0	3
2.	3077	4	4	3.	4794	12	4	4.	5791	8	9
5.	4507	15	4	6.	4789	18	10	7.	4460	15	2
8.	6543	12	6	9.	7659	1	0	10.	5606	3	0
11.	8790	2	11	12.	4023	16	0				
								Gross Total	68036	10	5

XXI. Page 35.

1. £4. 4*s*. 10*d*. 2. £1. 12*s*. 7¾*d*. 3. £2. 0*s*. 5*d*. 4. £24. 17*s*. 9¾*d*.
5. £14. 18*s*. 4½*d*. 6. £57. 8*s*. 11¾*d*. 7. 18*s*. 11¼*d*. 8. £1. 18*s*. 9¼*d*.
9. £3. 18*s*. 10½*d*. 10. £12. 18*s*. 11¾*d*.

XXII. Page 36.

1. £343. 19*s*. 7¾*d*.; £355. 12*s*. 10¼*d*.; £518. 17*s*. 9½*d*.
2. £859. 8*s*. 0½*d*.; £2253. 0*s*. 6½*d*.; £1927. 16*s*. 11½*d*.
3. £4490. 8*s*. 2½*d*.; £4035. 13*s*. 8½*d*.; £4945. 2*s*. 8½*d*.
4. £1581. 16*s*. 1¾*d*.; £1613. 2*s*. 7¼*d*.; £1675. 15*s*. 6¼*d*.
5. £1923. 3*s*. 4½*d*.; £1993. 14*s*. 10½*d*.; £2099. 12*s*. 1½*d*.
6. £2080. 1*s*. 11¼*d*.; £2547. 10*s*. 8¼*d*.; £2968. 4*s*. 6¾*d*.
7. £6159. 4*s*. 0¾*d*.; £7120. 4*s*. 3¼*d*.; £8561. 14*s*. 7*d*.
8. £30046. 11*s*. 4½*d*.; £31806. 0*s*. 2¾*d*.; £39855. 3*s*. 2½*d*.
9. £18009. 14*s*. 2*d*.; £25213. 11*s*. 10*d*.; £49526. 13*s*. 11½*d*.
10. £75894. 16*s*. 1½*d*.; £113373. 14*s*. 5½*d*.; £70272. 19*s*. 4½*d*.
11. £481489. 2*s*. 6*d*.; £449389. 17*s*.; £513588. 8*s*.
12. £342274. 11*s*. 4*d*.; £825126. 3*s*. 9*d*.; £2292017. 3*s*. 9*d*.

XXIII. Page 37.

1. £1192. 8*s*. 1½*d*.; £687. 1*s*. 0¾*d*. 2. £148. 11*s*. 2½*d*.; £424. 9*s*. 2*d*.
3. £30. 11*s*. 2½*d*.; £25. 0*s*. 0¾*d*. 4. £191. 10*s*. 2½*d*.; £275. 15*s*. 6*d*.
5. £64. 18*s*. 5¾*d*. 6. £58. 7*s*. 3½*d*. 7. £23. 15*s*. 6¼*d*.
8. £148. 4*s*. 8½*d*. 9. £157. 3*s*. 4½*d*. 10. £175. 17*s*. 4½*d*.
11. £281. 5*s*. 7½*d*. 12. £1473. 2*s*. 9¾*d*.

XXIV. Page 41.

Arrangement of Aliquot Parts (Practice).

XXV A. Page 43.

1. £4486. 2. £5410. 10*s*. 3. £6645. 12*s*.
4. £2876. 13*s*. 4*d*. 5. £3961. 2*s*. 6*d*. 6. £5137. 4*s*.
7. £6354. 6*s*. 8*d*. 8. £4270. 18*s*. 8*d*. 9. £6955. 6*s*. 3*d*.
10. £3512. 11*s*. 11. £4444. 15*s*. 12. £9035. 8*s*. 8*d*.
13. £3706. 8*s*. 9*d*. 14. £4649. 15*s*. 6*d*. 15. £3650. 6*s*. 8*d*.
16. £1579. 7*s*. 8*d*. 17. £5299. 17*s*. 6*d*. 18. £4708. 8*s*. 0*d*.
19. £1220. 3*s*. 1½*d*. 20. £1382. 7*s*. 8*d*. 21. £1108. 0*s*. 7½*d*.
22. £1422. 1*s*. 23. £1545. 3*s*. 1½*d*. 24. £1739. 7*s*. 4½*d*.

XXV B. Page 44.

1. £30. 15*s.* 10*d.*
2. £53. 10*s.* 7½*d.*
3. £100. 16*s.* 6¾*d.*
4. £49. 9*s.* 4*d.*
5. £97. 0*s.* 9*d.*
6. £30. 3*s.* 10¼*d.*
7. £70. 17*s.* 11*d.*
8. £143. 4*s.* 9¾*d.*
9. £97. 19*s.* 11½*d.*
10. £75. 6*s.* 11⅝*d.*
11. £125. 3*s.* 8¼*d.*
12. £164. 10*s.* 11½*d.*
13. £345. 16*s.* 1⅝*d.*
14. £771. 11*s.* 6*d.*
15. £30. 0*s.* 7½*d.*
16. £73. 11*s.* 11*d.*
17. £231. 8*s.* 0⅞*d.*
18. £313. 10*s.* 8⅛*d.*
19. £206. 18*s.* 0$\frac{15}{32}$*d.*
20. £538. 13*s.* 3$\frac{9}{16}$*d.*
21. £342. 18*s.* 7⅜*d.*
22. £55. 13*s.* 5$\frac{3}{10}$*d.*
23. £101. 8*s.* 5$\frac{1}{10}$*d.*
24. £183. 1*s.* 5⅝*d.*
25. £114. 7*s.* 0$\frac{7}{8}$*d.*
26. £297. 12*s.* 1⅞*d.*
27. £470. 9*s.* 1$\frac{3}{32}$*d.*
28. £813. 4*s.* 9$\frac{1}{32}$*d.*
29. £149. 13*s.* 6$\frac{7}{10}$*d.*
30. £163. 18*s.* 4$\frac{1}{20}$*d.*

XXV C. Page 44.

1. £1377. 12*s.* 6*d.*
2. £1093. 10*s.*
3. £6775. 2*s.* 2½*d.*
4. £42385. 8*s.* 4*d.*
5. £45197. 17*s.* 9*d.*
6. £69339. 19*s.* 4*d.*
7. £119662. 6*s.* 2*d.*
8. £18789. 15*s.*
9. £41151.
10. £65128. 16*s.* 8*d.*
11. £113480. 17*s.*
12. £98573. 13*s.* 3½*d.*
13. £109802. 1*s.* 10*d.*
14. £99488. 19*s.* 6*d.*
15. £7721. 14*s.* 4½*d.*
16. £4011. 6*s.* 7*d.*
17. £8533. 8*s.* 9*d.*
18. £6928. 1*s.* 5*d.*
19. £96100. 13*s.* 10*d.*
20. £72386. 1*s.* 8*d.*

XXVI. Page 45.

1. £5934. 1*s.* 10½*d.*
2. £39478. 1*s.* 6*d.*
3. £4549. 19*s.* 6*d.*
4. £2648. 4*s.* 11*d.*
5. £780. 2*s.* 7½*d.*
6. £16072. 1*s.* 9*d.*
7. £69.
8. £2288. 19*s.* 8*d.*
9. £11. 16*s.* 3*d.*
10. £1197. 17*s.* 3*d.*

XXVII. Page 47.

1. £1. 12*s.* 10*d.*
2. £2. 1*s.* 1½*d.*
3. £3. 2*s.* 7*d.*
4. £33. 19*s.* 2*d.*
5. £279. 16*s.* 6*d.*
6. £48. 18*s.* 6½*d.*
7. £87. 0*s.* 9*d.*
8. £16. 15*s.* 6*d.*
9. £8. 12*s.* 5*d.*
10. £228. 13*s.* 6*d.*
11. £1. 6*s.* 9*d.*
12. £75. 10*s.* 4½*d.*
13. £71. 17*s.* 2*d.*
14. £1. 17*s.*
15. £16. 5*s.* 7*d.*
16. £412. 10*s.* 1*d.*
17. £2734. 15*s.* 3*d.*
18. £467. 1*s.* 7*d.*
19. £155. 15*s.*
20. £300. 15*s.* 10*d.*

XXVIII. Page 47.

1. £21. 18*s.* 10*d.*
2. £20. 6*s.* 2*d.*
3. £253. 0*s.* 4*d.*
4. £94. 2*s.* 8½*d.*
5. £26. 17*s.* 10½*d.*
6. £28. 18*s.* 6*d.*
7. £28. 4*s.* 8*d.*
8. £148. 16*s.* 8*d.*
9. £38. 16*s.* 3*d.*
10. £33. 0*s.* 4½*d.*
11. £22. 11*s.* 8*d.*
12. 9*s.* 8*d.*
13. £6. 14*s.* 5½*d.*
14. £21. 10*s.* 1*d.*
15. £3. 7*s.* 1*d.*
16. £19. 11*s.*
17. £16. 18*s.* 6*d.*
18. £29. 12*s.* 10*d.*
19. £24. 5*s.* 7½*d.*
20. £45. 17*s.* 10*d.*

XXIX. Page 49.

1. £910. 3*s.* 1½*d.*
2. £113. 4*s.* 7½*d.*
3. £180. 19*s.* 10¾*d.*
4. £5393. 1*s.* 11*d.*
5. £631. 10*s.*
6. £613. 2*s.* 8*d.*
7. £5479. 17*s.* 8¼ *l.*
8. £1980.
9. £4595. 8*s.* 4*d.*
10. £10011. 0*s.* 6¾*d.*

XXX. Page 50.

1. $\frac{1}{5^0}$. **2.** $\frac{3}{1^0}$. **3.** $\frac{1}{2^0}$. **4.** $\frac{2}{2^5}$. **5.** $\frac{1}{1^0}$. **6.** $\frac{1}{5}$. **7.** $\frac{1}{4}$.
8. $\frac{2}{5}$. **9.** $\frac{1}{2}$. **10.** $\frac{3}{4}$. **11.** $\frac{1}{4^0}$. **12.** $\frac{1}{8}$. **13.** $\frac{1}{8}$. **14.** $\frac{3}{8}$.
15. $\frac{1}{2^0}$. **16.** $\frac{1}{4^0}$. **17.** $\frac{1}{3^0}$. **18.** $\frac{1}{5^0}$. **19.** $\frac{3}{5^0}$. **20.** $\frac{3}{1^0}$.

XXXI. Page 50.

1. 50. **2.** 25. **3.** 20. **4.** 10. **5.** 5. **6.** 15. **7.** 4
8. 28. **9.** 2. **10.** 26. **11.** $9\frac{1}{11}$. **12.** $8\frac{1}{3}$. **13.** $3\frac{1}{3}$. **14.** $2\frac{1}{3}$.
15. $7\frac{1}{2}$. **16.** 60. **17.** 70. **18.** 75. **19.** 125. **20.** 101.

XXXII. Page 51.

1. £1; £2; £5; £3. 10*s.*; £7. 10*s.* **2.** £1. 10*s.*; £3; £4. 10*s.*; £6.
3. 5*s.*; £1; £1. 10*s.*; £2. 10*s.* **4.** £4; £8; £6.
5. £2; £5; £8; £2. 10*s.*; £17. 10*s.* **6.** £2; £10; £20; £30.
7. £57. 10*s.*; £93. 15*s.* **8.** £3. 14*s.* **9.** 3 oz. 12 drs.
10. 3 cwt. 3 qrs. 12 lb. 4 oz.

XXXIII. Page 51.

1. 50. **2.** 20. **3.** 60. **4.** $66\frac{2}{3}$. **5.** 5. **6.** $3\frac{3}{4}$.
7. $11\frac{1}{4}$. **8.** 3. **9.** $3\frac{1}{3}$. **10.** $\frac{5}{8}$. **11.** $\frac{3}{4}$. **12.** 30.

XXXIV. Page 52.

1. £1600. **2.** £2880. **3.** £2400. **4.** £26. **5.** £1040
6. £1500. **7.** £3532. 10*s.* **8.** £13750. **9.** £402. **10.** £36291. 13*s.* 4*d.*

XXXV. Page 52.

1. £1425. **2.** £1940. **3.** 7374. **4.** $33\frac{1}{3}$. **5.** $7\frac{1}{2}$. **6.** $14\frac{1}{4}$.
7. £90. **8.** 63 gal. **9.** £1240. **10.** 9740. **11.** 9600. **12.** $1\frac{35}{15}\frac{9}{8}\frac{9}{1}$.

XXXVI. Page 55.

1. £50. **2.** £12. 0*s.* 9*d.* **3.** £96. 5*s.* **4.** £202. 10*s.* **5.** £23.
6. £10526. 5*s.* **7.** £8650. **8.** £78. 12*s.* $9\frac{3}{4}d.$ **9.** £765.
10. £1600. **11.** £548. 2*s.* $11\frac{5}{8}d.$ **12.** £447. 6*s.* 8*d.*
13. £1362. 3*s.* $2\frac{3}{7}\frac{4}{1}d.$ **14.** (i.) £2. 6*s.* 9*d.*; (ii.) £4. 19*s.* $10\frac{1}{2}d.$;
(iii.) £9. 0*s.* $7\frac{1}{2}d.$ **15.** 1*s.* $0\frac{3}{4}\frac{2}{0}d.$ (correct discount £1. 4*s.* 5*d.*).

XXXVII A. Page 57.

1. 27. **2.** 22. **3.** [illegible] **4.** 196. **5.** 22·478. **6.** 12·36.
7. 23·9554. **8.** 187·04875. **9.** 228·3. **10.** 53·925.

XXXVII B. Page 57.

1. 21990. **2.** 185901. **3.** 63·6. **4.** 1·05. **5.** 691.
6. $8\frac{13}{35}$ years. **7.** 1·16. **8.** $8\frac{1}{4}$. **9.** $25\frac{3}{4}$. **10.** 11 st. 13 lb. 11 oz.
11. 1 ton. 2 qrs. **12.** $69\frac{1}{3}$ and 63. **13.** $3\frac{1}{2}\%$. **14.** 26719.

XXXVIII. Page 58.

1.

	1892.	1893.	1894.	1895.	1896.
(A)	5759	6974	7682	8323	8711
(B)	3323	3996	4620	4973	4966
(C)	59·12	57·75	60·45	60·36	57·71
(D)	53·51	55·88	59·15	57·87	54·93
(E)	57·70	57·29	60·14	59·75	57·00

2.

(i.) 52·20.
(ii.) 54·26.
(iii.) 49·125.
(iv.) 52·13.
(v.) 51·92.
(vi.) 51·43.
(vii.) 49·41.
(viii.) 48·80.
(ix.) 42·11.
(x.) 50·87.

3.

Year.	Total Population.	Increase.	
		Decennial.	Per Cent.
1801	8892536	—	—
1811	10164256	1271720	14·31
1821	12000236	1835980	18·06
1831	13896797	1896561	15·80
1841	15914148	2017351	14·52
1851	17927609	2013461	12·65
1861	20066224	2138615	11·93
1871	22712266	2646042	13·14
1881	25974439	3262173	14·41
1891	29002525	3028086	11·66

4.

Year.	Mileage.	Working Expenses.	Year.	Mileage.	Working Expenses.
	£.	Per Cent.		£.	Per Cent.
1880	3511	53	1889	3696	54
1881	3516	59	1890	3813	56
1882	3605	54	1891	3881	58
1883	3651	55	1892	3864	58
1884	3589	55	1893	3722	59
1885	3477	55	1894	3820	59
1886	3446	55	1895	3844	59
1887	3469	55	1896	4009	59
1888	3520	54			

	Companies.	Per Mile.	Per Cent. on Gross Receipts.	Net Receipts.
		£	£	£
5.	Cambrian...	1133	... 59 ...	117650
	Furness	3389	... 49 ...	229481
	Gt. Central	6694	... 55 ...	1143428
	Gt. Eastern	4319	... 57 ...	2073794
	Gt. Northern	5660	... 62 ...	1741370
	Gt. Western	3781	... 55 ...	4299688
	Lancashire and Yorkshire ...	9371	... 56 ...	2175814
	London and N. W.	6461	... 55 ...	5579468
	London and S. W.	4802	... 57 ...	1810969
	London, Brighton and S. Coast	6620	... 54 ...	1323610
	London, Chatham and Dover	8463	... 54 ...	735620
	London, Tilbury and Southend	3896	... 55 ...	137924
	Maryport and Carlisle ...	2477	... 50 ...	51177
	Metropolitan	12300	... 44 ...	455991
	Metropolitan District	24172	... 46 ...	247259
	Midland	6967	... 55 ...	4340878
	North Eastern	4756	... 57 ...	3327413
	North London	43203	... 51 ...	255026
	North Staffordshire	4178	... 53 ...	376439
	South Eastern	6046	... 52 ...	1198719
	Taff Vale...	6370	... 54 ...	354049
	Total of above railways ...	£5480	£56	£31975767
	Total miles in work	13120		
	Total gross receipts	£71904144		
	Total working expenses ...	£39928377		

XXXIX. Page 65.

	£	s.	d.
1.		8.	3
	1.	9.	2
		14.	9½
	2.	4.	0
	1.	2.	9
	5.	18.	11½

	£	s.	d.
2.	107.	16.	0
	30.	0.	0
		7.	10½
	15.	17.	4
	2.	0.	3
	156.	1.	5½

	£	s.	d.
3.		8.	7. 10½
	38.	9.	6
	254.	2.	0
	3.	6.	8
	1.	8.	1
	305.	14.	1½

	£	s.	d.
4.	9.	14.	9
	13.	6.	0
		3.	0
	1.	19.	0
		14.	9½
	25.	17.	6½

	£	s.	d.
5.		9.	2
	1.	3.	5½
	1.	6.	8
		6.	3
	1.	1.	0
	4.	6.	6½

	£	s.	d.
6.	1.	10.	11¼
	2.	14.	9
	29.	7.	9¾
	2.	2.	0
		18.	5¼
	36.	13.	11¼

7.

£	s.	d.
65	12	6
3	12	6
7	8	6
4	18	5¼
1	1	0
82	12	11¼

8.

£	s.	d.
2	3	9
	3	8
2	12	1
	4	9½
2	5	0
7	9	3½

9.

£	s.	d.
	16	8
	13	3¼
4	18	0
	10	8¼
	10	6
7	9	1½

10.

£	s.	d.
16	12	6¾
	7	2
3	3	0
	9	1¼
1	8	6
22	0	4

11.

£	s.	d.
1	0	0
	1	1½
5	18	9
	9	2
	14	8
8	3	8½

12.

£	s.	d.
33	15	0
37	16	0
6	6	0
9	15	0
2	2	0
89	14	0

13.

£	s.	d.
	14	3
1	10	0
3	4	8¼
15	9	9
41	12	6
62	11	2¼

14.

£	s.	d.
	16	9¼
13	6	0
1	13	6½
2	5	6
	8	1½
18	9	11¼

15.

£	s.	d.
	9	7½
		7½
	14	9¾
	9	9
1	1	10½
2	16	8¼

16.

£	s.	d.
	6	3
1	8	4¾
	11	3
	2	0
8	10	0
10	17	10¾

17.

£	s.	d.
	8	2
1	0	6½
	2	7¼
5	16	8
1	9	4
8	17	3¾

18.

£	s.	d.
2	5	0
2	0	3
4	11	0
3	4	1¼
	10	5
12	10	9¼

19.

£	s.	d.
3	18	9
3	2	6
1	3	9
	11	0
	11	11¾
9	7	11¾

20.

£	s.	d.
	8	10¼
	4	9¾
	3	2¼
	6	7¾
	5	5¼
1	8	11¼

21.

£	s.	d.
2	9	5 2/3
3	12	2 3/16
1	1	8¾
2	11	2¼
	10	5⅜
10	5	1 3/3

22.

£	s.	d.
24375	0	0
1987	10	0
8960	0	0
4620	0	0
8424	0	0
780	0	0
49146	10	0
2457	6	6
46689	3	6

23.

£	s.	d.
36	0	0
9	5	$7\frac{1}{2}$
2	17	9
10	13	9
24	7	6
83	4	$7\frac{1}{2}$
6	4	$10\frac{13}{80}$
76	19	$9\frac{27}{80}$

24.

£	s.	d.
525	0	0
300	0	0
418	15	0
218	15	0
18	0	0
34	10	0
1515	0	0
37	17	6
1477	2	6

25.

£	s.	d.
81	4	0
130	13	4
12	8	3
23	12	0
247	17	7
24	15	$9\frac{1}{10}$
223	1	$9\frac{9}{10}$

26.

£	s.	d.
1312	8	3
451	10	0
255	18	9
9	19	6
67	6	$6\frac{4}{5}$
7	7	3
125	0	0
2229	10	$3\frac{4}{5}$
55	14	$9\frac{19}{200}$
2173	15	$6\frac{141}{200}$

27.

£	s.	d.
1	13	$6\frac{1}{2}$
1	13	9
134	15	0
45	0	0
91	13	$10\frac{3}{4}$
11	8	$1\frac{1}{4}$
275	7	$10\frac{1}{2}$
6	17	$8\frac{29}{80}$
268	10	$2\frac{11}{80}$

28.

£	s.	d.
	12	$6\frac{1}{2}$
194	15	$7\frac{1}{2}$
12	6	6
8	5	$4\frac{17}{24}$
56	2	$4\frac{1}{2}$
272	2	$5\frac{5}{24}$
13	12	$1\frac{221}{480}$
258	10	$3\frac{353}{480}$

29.

£	s.	d.
16	18	3
35	2	0
12	15	0
76	10	0
19	0	0
160	5	3
12	0	$4\frac{23}{40}$
148	4	$10\frac{17}{40}$

30.

£	s.	d.
2	1	3
13	5	6
2	10	$9\frac{1}{2}$
5	3	$1\frac{1}{2}$
	9	9
5	6	6
1	1	$3\frac{3}{4}$
29	18	$2\frac{3}{4}$
14	11	$\frac{15}{32}$
29	3	$3\frac{9}{32}$

31.

£	s.	d.
4169	12	0
15	9	3
110	7	4
104	4	$9\frac{3}{5}$
3939	10	$7\frac{2}{5}$

32.

£	s.	d.
2450	0	0
3468	15	0
4700	0	0
2450	0	0
13068	15	0
326	14	$4\frac{1}{2}$
653	8	9
12088	11	$10\frac{1}{2}$

33.

£	s.	d.
300	0	0
80	0	0
240	0	0
440	0	0
1060	0	0
80	6	8
67	10	0
10	12	0
53	0	0
848	11	4

34.

£	s.	d.
28	10	0
54	0	0
31	10	0
43	0	0
157	0	0
18	0	0
7	17	0
131	3	0

35.

£	s.	d.
28	4	0
33	12	0
34	10	0
40	10	0
136	16	0
15	10	0
3	8	$4\frac{4}{5}$
117	17	$7\frac{1}{5}$

36.

£	s.	d.
92	10	0
115	2	9
138	18	$5\frac{1}{4}$
156	16	0
503	7	$2\frac{1}{4}$
1	5	$2\frac{3}{64}$
12	11	$8\frac{5}{32}$
489	10	$4\frac{5}{64}$

XL. Page 70.

1. £30. 17*s.* **2.** £42. 3*s.* **3.** £46. 17*s.* **4.** £37. 1*s.* **5.** £29. 6*s.*
6. £33. 12*s.* **7.** £237. 10*s.* **8.** £8. 15*s.* **9.** £16. 5*s.* **10.** £13. 10*s.*
11. £11. 10*s.* **12.** £23. 13*s.* **13.** £6. **14.** £30. **15.** £12.
16. £44. **17.** £36. **18.** £45. **19.** £28. 18*s.* 6*d.* **20.** £83. 13*s.* 6*d.*
21. £47. 1*s.* 9*d.* **22.** £31. 17*s.* 9*d.* **23.** £49. 4*s.* 9*d.* **24.** £88. 2*s.* 3*d.*
25. £159. 17*s.* 3*d.* **26.** £259. 4*s.* $1\frac{1}{2}d.$ **27.** £314. 7*s.* $1\frac{1}{2}d.$
28. £140. 14*s.* $4\frac{1}{2}d.$ **29.** £281. 3*s.* $10\frac{1}{2}d.$ **30.** £293. 1*s.* $7\frac{1}{2}d.$
31. £1. 13*s.* 4*d.* **32.** £2. 8*s.* **33.** £2. 10*s.* **34.** £1. 16*s.* **35.** £3. 6*s.*
36. £1. 16*s.*

XLI. Page 72.

1. £14. 14*s.* **2.** £26. 14*s.* **3.** £91. 17*s.* 6*d.* **4.** £221. **5.** £259. 4*s.*
6. £91. **7.** £32. 11*s.* **8.** £250. 2*s.* 6*d.* **9.** £114. 1*s.* 6*d.*
10. £21. 3*s.* $11\frac{1}{4}d.$ **11.** £154. 18*s.* 3*d.* **12.** £50. 12*s.* 9*d.*
13. £197. 0*s.* 2*d.* **14.** £220. 14*s.* 0·52*d.* **15.** £339. 18*s.* **16.** £621. 13*s.* 6*d.*
17. £897. 18*s.* $4\frac{1}{2}d.$ **18.** £371. 14*s.* $10\frac{1}{2}d.$ **19.** £105. 0*s.* $4\frac{1}{2}d.$
20. £248. 11*s.* 9*d.* **21.** £187. 11*s.* $10\frac{1}{2}d.$ **22.** £248. 19*s.* $2\frac{1}{2}d.$
23. £257. 5*s.* 3*f.* **24.** £897. 7*s.* $1\frac{1}{2}d.$ **25.** £22. 7*s.* 2*d.*
26. £80. 14*s.* 8*d.* **27.** £124. 6*s.* 3*d.* **28.** £288. 8*s.* **29.** £10. 5*s.* $1\frac{1}{4}d.$
30. £6. **31.** £123. 17*s.* $5\frac{1}{4}d.$ **32.** £46. 11*s.* $10\frac{1}{2}d.$ **33.** £82. 9*s.*
34. £1. 5*s.*

XLII. Page 75.

1. £48. 1*s.* 9·6*d.* **2.** £129. 12*s.* 9*d.* **3.** £6. 8*s.* **4.** £10. 18*s.* 2*d.*
5. £265. 12*s.* $11\frac{1}{4}d.$ **6.** £8. 2*s.* 11*d.* **7.** £11. 13*s.* $5\frac{1}{2}d.$ **8.** £83. 19*s.* 10*d.*
9. £107. 18*s.* $10\frac{3}{4}d.$ **10.** £171. 8*s.* $5\frac{1}{2}d.$ **11.** £39. 12*s.* $10\frac{3}{4}d.$
12. £140. 15*s.* 1*d.* **13.** £409. 8*s.* 5*d.* **14.** £886. 11*s.* 7*d.* **15.** £5. 9*s.* 4*d.*
16. £193. 0*s.* 6*d.* **17.** £10. 14*s.* 7*d.* **18.** £85. 2*s.* 2*d.* **19.** £319. 13*s.* 7*d.*
20. £61. 4*s.* 11*d.* **21.** £347 + £7. 13*s.* 3*d.* = £354. 13*s.* 3*d.*
22. £398 + £17. 10*s.* 3*d.* = £415. 10*s.* 3*d.*

XLIII. Page 76.

1. £350. **2.** £720. **3.** £175. **4.** £370. **5.** £275. **6.** £258. 17*s.* 2*d.*
7. £4328. 2*s.* 6*d.* **8.** £906. 13*s.* 4*d.* **9.** £6307. 15*s.* **10.** £3245. 10*s.*
11. £5768. 15*s.* **12.** £3562. 10*s.*

XLIV. Page 77.

1. 25 years. **2.** 7 years. **3.** 8 years. **4.** $3\frac{1}{2}$ years. **5.** 7 years. **6.** 3 years. **7.** 2 years. **8.** 5 years. **9.** $3\frac{1}{4}$ years. **10.** $4\frac{3}{4}$ years. **11.** $7\frac{1}{2}$ years. **12.** $7\frac{1}{2}$ years. **13.** $4\frac{1}{4}$ years. **14.** 8 years. **15.** 4 years. **16.** 5 years. **17.** $4\frac{3}{4}$ years. **18.** $4\frac{1}{2}$ years.

XLV. Page 78.

1. 9. **2.** $8\frac{1}{3}$. **3.** $3\frac{1}{3}$. **4.** $4\frac{1}{4}$. **5.** $3\frac{1}{2}$. **6.** $10\frac{1}{2}$. **7.** $3\frac{1}{4}$. **8.** $4\frac{1}{2}$. **9.** $9\frac{1}{2}$. **10.** 10. **11.** $2\frac{1}{2}$. **12.** $4\frac{1}{2}$. **13.** 5. **14.** $2\frac{1}{2}$. **15.** $5\frac{1}{4}$. **16.** $4\frac{1}{2}$. **17.** 4. **18.** $5\frac{5}{16}$.

XLVI. Page 80.

1. 6. **2.** 30. **3.** 42. **4.** 84. **5.** 105. **6.** 132. **7.** 231. **8.** 252. **9.** 315. **10.** 350. **11.** 385. **12.** 495. **13.** 693. **14.** 756. **15.** 825. **16.** 1575. **17.** 3528. **18.** 4312. **19.** 4725. **20.** 17325.

XLVII. Page 83.

1. 19. **2.** 23. **3.** 41. **4.** 59. **5.** 83. **6.** 93. **7.** 119. **8.** 149. **9.** 151. **10.** 253. **11.** 383. **12.** 476. **13.** 499. **14.** 503. **15.** 613. **16.** 739. **17.** 1123. **18.** 1059. **19.** 3852. **20.** 6304. **21.** 4701. **22.** 8027. **23.** 8513. **24.** 3254.

XLVIII. Page 86.

1. 1·4. **2.** 9·3. **3.** 1·19. **4.** 18·2. **5.** 3·47. **6.** 31·87. **7.** 13·05. **8.** 2·946. **9.** 10·043. **10.** 12·025. **11.** 90·909. **12.** 5·1034. **13.** ·3443. **14.** ·207. **15.** ·0031. **16.** ·0101. **17.** ·0219. **18.** ·00043. **19.** ·00148. **20.** ·00251. **21.** 1·4142.... **22.** 2·2360.... **23.** 3·4641.... **24.** 3·6055.... **25.** 1·2649.... **26.** ·6324.... **27.** ·8366.... **28.** ·0447....

XLIX. Page 87.

1. $\frac{11}{13}$. **2.** $\frac{16}{29}$. **3.** $\frac{7}{8}$. **4.** $\frac{15}{32}$. **5.** $1\frac{13}{126}$. **6.** $1\frac{1}{3}$. **7.** $6\frac{1}{4}$. **8.** $14\frac{1}{2}$. **9.** $6\frac{1}{4}$. **10.** $48\frac{1}{3}$. **11.** $8\frac{1}{3}$. **12.** $16\frac{3}{7}$. **13.** $2\frac{3}{17}$. **14.** $2\frac{7}{23}$. **15.** $3\frac{5}{29}$. **16.** $6\frac{3}{29}$.

L. Page 88.

1. 70. **2.** 440. **3.** 770. **4.** 8030. **5.** £35. 15*s*. **6.** £71.

LI. Page 91.

1. 170 sq. ft. 6 sq. in. **2.** 529 sq. ft. 4 sq. in. **3.** 190 sq. ft. 132 sq. in. **4.** 208 sq. ft. 48 sq. in. **5.** 293 sq. ft. 70 sq. in. **6.** 98 sq. ft. $15\frac{3}{4}$ sq. in. **7.** 87 ac. 1 ro. 26 per. **8.** 16 ac. 2 ro. 10 per. **9.** 4 ac. 1 ro. $28\frac{3}{4}$ per. **10.** 9 ac. 1 ro. 14 per. **11.** 23 ac. 1 ro. 30 per. **12.** 13 ac. 0 ro. 34 per.

LII. Page 91.

1. 12*s*. $4\frac{2}{3}d$. **2.** £67. 12*s*. $0\frac{3}{11}d$. **3.** 864; £233. 17*s*. 9*d*. **4.** 4000 turfs; £13. 10*s*. **5.** 10000; £208. 6*s*. 8*d*. **6.** 2916 sq. ft. **7.** £16. 8*s*. $1\frac{1}{2}d$. **8.** £1. 1*s*. $11\frac{1}{3}d$. **9.** (i.) 3527 sq. ft. 4 sq. in. (ii.) 1151 sq. ft. 36 sq. in. (iii.) £9. 11*s*. $10\frac{1}{2}d$. **10.** £1498. 7*s*. 6*d*. **11.** £2. 0*s*. $1\frac{1}{2}d$. + £1. 5*s*. $10\frac{1}{2}d$. = £3. 5*s*. $11\frac{3}{4}d$. **12.** 32.

LIII. Page 94.

1. 23⅓. **2.** 87. **3.** 70. **4.** 25. **5.** 43¾ yds.; £8. 4*s.* 0¾*d.*
6. 52$\frac{5}{7}$ yds.; £8. 18*s.* 9*d.* **7.** 40$\frac{1}{9}$; £9. 0*s.* 6*d.* **8.** 25½; £4. 13*s.* 6*d.*
9. £29. 11*s.* 11½*d.* (£27. 6*s.* + £2. 8*s.* 11½*d.*) **10.** £26. 17*s.* 6*d.*

LIV. Page 96.

1. 1260 sq. ft. **2.** 858 sq. ft. **3.** 800 sq. ft. **4.** 941 sq. ft. 60 in.
5. 876 sq. ft. 18 in. **6.** 504 sq. ft.

LV. Page 97.

1. £20. 2*s.* 6*d.* **2.** £7. 8*s.* 8¾*d.* **3.** Walls and doors, £5. 3*s.* 9*d.*; ceiling, £1. 6*s.* 3*d.*; total £6. 10*s.* **4.** £1. 2*s.* 11$\frac{7}{9}$*d.* **5.** £4. 4*s.* 8$\frac{3}{8}$*d.*
6. £9. 18*s.* 0$\frac{13}{16}$*d.* **7.** Walls, 16*s.* 1*d.*; ceilings, £1. 1*s.* 4*d.*; total, £1. 17*s.* 5*d.* **8.** 16. **9.** £47. 16*s.* 3*d.* **10.** £168. 12*s.*

LVI. Page 98.

1. 97$\frac{7}{9}$ yds. **2.** 216 yds. **3.** 115 yds.; £2. 3*s.* 1½*d.* **4.** £11. 4*s.* 6*d.*
5. £3. 10*s.* **6.** £1. 16*s.* 3*d.* **7.** £1. 5*s.* 3¾*d.* **8.** £3. 12*s.*
9. £4. 5*s.* 3*d.* **10.** £2. 8*s.* 9*d.*

LVII. Page 101.

1. 217 cu. ft. **2.** 58 cu. ft. 162 in. **3.** 139 cu. ft. 519 in.
4. 12 cu. ft. 1304 in. **5.** 35 cu. ft. 720 in. **6.** 283 cu. ft. 1176 in.
7. 2809 cu. ft. 198 in. **8.** 14629 cu. ft. 54 in.

LVIII. Page 101.

1. 12 ft. **2.** 1¼ in. **3.** 12½ cu. ft. **4.** 3229 cu. ft. 1044 in.
5. 15*s.* 2*d.* **6.** £1. 6*s.* 6*d.* **7.** 10 ft. 5 in.
8. 192 cu. ft.; 5 tons 7 cwt. 16 lb. **9.** £60. 0*s.* 6*d.*
10. £63. 7*s.* 11$\frac{35}{48}$*d.* **11.** 3 tons 4 cwt. 3 qrs. 4 lb. 13 oz.
12. 3 ft. 8 in. **13.** 125 gallons. **14.** £19. 8*s.* 11*d.*
15. 33$\frac{17}{36}$ lb. **16.** £2. 16*s.*

LIX. Page 105.

1. 2 ft. 6′. **2.** 25 ft. 10′ 6″. **3.** 14 ft. 5′ 3″. **4.** 10 ft. 4′ 4″.
5. 12 ft. 2′ 1″ 6‴. **6.** 15 sq. ft. 2′ 4″. **7.** 23 sq. ft. 3′ 11″.
8. 154 sq. ft. 9′ 9″ **9.** 108 sq. ft. 9′ 5″ 6‴. **10.** 121 sq. ft. 11′ 0″ 4‴.
11. 276 cu. ft. 1′ 3″ 3‴. **12.** 42 cu. ft. 2′ 3″ 10‴. **13.** 47 cu. ft. 6′ 2″ 4‴.
14. 39 cu. ft. 7′ 9″ 4‴ 6^{iv}. **15.** 156 cu. ft. 11′ 10″ 10‴ 4^{iv}.

LX. Page 107.

1. 72 sq. in. **2.** 20 sq. in. **3.** 28 sq. in. **4.** 170 sq. ft. 6 in.
5. 95 sq. ft. 100 in. **6.** 230 sq. ft. 90 in. **7.** 41 sq. ft. 142½ in.
8. 137 sq. ft. 136$\frac{1}{8}$ in. **9.** 244 sq. ft. 43$\frac{17}{18}$ in. **10.** 171 sq. ft. 126 in.
11. 344 sq. ft. 32$\frac{5}{9}$ in. **12.** 526 sq. ft. 107$\frac{2}{3}$ in.

LXI. Page 108.

1. 139 cu. ft. 519 in. **2.** 2809 cu. ft. 198 in. **3.** 15 cu. ft. $1182\frac{107}{144}$ in.
4. $98\frac{613}{1152}$ cu. ft. **5.** 92 cu. ft. $638\frac{7}{12}$ in. **6.** 170 cu. ft. $542\frac{7}{9}$ in.
7. 450 cu. ft. $766\frac{17}{24}$ in. **8.** 899 cu. ft. $206\frac{1}{48}$ in.
9. 1090 cu. ft. $1010\frac{25}{48}$ in. **10.** 939 cu. ft. $1255\frac{1}{2}$ in.

LXII. Page 108.

1. 12600 sq. ft. 0′ 1″ 4‴ 6^{iv}. **2.** 246 cu. ft. 162 in. **3.** 4 sq. ft. 54 in.
4. £4 12*s.* $5\frac{1}{4}d.$ **5.** £4. 7*s.* $6\frac{5}{8}d.$ **6.** £6 9*s.* 10*d.*
7. £6 11*s.* 3*d.* **8.** £3 11*s.* $11\frac{3}{64}d.$ **9.** £63 2*s.* $0\frac{1}{2}d.$
10. 541 cu. yds. 22 ft. 54 in.; £975 5*s.* $4\frac{1}{2}d.$

LXIII. Page 111.

1. $71\frac{863}{972}$ sq. yds. **2.** $142\frac{61}{1008}$ sq. yds. **3.** 6 rods $218\frac{35}{88}$ ft.
4. 6 rods $128\frac{5}{12}$ ft. **5.** 3 rods 222 ft. **6.** 31 rods 118 ft. **7.** 55296.
8. 30000. **9.** 2160. **10.** £10. 16*s.*

LXIV. Page 113.

£9. 3*s.* 4*d.*+£3. 1*s.* 6*d.*+£8. 2*s.* $8\frac{3}{5}d.$+£25. 18*s.* $2\frac{3}{8}d.$+£33. 0*s.* 0*d.*+£4. 14*s.* 6*d.*+£11. 0*s.* 0*d.*+16*s.* 3*d.*+£301. 7*s.* $9\frac{9}{17}d.$+£166. 18*s.* $7\frac{4}{17}d.$+£1. 16*s.* 0*d.*+£13. 17*s.* 0*d.*+£3. 18*s.* $4\frac{1}{2}d.$+£6. 15*s.* 0*d.*+£4. 13*s.* 0*d.*+£4. 10*s.* 0*d.*+£11. 14*s.* 0*d.*+£6. 13*s.* 6*d.*+£5. 4*s.* 3*d.*+£1. 10*s.* 0*d.*= £624. 14*s.* $0\frac{11}{96}d.$

LXV. Page 114.

£63. 6*s.* 9*d.*+£101. 1*s.* 3*d.*+£3. 0*s.* 0*d.*+£1. 10*s.* 0*d.*+£1. 14*s.* 8*d.*+£5. 4*s.* $1\frac{1}{2}d.$+£19. 18*s.* 3*d.*+£4. 16*s.* 0*d.*+£7. 6*s.* 8*d.*+£2. 11*s.* 0*d.*+£16. 7*s.* 3*d.*+£6. 7*s.* 6*d.*+£3. 12*s.* 0*d.*+£7. 0*s.* 0*d.*+£6. 15*s.* 0*d.*+£84. 1*s.* 6*d.*+£7. 4*s.* 11*d.*+£18. 11*s.* 3*d.*+£4. 8*s.* $10\frac{1}{2}d.$+£4. 4*s.* 0*d.*+£4. 13*s.* 9*d.*+£10. 12*s.* 6*d.*+£5. 8*s.* 0*d.*+£4. 10*s.* 0*d.*+£6. 0*s.* 0*d.*+£8. 2*s.* 0*d.*=£408. 7*s.* 3*d.*

LXVI. Page 115.

£65. 3*s.* $10\frac{1}{2}d.$+£1. 16*s.* 8*d.*+£9. 1*s.* 0*d.*+£9. 4*s.* $7\frac{1}{2}d.$+£5. 14*s.* $4\frac{1}{2}d.$+10*s.* 0*d.*=£91. 10*s.* $6\frac{1}{2}d.$

LXVII. Page 115.

£60. 2*s.* 8*d.*+£4. 16*s.* 0*d.*+£44. 13*s.* 0*d.*+£4. 15*s.* 0*d.*+15*s.* 0*d.*+£1. 7*s.* 0*d.*+£15. 13*s.* 6*d.*+10*s.* 0*d.*=£132. 12*s.* 2*d.*

LXVIII. Page 116.

£27. 2*s.* 8*d.*+£2. 0*s.* 0*d.*+£1. 4*s.* 0*d.*+18*s.* 0*d.*+£4. 10*s.* 0*d.*+£1. 10*s.* 0*d.*+£38. 4*s.* 0*d.*+£41. 15*s.* 6*d.*+£3. 0*s.* 0*d.*+£2. 5*s.* 0*d.*+£17. 0*s.* 0*d.*+£4. 10*s.* 0*d.*+£18. 8*s.* $5\frac{1}{4}d.$+£27. 0*s.* 0*d.*+15*s.* 0*d.*= £190. 2*s.* $7\frac{1}{4}d.$

LXIX. Page 122.

1. 10 decam.; 1400000 cm. **2.** 10440·539. **3.** 7·92 kilom. per hour.
4. 45 min. 41·2 sec. **5.** 4·5749279.... **6.** 36 hrs.
7. 4 hrs. 49 min. 51 sec. **8.** 330 m. 4 fur. 24·139776 poles. **9.** 633·6.
10. Difference 1·6 in. **11.** 4389·039. **12.** 297600000.

LXX. Page 123.

1. 100. **2.** 14·1804. **3.** 3380. **4.** 636. **5.** 70.
6. 2 hectares 25 ares. **7.** 320 m. **8.** 148000. **9.** 2·471 acres.
10. 929. **11.** 6·45148 m. **12.** 370·661157 acres. **13.** 1·09363 yds.
14. 7·7221.

LXXI. Page 124.

1. 1·26954576 cu. m. **2.** 7·68 cu. m.; 7680 litres.
3. 6·453 kilol.; 415 centilitres. **4.** 1871992 litres. **5.** 1·274216.
6. 1529·0746. **7.** 220000000 hectol.; 38720000000 pints.
8. 17·608; 16·0673.

LXXII. Page 124.

1. 30000 kilos. **2.** 145000 gr. **3.** 2700 kilos. **4.** 270 grams.
5. 68·15... kilos. **6.** 503·32.... **7.** 6562·5 litres; 5998·125 kilos.
8. 750 litres; 1380 kilos. **9.** 1·452 kilos. **10.** 4754·7808 kilos.
11. olive oil, 91·5 kilos; sea water, 102·6 kilos. **12.** 1024 kilos.
13. 882·86 kilos. **14.** ·0319... millim. **15.** 7·31....

LXXIII. Page 126.

1. 350·1 fr. **2.** 31·61 fr. **3.** 40·35 fr. **4.** 62·34 fr. **5.** 156·01 fr.
6. 340·01 fr. **7.** ·15 fr. **8.** ·57 fr. **9.** ·04 fr. **10.** ·03 fr.
11. ·01 fr. **12.** ·025 fr. **13.** 5631 c. **14.** 415 c. **15.** 382 c.
16. 509 c. **17.** 2503 c. **18.** 3600 c. **19.** 1842 fr. 25 c.
20. 1417 fr. 45 c. **21.** 492 fr. 28 c. **22.** 1279 fr. 35 c.
23. 3897 fr. 85 c. **24.** 685039 fr. 30 c. **25.** 156 fr. 75 c.
26. 76 fr. 82 c.

LXXIV. Page 128.

1. £20. 2*s.* 6*d.* **2.** £754. 12*s.* 6*d.* **3.** £500. 19*s.* 8*d.* **4.** 6211·70 francs.
5. 6500 francs. **6.** 8001·45 francs. **7.** £6. 6*s.* 1*d.* **8.** 483.
9. £43564. 7*s.* 1½*d.* **10.** 31 fr. 75 c.

LXXV. Page 128.

1. 282·51 fr. **2.** 505·05 fr. **3.** 600 fr. **4.** 54 fr. 84 c.
5. 141 fr. 31¼ c. **6.** 602·15 c. **7.** 5*s.* 4*d.* **8.** ·4125 fr. per litre.
9. 53748·99 fr. **10.** £2. 2*s.* 1*d.* **11.** £20. 12*s.* 9½*d.*
12. 6·66225 fr.; 5*s.* 4*d.* **13.** £91. 17*s.* 4*d.* **14.** £691. **15.** English.
16. 1½*d.* per kilom. in France; 2½*d.* per mile in England: Ratio= 31680 : 32809.

LXXVI. Page 132.

1. 404 as. **2.** 2572 p. **3.** 256. **4.** 92. **5.** Rs. 22472. 1 p.
6. Rs. 576. 2 as. **7.** Rs. 214. 15 as. **8.** Rs. 1904. 10 as.
9. Rs. 58467. 13 as. 10 p. **10.** Rs. 958. 15 as. 9 p.
11. Rs. 1001. 0 as. 11 p. **12.** Rs. 11. 3 as. 4 p.
13. Rs. 125. 15 as. 1 p. **14.** Rs. 511. 8 as. 10 p.
15. Rs. 251. 8 as. 3 p.; Rs. 586. 13 as. 11 p.; Rs. 922. 3 as. 7 p.
16. Rs. 481501. 4 as.; Rs. 449401. 2 as. 8 p.; Rs. 513601 5 as. 4 p.
17. Rs. 37. 12 as. 6 p.; Rs. 12. 9 as. 6 p.
18. Rs. 317. 6 as. 8 p.; Rs. 268. 9 as. 4 p.
19. £32. 5*s.* 10*d.* **20.** £145. 2*s.* 3*d.* **21.** £158. 6*s.* 8*d.*
22. Rs. 1905. 12 as. **23.** Rs. 2859. 12 as. **24.** Rs. 6090.

LXXVII. Page 133.

1. Rs. 1632. 0 as. 8 p. **2.** Rs. 8650. **3.** Rs. 23.
4. Rs. 202. 8 as. **5.** Rs. 85. 6 as. 3·12 p. **6.** Rs. 468.

LXXVIII. Page 134.

1. $22\frac{1}{2}$. **2.** 23. **3.** 20. **4.** $34\frac{8}{13}$. **5.** $29\frac{3}{8}$. **6.** $9\frac{1}{11}$.
7. 12*s.* 6*d.*; $19\frac{3}{13}$. **8.** $11\frac{1}{8}$. **9.** 25. **10.** $8\frac{1}{3}$. **11.** $17\frac{1}{2}\frac{3}{1}$.
12. 60 % gain. **13.** *A* 20; *B* $15\frac{1}{2}$. **14.** $40\frac{5}{8}$. **15.** $18\frac{1}{1}\frac{0}{5}\frac{2}{1}$.
16. $34\frac{6}{0}\frac{0}{4}\frac{2}{7}$. **17.** 60. **18.** 33. **19.** 125. **20.** 40. **21.** $44\frac{4}{9}$.
22. $63\frac{7}{11}$. **23.** $4\frac{1}{6}$ % gain. **24.** $16\frac{2}{3}$ % loss.

LXXIX. Page 137.

1. £510; £4760. **2.** 1*s.* $10\frac{1}{2}d.$ **3.** £58. 10*s.* **4.** 11*s.* 6*d.*
5. £11. 18*s.* $0\frac{1}{2}d.$ **6.** £41. 17*s.* **7.** £1. 6*s.* **8.** $7\frac{1}{2}d.$
9. £1191. 9*s.* **10.** £109. 2*s.* 6*d.* **11.** 6*d.* **12.** 18*s.* **13.** 1*s.* 3*d.*

LXXX. Page 139.

1. £5. 5*s.* **2.** £4. 10*s.* **3.** £4. 2*s.* 6*d.* **4.** £130. 10*s.*
5. £3. 16*s.* 8*d.* **6.** 18*s.* 9*d.* **7.** £11. 17*s.* 6*d.* **8.** £4725.
9. £988. 19*s.* 6*d.* **10.** $6\frac{3}{4}d.$ **11.** $10\frac{1}{2}$. **12.** $15\frac{9}{1}\frac{}{0}$. **13.** 10.
14. 25. **15.** $26\frac{6}{1}\frac{}{9}$. **16.** $37\frac{1}{2}$. **17.** 65.

LXXXI. Page 141.

1. £26. **2.** £10. 5*s.* $2\frac{1}{2}d.$ **3.** £1812. 10*s.* **4.** £120. **5.** £240.
6. £24. **7.** £446. 10*s.* **8.** 16*s.* $10\frac{1}{2}d.$ **9.** £1. 13*s.* 4*d.*
10. £3. 10*s.* **11.** £120. **12.** 8*s.* 4*d.* **13.** £5. **14.** £60.
15. 16*s.* 8*d.* **16.** £2. 6*s.* 3*d.* **17.** £1. 2*s.* 2*d.* **18.** £8. 6*s.* 8*d.*

LXXXIA. Page 142A.

1. £1 3*s.* 9*d.* **2.** 14*s.* 4*d.* **3.** $6\frac{2}{5}d.$ **4.** 1*s.* $7\frac{1}{2}d.$
5. £1 5*s.* 4*d.* **6.** 2*s.* $2\frac{1}{2}d.$ **7.** 19*s.* 2·86*d.*
8. At not less than $2\frac{11}{2}d.$ **9.** At not less than 7*d.* **10.** 7*s.* 10*d.*
11. 3*s.* 1*d.* **12.** 17*s.* 6*d.* **13.** $12\frac{5}{8}\frac{}{}d.$ **14.** 1. 9. 11·05*d.*
15. $8\frac{1}{3}$. **16.** $118\frac{4}{7}$. **17.** 1*s.* 3*d.* **18.** $9\frac{3}{2}\frac{0}{5}\frac{2}{4}d.$; $30\frac{7}{7}\frac{0}{1}\frac{1}{8}\frac{9}{3}$; $23\frac{1}{2}\frac{5}{3}\frac{2}{5}\frac{8}{2}$.

LXXXIB. Page 142C.

1. 3 : 1. **2.** 5 : 9. **3.** 1 : 2. **4.** 7 : 2. **5.** 1 : 3 : 1 : 2.
6. 3 : 7 : 2. **7.** 4 : 5 : 1. **8.** 1 . 3 : 4. **9.** 1 : 1 : 1 : 1.
10. 3 : 5 : 1 : 2. **11.** 10 gallons. **12.** 1 quart. **13.** $\frac{1}{2}$ pint.
14. 144. **15.** $11\frac{1}{1}\frac{}{7}$ dwt.

LXXXII. Page 143.

1. £70. 8*s.*; £85. 16*s.* **2.** £1000; £1500; £500.
3. £793; £549; £305. **4.** £329; £121. **5.** £350; £450.
6. £205. 7*s.* 6*d.*; £240. 19*s.* 4½*d.*; £329. 6*s.* 3*d.* **7.** £350; £210; £280.
8. £63; £218. 8*s.*; £138. 12*s.* **9.** £240; £135.
10. £240; £220; £200; £158. **11.** £245; £175; £210.
12. £295; £177. **13.** £599. 5*s.* 8⅓*d.*; £3920. 14*s.* 3⅜*d.*
14. £1228; £1542.

LXXXIII. Page 146.

1. 9*s.* 8½*d.* **2.** 5*s.* **3.** 8*s.* 5*d.* **4.** 4*s.* 9⅜*d.*; £252.
5. 6*s.* 3*d.*; £249. 10*s.* 7½*d.* **6.** 15*s.* 3*d.* **7.** No; 13*s.* 4*d.*
8. £675. 18*s.* 9*d.* **9.** £1871. 3*s.* 4*d.* **10.** £2933 6*s.* 8*d.*
11. £722. 13*s.* 4*d.* **12.** £5685. 19*s.* 8*d.* **13.** £864. 5*s.* 7½*d.*
14. £3. 17*s.* 11$\frac{5}{7}$. **15.** 13*s.* 4*d.*
16. 4*s.* 1⅔*d.*; £108. 1*s.* 3*d.*; £129. 13*s.* 6*d.*; £163. 12*s.* 9*d.* **17.** 15*s.*

LXXXIV. Page 149.

1. 1*s.* 0½*d.* **2.** 2*s.* 7½*d.* **3.** 3*d.* **4.** £879. 14*s.* 8¼*d.*
5. £12. 8*s.* 4*d.* **6.** £631. 10*s.* **7.** £5676. 13*s.* 4*d.* **8.** £87. 10*s.*
9. £70. **10.** 1*s.* 2*d.* **11.** £500. **12.** £502.

LXXXV. Page 158.

1. £165. **2.** £171. **3.** £28. **4.** £323. 5*s.* **5.** £344. 5*s.*
6. £205. 12*s.* 6*d.* **7.** £194. 10*s.* 3*d.* **8.** £32. 10*s.*
9. £108. 3*s.* 3*d.* **10.** £238. 14*s.* 5¼*d.*

LXXXVI. Page 159.

1. £5000. **2.** £2280. **3.** £4200. **4.** £3610. **5.** £14400.
6. £3725. **7.** £3600. **8.** £900. **9.** £1000. **10.** £1300.

LXXXVII. Page 160.

1. £1350. **2.** £4095. **3.** £5062. 10*s.* **4.** £2092. 10*s.*
5. £1710. **6.** £3273. 4*s.* **7.** £8308. 15*s.* **8.** £1045. 16*s.*
9. £6909. 18*s.* **10.** £6897. 16*s.* **11.** £2314. **12.** £1845.

LXXXVIII. Page 160.

1. £583. 6*s.* 8*d.* **2.** £49. **3.** £864. 10*s.* **4.** £126. **5.** £210.
6. £159. 12*s.* **7.** £140. **8.** £40. 6*s.* 8*d.* + £66. 13*s.* 4*d.* = £107.
9. £58. 6*s.* 8*d.* − £56. 5*s.* = £2. 1*s.* 8*d.*
10. £5. 19*s.* + £5. 8*s.* = £11. 7*s.* **11.** £500; £511. 13*s.* 4*d.*
12. 1st year's dividend, £864. 10*s.*; 2nd year's dividend, £893; 3rd year's dividend, £921. 4*s.*
13. £52. 10*s.* **14.** £41. 13*s.* 4*d.*

LXXXIX. Page 161.

1. The former. **2.** The latter. **3.** The latter. **4.** The latter.
5. The latter. **6.** The latter. **7.** Equal. **8.** The former.

XC. Page 162.

1. $3\frac{9}{17}$. **2.** $3\frac{4}{7}$ **3.** $3\frac{11}{13}$. **4.** $2\frac{7}{9}$. **5.** $3\frac{1}{3}$. **6.** $4\frac{13}{37}$. **7.** $3\frac{1}{5}$

XCI. Page 163.

1. 90. **2.** $91\frac{3}{7}$. **3.** 80. **4.** $72\frac{2}{3}$. **5.** $86\frac{1}{4}$. **6.** $97\frac{11}{12}$.

XCII. Page 164.

1. £107. 5*s*. **2.** £135. **3.** £15 loss.
4. £130. **5.** £75 decrease. **6.** £5 loss.

XCIII. Page 165.

1. 612. **2.** £141. 5*s*. **3.** £9858. **4.** £2000.
5. £1680. **6.** £25000.

XCIV. Page 166.

1. £15. 10*s*. **2.** £19 increase. **3.** £48. 15*s*. increase. **4.** £20.
5. £576; £680. **6.** £1250; £2. 10*s*. decrease. **7.** £285. 4*s*. increase.
8. No change. **9.** £20 increase. **10.** £27. 4*s*.
11. £1. 12*s*. increase. **12.** £31. 5*s*. **13.** £18 increase.
14. £105. 8*s*. $5\frac{63}{143}d$. increase.

XCV. Page 167.

1. £4262. 10*s*. **2.** £8201. 8*s*. $1\frac{1}{2}d$. **3.** £2410. 15*s*.
4. £324. **5.** £31833. 6*s*. 8*d*. **6.** $4\frac{1}{8}$. **7.** £127. 10*s*.
8. No change. £96 from each investment.
9. £502. 10*s*. $3\frac{3}{199}d$.; £15. 1*s*. $6\frac{148}{199}d$. **10.** £148. **11.** 86.
12. £14520. **13.** 4%. **14.** £18. 11*s*. 3*d*. increase.
15. £27. 10*s*. increase. **16.** £201. 12*s*. **17.** $3\frac{43}{119}$; £655. 9*s*. $2\frac{110}{119}d$.
18. £4500.

XCVI. Page 177.

1. Charles Field, Jan. 7, 1899. **2.** Alfred Allen, Nov. 5, 1898.
3. John Walker, Aug. 25, 1896. **4.** Robert Smith, Nov. 6, 1898.
5. Charles Field, Jan. 15, 1899. **6.** John Walker, Nov. 13, 1898.

XCVII. Page 179.

1. £168. 16*s*. 8*d*.; £8. 8*s*. 10*d*. **2.** £5600; £147.
3. £216. 17*s*. 6*d*.; £8. 13*s*. 6*d*. **4.** £4000. 8*s*. 4*d*.; £120. 0*s*. 3*d*.
5. £1820.; £22. 15*s*. **6.** £356. 5*s*.; £21. 7*s*. 6*d*.
7. £524. 8*s*. $10\frac{3}{5}d$.; £26. 4*s*. $5\frac{1}{3}d$. **8.** £241. 10*s*. $0\frac{8}{11}d$.; £7. 10*s*. $11\frac{3}{11}d$.
9. £200. 15*s*.; 9*s*. $7\frac{1}{2}d$. **10.** £73. 17*s*. 4·12*d*.; £1. 2*s*. 9·21*d*.

XCVIII. Page 180.

1. £25. 10*s.* **2.** £8. 11*s.* 2·79*d.* **3.** £2. 1*s.* 4·44*d.* **4.** £6. 0*s.* 1·54*d.*
5. £13. 4*s.* 9·37*d.* **6.** £1893. 7*s.* 2*d.* **7.** £6. 19*s.* 9·87*d.*
8. £6. 18*s.* 2·47*d.* **9.** £12. 1*s.* 3·88*d.* **10.** £24. 0*s.* 5·17*d.*
11. £1. 2*s.* 9·93*d.* **12.** £18. 19*s.* 6·68*d.*

XCIX. Page 184.

1. Period=2·866 months. **2.** Period=4·833 months.
3. Period=3·75 months. **4.** Period=2·844 months.
5. Period=15·818 months. **6.** Period=4·239 months.
7. Period, 294·2 days; Date, 23rd March, 1895.
8. Period, 314·1 days; Date, 13th Dec., 1893.
9. Period, 389·2 days; Date, 26th Jan., 1898.

C. Page 186.

1. £1428. 10*s.* 2·22*d.* **2.** £689. 12*s.* 3·3*d.* **3.** £1069. 10*s.* 0·18*d.*
4. £966. 11*s.* 2·49*d.* **5.** £343. 7*s.* 11·52*d.* **6.** £2523. 4*s.* 1·55*d.*
7. £3978. 3*s.* 6·19*d.* **8.** £3727. 18*s.* 8·84*d.* **9.** £1071. 6*s.* 5·66*d.*
10. £476. 11*s.* 2·95*d.*

CI. Page 187.

1. £1333. 0*s.* 5*d.*; £76. 10*s.* 5*d.* **2.** £7868. 6*s.* 3*d.*; £663. 1*s.* 3*d.*
3. £3567. 5*s.* 10*d.*; £302. 14*s.* 3*d.* **4.** £3664. 9*s.* 1*d.*; £406. 15*s.* 4*d.*
5. £2853. 14*s.* 11*d.*; £279. 16*s.* 7*d.* **6.** £5794. 10*s.* 7*d.*; £530. 2*s.* 9*d.*
7. £3174. 16*s.* 4*d.*; £205. 0*s.* 1*d.* **8.** £1810. 13*s.* 2*d.*; £223. 19*s.* 8*d.*
9. £2892. 5*s.* 9*d.*; £206. 6*s.* 4*d.* **10.** £2067. 8*s.* 3*d.*; £170. 13*s.* 1*d.*
11. £1853. 11*s.* 7*d.*; £175. 1*s.* 8*d.* **12.** £4089. 7*s.* 3*d.*; £399. 19*s.* 0*d.*
13. £2554. 0*s.* 5*d.*; £239. 3*s.* 3*d.* **14.** £7853. 7*s.* 11*d.*; £652. 6*s.* 2*d.*
15. £3989. 17*s.* 1*d.*; £361. 9*s.* 8*d.*

CII. Page 188.

1. £338. 7*s.* 6*d.* **2.** £520. 10*s.* 3*d.* **3.** £718. 16*s.* 11*d*
4. £1299. 17*s.* 8*d.* **5.** £2788. 15*s.* 3*d.*

CIII. Page 190.

(In these answers a fraction of a penny is reckoned as a whole penny.)

1. £90. 14*s.* 1*d.* **2.** £340. 2*s.* 9*d.* **3.** £399. 9*s.* 3*d.*
4. £460. 14*s.* 6*d.* **5.** £656. 0*s.* 3*d.* **6.** £351. 7*s.* 1*d.*
7. £1197. 9*s.* 3*d.* **8.** £1974. 12*s.* 0*d.* **9.** £6942. 2*s.* 10*d.*
10. £4701. 12*s.* 1*d*

CIV. Page 190.

1. £90. 14*s.*; £9. 6*s.*
2. £530. 18*s.* 10*d.*; £69. 1*s.* 2*d.*
3. £431. 18*s.* 4*d.*; £68. 1*s.* 8*d.*
4. £732. 6*s.* 5*d.*; £67. 13*s.* 7*d.*
5. £933. 10*s.* 7*d.*; £66. 9*s.* 5*d.*
6. £1244. 10*s.*; £105. 10*s.*
7. £516. 1*s.* 4*d.*; £46. 8*s.* 8*d.*
8. £344. 0*s.* 5*d.*; £27. 14*s.* 7*d.*
9. £514. 17*s.* 3*d.*; £32. 0*s.* 3*d.*
10. £755. 0*s.* 9*d.*; £106. 11*s.* 9*d.*

CV. Page 195.

	l.	c.		£	s.	d.		fl.	c.		£	s.	d.
1.	8442	0	**3.**	25	8	4	**25.**	10200	0	**27.**	472	13	4
2.	11601	20	**4.**	149	0	0	**26.**	2648	46	**28.**	350	0	0

	dr.	l.						kr.	öre.				
5.	9287	0	**7.**	410	0	0	**29.**	22712	50	**31.**	62	5	$8\frac{1}{2}$
6.	3769	70	**8.**	372	10	0	**30.**	65118	28	**32.**	232	10	8·9

	ps.							$.	c.				
9.	2160		**11.**	505	11	6	**33.**	760	42	**35.**	608	15	4
10.	2468		**12.**	840	8	9	**34.**	2352	$24\frac{1}{2}$	**36.**	1326	10	7

	milr.	reis.						r.	co.				
							37.	2312	50	**39.**	229	0	0
13.	1849	500	**15.**	428	1	$18\frac{8}{9}$	**38.**	4160	25	**40.**	645	0	0
14.	2333	45	**16.**	613	11	0							

								pias.	paras.				
	mk.	pf.					**41.**	71682	0	**42.**	36	10	7·1
17.	12883	70	**19.**	130	18	8	**43.**	70600	50	**44.**	55	4	0·07
18.	15330	36	**20.**	159	11	5							

								taels.	mace.				
	fl.	kr.					**45.**	3296	0	**46.**	541	9	$8\frac{1}{4}$
21.	3692	80	**23.**	658	0	0							
22.	8625	80	**24.**	724	0	0		yens.	sens.				
							47.	7964	0	**48.**	290	6	2

CVI. Page 204.

1. £2569. 3*s.* 4·8*l.*
2. £4174. 15*s.* 1·74*d.*
3. 4320. 19*s.* 9*d.*
4. £5637. 6*s.* 1·4*d.*
5. £4076. 1*s.* 9*d.*
6. £2329. 18*s.* 4*d.*
7. £8412. 7*s.* 5*d.*
8. £501. 17*s.* 4*d.*
9. £982. 16*s.* 3*d.*
10. £1236. 11*s.* 8*d.*
11. £491. 15*s.* 10*d.*
12. £465. 13*s.* 4*d.*
13. 15*s.* 6*d.*
14. 6*s.* $11\frac{1}{2}d.$
15. The latter $\frac{3}{4}$ per cent. better.
16. 25·50$\frac{1}{2}$ fr.
17. $35\frac{7085}{12800}d.$ per milreis : 6750·358 milreis.
18. The former.
19. 24983·75 fr.
20. £1011. 12*s.* 6·7*d.*
21. £500. 15*s.* 1·52*d.*
22. £1050.
23. £250.
24. $4\frac{16}{21}$.
25. $\frac{81}{1009}$ % premium.
26. 1 ; $\frac{100}{101}$.

CVII. Page 207.

1. 1512.
2. 6000.
3. Rs. 828$\frac{1}{3}$.
4. 1*s.* 6*d.*=Re. 1.
5. £1=24·97 fr.
6. 54·50d. per milreis.
7. £1=25·92 fr.
8. Direct=Rs. 6562·8 as.; Circuitous=Rs. 6400.
9. Buying Dutch Bills in Paris.
10. Direct, £1=24·6 fr.; Indirect, £1=26·24 fr.; ∴ Circuitous route is the more advantageous.

11. £12. 18*s.* $7\frac{13}{20}d.$ **12.** £1 19*s.* $4\frac{1}{2}d.$ **13.** Gains £2. 3*s.* 2·62*d.*
14. Gains £11. 5*s.* **15.** 10 %. **16.** 136 fr. 80 c. **17.** 25·58 fr.
18. Rs. $613\frac{41}{3}$. **19.** $37·40. **20.** 996·87 fr. **21.** 1079·4.
22. $9\frac{3}{8}d.$ per £.; $11\frac{233}{392}$ fr. in 300 fr. **23.** 25·23 fr. per £.
24. About $56\frac{1}{4}$. **25.** £1. 5*s.* $9\frac{3}{25}$.

EXAMINATION PAPERS.

1. £20. 2*s.* $7\frac{1}{3}d.$ **2.** 17578·546266. **3.** 12240. **4.** £9. 3*s.* $4\frac{5}{6}d.$
5. 12*s.* $6\frac{1}{2}d.$ + £10. 8*s.* 4*d.* + 5*s.* + £2. 6*s.* 9*d.* + 19*s.* 6*d.* + 1*s.* 8*d.* + £5. 5*s.* + £13 + £1 6*s.* 3*d.* + £1. 4*s.* $5\frac{1}{4}d.$ = £35. 9*s.* $5\frac{3}{4}d.$ **6.** £79. 13*s.* $3\frac{3}{4}d.$; $\frac{1}{2}d.$
7. £16. 17*s.* 6*d.* **8.** 2200. **9.** ·02665; 432337125.
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